be used directly to test the consistency of $\pi^{\pm}p$ data with forward dispersion relations and to constrain the parameters in theoretical models of high-energy scattering amplitudes.

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Possible non-Regge behavior of electroproduction structure functions*

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The large-ω behavior of deep-inelastic structure functions, e.g., $F_2(\omega, q^2)$, is studied in the framework of asymptotically free field theories. On the basis of certain uniformity assumptions we predict an unbounded growth with ω: slower than any power of ω but faster than any power of $\log\omega$.

The discovery that non-Abelian gauge theories are asymptotically free[1] has attracted a great deal of interest, especially in connection with the search for a field-theoretic explanation of Bjorken scaling. In fact, theories of this class do not quite scale, but they come close in a sense that we shall presently recall. Further development of the subject hinges on the observation of departures from scaling. Does scaling break down in the ways that are characteristic of asymptotically free theories? What is most sharply characteristic of these theories is the large-q^2 behavior of the moments of deep-inelastic structure functions. But it is also natural to consider the implications for the structure functions themselves. Discussion along these lines has been initiated in several recent publications, which deal especially with the threshold region, $\omega \geq 1$.[2] Here we want to focus on the behavior in the limit of large ω.[3]

For definiteness, let us start with the structure function $F_2(\omega, q^2)$ of deep-inelastic electron scattering, where q^2 is minus the invariant momentum transfer squared and $\omega = 2m\nu/q^2$ is the Bjorken scaling variable. The moments of the structure functions are defined by

$$F_2^n(q^2)=\int_1^\infty \frac{d\omega}{\omega^2}\,\omega^{-n}F_2(\omega,q^2)\,. \tag{1}$$

Exact scaling is the statement that $F_2(\omega,q^2)$ approaches a finite nonvanishing limit as $q^2\to\infty$ for fixed ω. Hence the moments, F_2^n, would approach finite limits as $q^2\to\infty$. In asymptotically free theories, deviations from scaling take the form of a logarithmic decrease of the moments:

$$F_2^n(q^2)\xrightarrow[q^2\to\infty]{} c_n[\lambda(q^2)]^{-a_n}\left[1+\alpha_n^{(1)}\left(\ln\frac{q^2}{\mu^2}\right)^{-1}+\alpha_n^{(2)}\left(\ln\frac{q^2}{\mu^2}\right)^{-2}+\cdots\right], \tag{2}$$

where

$$\lambda(q^2)=\frac{\ln(q^2/\mu^2)}{\ln(q_0^{\,2}/\mu^2)}\,. \tag{3}$$

The scale parameter μ and the coefficients c_n are unspecified. The parameter $q_0^{\,2}\gg\mu^2$ is introduced for later convenience as an arbitrary choice of reference momentum transfer. The exponents a_n in Eq. (3) are related to the anomalous dimensions of the dominant operators of spin $n+2$ in the Wilson expansion of a product of currents. They depend on the gauge group and the quark content of the theory and can be explicitly calculated from this information. There are three operators of a given spin that contribute to the leading term in Eq. (2), each with its own exponent a_n; in quark-gluon models with a global SU(3), there are two singlets and one octet. For each moment the term with the smallest exponent a_n will eventually dominate for large q^2.

Suppose that the structure function $F_2(\omega,q^2)$ were known empirically in its dependence on ω at some $q^2=q_0^{\,2}$ which is sufficiently large that the "subdominant" terms with coefficients $\alpha_n^{(i)}$ can be neglected. We would then know the $F_2^n(q_0^{\,2})$, and hence the coefficients c_n in Eq. (2). We could then determine $F_2^n(q^2)$ for larger q^2 and reconstruct the whole structure function $F_2(\omega,q^2)$. This procedure relies on the assumption that, for $q^2\gg\mu^2$, the subdominant terms are negligible for all n—a delicacy that we set aside till later. A convenient technique for effecting this reconstruction has been discussed by Gross.[2] In practice, a full analysis along these lines would require detailed starting data at some (large enough) $q_0^{\,2}$, and some rather complicated reconstruction mathematics. Here we want to see what kinds of qualitative things can be said in advance, without resort to the full machinery.

Since the integral of Eq. (1) converges for $n=0$, this equation serves to define F_2^n as a function of complex n, analytic for Re $n>0$.[4] If the continued function has no singularities to the right of the line Re $n=-n_0$ then it follows that $F_2(\omega,q^2)$ is bounded for large ω by $F_2<B\omega^{1-n_0+\epsilon}$ (ϵ arbitrarily small). The SLAC-MIT experiments[5] suggest that the proton and neutron structure functions approach constant limits as $\omega\to\infty$, although a slow growth (or falloff) cannot be excluded in the region of q^2 relevant for these experiments. Taking $q_0^{\,2}$ to be a representative momentum transfer in this region, we may then conjecture that c_n has no singularities to the right of Re$n=-1$.

The behavior of $F_2(\omega,q^2)$ for $\omega\to\infty$ is governed by the singularities of a_n as well as c_n. Recall that several operators contribute to the moments —two singlets and a nonsinglet. The key observation is now this: The rightmost singularity comes from one of the singlet terms, which has a simple pole in a_n at $n=-1$ with negative residue. For $n\to-1$ we have

$$a_n\to-\frac{a}{n+1}+b\,, \tag{4}$$

and therefore

$$F_2^n\to c_n\lambda^{a/(n+1)-b}\,. \tag{5}$$

This represents an essential singularity in F_2^n at $n=-1$. Except perhaps for a unique choice of the parameter $q_0^{\,2}$, we must expect that c_n also has this same singularity. We therefore write

$$c_n=M(n+1)K^{a/(n+1)}\,, \tag{6}$$

where K is a constant bigger than unity.[6] From the SLAC-MIT results we have inferred that the function $M(n+1)$ is regular for Re$n>-1$. With the constant K suitably chosen M is also supposed to be free of essential singularities at $n=-1$. The remaining properties of this function will not much matter for what follows: For large enough q^2, the behavior of F_2 as $\omega\to\infty$ will be governed chiefly by the essential singularities in Eqs. (5) and (6). To determine the large-ω behavior we approximate F_2^n using Eqs. (5) and (6), so that

$$F_2\xrightarrow[\omega\to\infty]{}\frac{\lambda^{-b}}{2\pi i}\int_{-i\infty}^{i\infty}dn\,M(n+1)\exp\left[(n+1)\ln\omega+\frac{z}{n+1}\right], \tag{7}$$

where $z=a\ln(K\lambda)$. The asymptotic behavior for large ω can be determined by the method of steepest descent. We find

$$F_2\xrightarrow[\omega\to\infty]{}\frac{1}{2\sqrt{\pi}}\lambda^{-b}M[(z/\ln\omega)^{1/2}](z/\ln^3\omega)^{1/4}e^{2(z\ln\omega)^{1/2}}\,. \tag{8}$$

For large ω we need to know the function M only in the region where its argument goes to zero. Thus if

$$M(n+1) \to A(n+1)^{\beta}$$

we have $M[(z/\ln\omega)^{1/2}] \to A(z/\ln\omega)^{\beta/2}$ in Eq. (8). The ω dependence in Eq. (8) is chiefly governed by the exponential factor. We are led to the prediction that F_2 must grow with ω, contrary to the usual expectations based on analogy with Regge behavior. The rate of growth increases with increasing q^2. It is always weaker than a power law in ω but more rapid than any power of $\log\omega$. For fixed q^2, this implies that F_2 grows faster than any power of $\log\nu$, a growth which is more rapid than is allowed by the Froissart theorem for purely hadronic cross sections. So far as we know, this is not a contradiction. The Froissart theorem makes essential use of unitarity, whereas we are considering an absorptive amplitude in lowest electromagnetic order.

Nevertheless the growth implied by Eq. (8) is undoubtedly surprising. However, it must perhaps be taken *cum grano salis*. We have assumed that analytic properties can be inferred from the leading terms of the perturbation expression of the moments. However, subdominant terms in $[\ln(q^2/\mu^2)]^{-1}$ are not necessarily negligible in determining the singularities in n of F_2^n. It is conceivable that these, along with the leading terms, are also singular at $n=-1$, and that the effects combine to produce a totally different singularity structure, e.g., a moving singularity which approaches $n=-1$ from the left as $q^2 \to \infty$. It is certainly not difficult to construct functions whose moments agree with Eq. (2) as $q^2 \to \infty$ but which have finite limits as $\omega \to \infty$ for any q^2; e.g., if we replace ω in Eq. (8) by $\omega q^2(\omega\mu^2+q^2)^{-1}$ the new function has no essential singularity at $n=-1$.

For the remainder of the discussion we return to the leading effects exclusively, ignoring the subdominant contributions. The analysis carried out for F_2 can be repeated now for the longitudinal structure function F_L. The moments of F_L differ from those of F_2 by a factor which is proportional to $[\ln(q^2/\mu^2)]^{-1}$:

$$\frac{F_L^n}{F_2^n} \underset{q^2\to\infty}{\longrightarrow} \text{const} \times (n+3)^{-1}\left(\frac{\ln q^2}{\mu^2}\right)^{-1}. \qquad (9)$$

The n-dependent factor is regular at $n=-1$, so for large q^2 and for $\omega \to \infty$, the two structure functions have the same ω behavior:

$$\frac{F_L(\omega,q^2)}{F_2(\omega,q^2)} \underset{\omega\to\infty}{\longrightarrow} \text{const} \times \left(\frac{\ln q^2}{\mu^2}\right)^{-1}. \qquad (10)$$

As we have noted several times, the leading behavior at large ω is governed by the SU(3) singlet operators, which contribute equally to the proton and neutron structure functions. These are each, separately, described by Eq. (8) in the large-ω limit—their ratio approaches unity in this limit. On the other hand, the difference $\Delta F_2 = F_2(\text{proton}) - F_2(\text{neutron})$ is of course governed by the nonsinglet operators.[7] The situation can again be represented as in Eq. (2), with new coefficients c_n' and a_n', where the primes denote nonsinglet. The important result here is that a_n' is regular at $n=-1$; its rightmost singularity is a simple pole at $n=-2$. If c_n' is similarly free of singularities to the right of Re $n=-2$, we would then expect that ΔF_2 should fall off as ω^{-1}, modified by an exponential factor of the sort appearing in Eq. (8). Standard Regge lore would suggest that ΔF_2 falls off roughly like $\omega^{-1/2}$. At present this would be attributed to a singularity in c_n' at $n \approx -\frac{3}{2}$, something for which we have no natural explanation here. If this pole is present, we can predict the q^2 dependence of ΔF_2. ΔF_2 is controlled by the value of the nonsinglet a_n' continued to $n \approx -\frac{3}{2}$. It turns out that a_n', which is positive for positive integer n, becomes negative for $n=-\frac{3}{2}$. Hence the coefficient of $\omega^{-1/2}$ grows with a power of $\log(q^2/\mu^2)$:

$$\Delta F_2(\omega,q^2) \to \text{constant} \times \omega^{-1/2}\left(\ln\frac{q^2}{\mu^2}\right)^c, \quad c>0. \qquad (11)$$

The parameters a and b in Eq. (8) and c in Eq. (11) are determined by the behavior of a_n and a_n'. If the strong gauge group is taken to be SU(3) with three quark triplets,[8] one finds

$$a=\tfrac{4}{3}, \quad b=\tfrac{101}{81}, \quad c \simeq 0.48. \qquad (12)$$

Moments of the weak structure functions $F_2^{(\nu,\bar\nu)}$ are determined by the same operators occurring in the discussion of electroproduction. Therefore, the leading q^2 dependence of a given moment is the same for analogous structure functions. Furthermore, each moment is proportional to the same hadronic matrix element of the dominant spin-$(n+2)$ operator. Since scattering off a given target includes singlet contributions, we predict that the large-ω behavior, also in its q^2 dependence, is identical for all such processes. For any target t, we find, as $\omega \to \infty$,

$$F_2^{\nu t} = F_2^{\bar\nu t} = 3F_2^{et}. \qquad (13)$$

A similar result is conventionally said to follow from the assumption of Pomeron dominance of the structure functions.[9]

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$$a=\frac{4C_2(G)}{11C_2(G)-4T(R)},$$

$$b=\frac{11C_2(G)+4T(R)-8C_2(R)T(R)/C_2(G)}{33C_2(G)-12T(R)},$$

$$c=3.2\,\frac{3C_2(R)}{11C_2(G)-4T(R)},$$

in the notation of Gross and Wilczek, Ref. 1. The numbers are not very sensitive to the quark content of the theory.

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π^+/π^- ratio in inclusive production and the triple-Regge formula

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Recently it has been reported in a photoproduction experiment that the ratio of π^+/π^- is high in the target fragmentation region, yielding a value of 10 near $x=-1$. The presence of this backward peak has been confirmed in πp and pp experiments. We explain this result by applying the triple-Regge formalism for π^+ and π^- production.

Single-particle inclusive distributions have been extensively studied with different beam and target particles. Recently, a study has been published on π^- inclusive production in a photoproduction experiment on deuterium.[1] From a comparison with the results of the SLAC-Berkeley-Tufts collaboration,[2] the ratio of π^+/π^- inclusive production was calculated as a function of the Feynman variable x (as described in Ref. 1). It is experimentally observed that this ratio is high in the target-fragmentation region, reaching a value of 10 for $-1.0<x<-0.8$; it drops off sharply at $x=-0.5$, reaching a value of unity in the pionization region ($x\sim 0$). The data available for π^+p (Ref. 3) and pp (Ref. 4) experiments seem to support the presence of this backward peak.

In this note we show that the observed high π^+/π^- ratio in the target-fragmentation region can be understood within the framework of the triple-Regge formalism. Consider the reactions

$$\gamma p \to \pi^{\pm} + \text{anything}.$$

For the fragmentation of the targets we can describe these reactions according to the triple-Regge diagrams shown in Figs. 1(a), 1(b), where P stands for Pomeron and the $\alpha(t)$ are the exchanged Regge trajectories. For π^+ production we can exchange either the neutron (N) or the Δ^0 trajectory. For π^- production only the Δ^{++} trajectory is allowed.