Squeezing more out of a laser

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A laser's intensity can be stabilized by negative feedback from a conventional photodetector. I propose extraction of sub-shot-noise light from the feedback loop at a beam splitter by illuminating the back side of the beam splitter with squeezed-state light.

Squeezed-state light has been generated in several laboratories.[1–4] Here I describe a technique that uses squeezed light together with negative feedback from a conventional photodetector to stabilize the intensity of a laser. The technique allows extraction of light that has sub-shot-noise photon statistics. Moreover, even if the unstabilized laser is shot-noise limited, the extracted light can have a better intensity signal-to-noise ratio; this signal-to-noise improvement requires both feedback and squeezing.

Machida and Yamamoto[5] used negative feedback from a conventional photodetector to stabilize a laser's intensity and thus to obtain sub-shot-noise photon statistics within the feedback loop. Out-of-loop light extracted at a beam splitter, however, displayed super-shot-noise statistics. This behavior can be understood both quantum mechanically[5–8] and semiclassically.[7–9] Quantum mechanically the culprit is the vacuum field that enters the beam splitter's second input port. Yamamoto *et al.*[6] proposed to extract sub-shot-noise light by replacing the conventional photodetector with a Kerr-effect quantum nondemolition photon counter[10] (see also Refs. 7 and 8). Here I suggest another alternative: Use feedback from a conventional photodetector, extract an out-of-loop beam at a beam splitter, but replace the culprit vacuum field with squeezed light generated by a nonlinear device—a "squeezer"—that is pumped by a portion of the laser light.

The technique is sketched in Fig. 1. Following Shapiro *et al.*,[8] I model the feedback by a variable attenuator (outside the laser), represented in Fig. 1 as a beam splitter with transmissivity $T(t)$. The laser light first traverses this variable beam splitter, after which it encounters a beam splitter with transmissivity $1 - \xi$, which removes a fraction ξ of the laser light to pump the squeezer. The remaining laser light (labeled a_1) is then combined with the squeezed light (labeled a_2) at an extraction beam splitter, which has transmissivity χ. One of the resulting outputs (labeled b_1) is directed onto a photodetector, which has quantum efficiency η_1 and produces a photocurrent $I_1(t)$. The feedback loop is closed by choosing $T(t) = T_0[1 - \zeta\delta I_1(t)]$, where T_0 is the fiducial transmissivity, ζ characterizes the strength of the feedback, and $\delta I_1(t)$ is the difference between $I_1(t)$ and a fixed reference level. [Shapiro *et al.*[8] assume feedback proportional to $I_1(t)$ without subtracting a reference level.] The second output (labeled b_2) yields the extracted light.

A heuristic analysis illustrates how the technique works when the feedback is perfect and $\eta_1 = 1$. Suppose that the a_1 light has a unity amplitude fluctuation in vacuum units, and suppose that the in-phase quadrature of the squeezed light has a fluctuation $\sqrt{R}$, where R measures the squeezing relative to vacuum in units of power. The photodetector sees an amplitude fluctuation $\Delta = \sqrt{\chi} \times 1 - \sqrt{1-\chi} \times \sqrt{R}$, which the feedback cancels by changing the amplitude of the a_1 light by $-\Delta/\sqrt{\chi}$. This leaves the a_1 light with an amplitude fluctuation $1 - \Delta/\sqrt{\chi} = \sqrt{(1-\chi)/\chi}\sqrt{R}$. The resulting amplitude fluctuation in the extracted light is $\sqrt{(1-\chi)} \times \sqrt{(1-\chi)/\chi}\sqrt{R} + \sqrt{\chi} \times \sqrt{R} = \sqrt{R/\chi}$; thus the extracted light has sub-shot-noise photon statistics if $R < \chi$. As far as the extracted light is concerned, feedback corrects the amplitude fluctuations arising from the a_1 light, but it anticorrects the amplitude fluctuations arising from the a_2 light. Squeezing reduces the in-phase a_2 fluctuations that feedback makes worse.

A rigorous analysis can be patterned after the recent work of Shapiro *et al.*[8] I ignore the propagation time τ around the feedback loop and refer all fields to a common point within the loop. Thus the analysis is valid only over a rf bandwidth $\lesssim \tau^{-1}$; more precisely, the

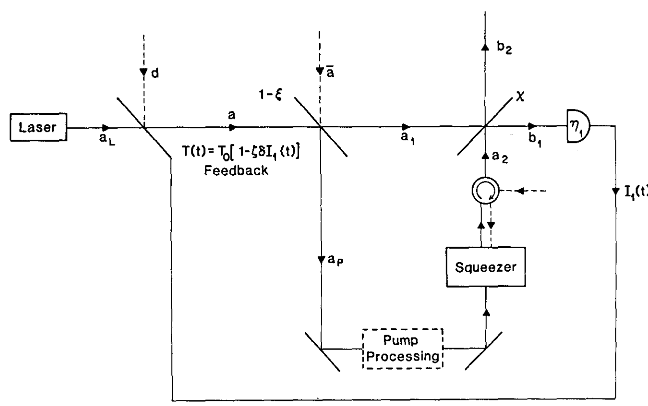

Fig. 1. Technique for stabilizing laser intensity and extracting sub-shot-noise light at a beam splitter.

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feedback loop can have high gain only over a rf bandwidth $\ll\tau^{-1}$.

Let Ω be the frequency of the laser. Since I am interested primarily in sub-shot-noise behavior, I assume that the laser light (labeled a_L) has shot-noise-limited amplitude fluctuations. The laser light has, in addition, phase fluctuations in excess of shot noise (e.g., phase-diffusion noise), but these excess phase fluctuations can be ignored so long as the squeezed light is referenced to the instantaneous phase of the laser. This condition is satisfied if the bandwidth over which the light is squeezed is much larger than the bandwidth of the laser's excess phase noise.

With these assumptions, the laser light can be described by a coherent state, and its positive-frequency photon-units electric-field operator can be written as

$$E_{a_L}{}^{(+)} = \int_+ \frac{d\omega}{2\pi}\, a_L(\omega) e^{-i\omega t} = \mathcal{P}_L{}^{1/2} e^{-i\Omega t} + \delta E_{a_L}{}^{(+)}. \quad (1)$$

Here $a_L(\omega)$ is a continuum annihilation operator, and the integral runs over a suitable range of positive frequencies. The quantity $\mathcal{P}_L$ is the laser power in photon units, and $\delta E_{a_L}{}^{(+)}$ is a vacuum field operator. (δ denotes the difference between an operator and its mean value.) The Hermitian quadrature phases of the laser light are defined by

$$E_{a_L}{}^{(+)} = (E_{a_L,1} + iE_{a_L,2})e^{-i\Omega t}. \quad (2)$$

In the quasi-monochromatic approximation, the field operators for the laser light obey free-field commutation relations

$$[E_{a_L,1}(t), E_{a_L,2}(t')] = \tfrac{1}{2}i[E_{a_L}{}^{(+)}(t), E_{a_L}{}^{(-)}(t')] = \tfrac{1}{2}i\delta(t-t'). \quad (3)$$

In Fig. 1 each beam of light is labeled by the symbol used for the corresponding continuum annihilation operators. The field operators for a particular beam are subscripted with the same symbol, and the quadrature phases are distinguished by a further subscript 1 or 2.

The fields drawn with dashed lines in Fig. 1 are vacuum inputs. The input vacuum fields $\delta E_{a_L}{}^{(+)}$, $E_d{}^{(+)}$, and $E_{\bar{a}}{}^{(+)}$ obey the free-field commutation relations (3); their quadrature phases are uncorrelated and have flat spectral densities equal to the vacuum level of $\tfrac{1}{2}$.

The light labeled a_p, after appropriate processing (frequency doubling for an optical parametric oscillator[3]), is used to pump the squeezer. I assume that the squeezer operates in a regime where the pump light can be treated classically; thus the fluctuations $\delta E_{a_p}{}^{(+)}$, which introduce feedback effects into the squeezing, can be neglected. The squeezer takes in a vacuum input through an isolator and produces squeezed light labeled a_2. With the assumption of a classical pump, the field operators associated with the squeezed light obey the free-field commutation relations (3). I assume that the squeezed light has no mean field, and I assume further that the quadrature phases $E_{a_2,1}$ and $E_{a_2,2}$ are uncorrelated and have spectral densities $S_{a_2,1}(\epsilon) = \tfrac{1}{2}R_{a_2,1}(\epsilon)$ and $S_{a_2,2}(\epsilon) = \tfrac{1}{2}R_{a_2,2}(\epsilon)$ as functions of the rf frequency ϵ. The spectra $R_{a_2,n}$ measure the squeezing in units of the vacuum level. Phases are adjusted so that, after the extraction beam splitter, the quadrature $E_{a_2,1}$ is in phase with the mean laser field. Squeezing the in-phase quadrature thus means $R_{a_2,1}(\epsilon) < 1$.

The analysis can now be sketched. The light leaving the variable beam splitter has $E_a{}^{(+)} = \sqrt{T(t)}E_{a_L}{}^{(+)} - \sqrt{1-T(t)}E_d{}^{(+)}$. Removal of the pump light leaves $E_{a_1}{}^{(+)} = \sqrt{1-\xi}E_a{}^{(+)} - \sqrt{\xi}E_{\bar{a}}{}^{(+)}$. Assuming high laser power and linearizing in the small fluctuations yields

$$E_{a_1}{}^{(+)} = [(1-\xi)T_0\mathcal{P}_L]^{1/2}e^{-i\Omega t} + \delta E_{a_1}{}^{(+)}, \quad (4a)$$

$$\delta E_{a_1}{}^{(+)} = \mathcal{E}^{(+)} - \tfrac{1}{2}\zeta[(1-\xi)T_0\mathcal{P}_L]^{1/2}\delta I_1(t)e^{-i\Omega t} \quad (4b)$$

(linearization sidesteps potential factoring-ordering problems), where

$$\mathcal{E}^{(+)} \equiv (1-\xi)^{1/2}[T_0{}^{1/2}\delta E_{a_L}{}^{(+)} - (1-T_0)^{1/2}E_d{}^{(+)}] - \xi^{1/2}E_{\bar{a}}{}^{(+)} \quad (5)$$

is the vacuum field whose quadrature $\mathcal{E}_1$ the feedback loop cancels. Notice that the feedback affects only the in-phase quadrature $E_{a_1,1}$. The extraction beam splitter produces the further transformation

$$E_{b_1}{}^{(+)} = \chi^{1/2}E_{a_1}{}^{(+)} - (1-\chi)^{1/2}E_{a_2}{}^{(+)}, \quad (6a)$$

$$E_{b_2}{}^{(+)} = (1-\chi)^{1/2}E_{a_1}{}^{(+)} + \chi^{1/2}E_{a_2}{}^{(+)}. \quad (6b)$$

Focus attention now on the photocurrent $I_1(t)$. Linearize again in the small fluctuations, and write $I_1(t)$ in units of the electronic charge. Then the mean (reference) photocurrent is $\langle I_1(t)\rangle = \eta_1\mathcal{P}_1$, where $\mathcal{P}_1 \equiv \chi(1-\xi)T_0\mathcal{P}_L$ is the photon intensity incident upon the photodetector. The deviation from the mean is

$$\delta I_1(t) = 2(\eta_1\mathcal{P}_1)^{1/2}[\eta_1{}^{1/2}\delta E_{b_1,1} + (1-\eta_1)^{1/2}E_{\mathcal{E}_1,1}], \quad (7)$$

where $E_{\mathcal{E}_1}{}^{(+)}$ is a vacuum field operator that accounts for fluctuations associated with subunity quantum efficiency. Using Eqs. (6a) and (4b), one can solve for

$$\delta I_1(t) = F(2(\eta_1\mathcal{P}_1)^{1/2}\{\eta_1{}^{1/2}[\chi^{1/2}\mathcal{E}_1 - (1-\chi)^{1/2}E_{a_2,1}] + (1-\eta_1)^{1/2}E_{\mathcal{E}_1,1}\}), \quad (8)$$

where $F \equiv (1+\zeta\eta_1\mathcal{P}_1)^{-1}$ characterizes the noise suppression resulting from the feedback. The spectrum of $I_1(t)$ is easily computed to be

$$S_{I_1}(\epsilon) = F^2(2\eta_1\mathcal{P}_1\{1 + \eta_1(1-\chi)[R_{a_2,1}(\epsilon) - 1]\}), \quad (9)$$

a result that agrees with Shapiro *et al.*[8] in the case of no squeezing ($R_{a_2,1} = 1$). The factor in bold parentheses is the spectrum in the absence of feedback ($F = 1$); as expected, feedback suppresses the in-loop photocurrent noise.

Turn now to the extracted light. Its mean field is $\langle E_{b_2}{}^{(+)}\rangle = \mathcal{P}_2{}^{1/2}e^{-i\Omega t}$, where $\mathcal{P}_2 \equiv (1-\chi)(1-\xi)T_0\mathcal{P}_L$ is its photon intensity. Using Eqs. (6b), (4b), and (8), one can show that the fluctuations in the quadrature phases of the extracted light are

$$\delta E_{b_2,1} = F(1-\chi)^{1/2}\mathscr{E}_1 + \frac{1-F(1-\chi)}{\chi^{1/2}} E_{a_2,1} - (1-F)\left(\frac{1-\chi}{\chi}\right)^{1/2}\left(\frac{1-\eta_1}{\eta_1}\right)^{1/2} E_{\mathscr{E}_1,1}, \qquad (10a)$$

$$\delta E_{b_2,2} = (1-\chi)^{1/2}\mathscr{E}_2 + \chi^{1/2}E_{a_2,2}. \qquad (10b)$$

Feedback affects only the in-phase quadrature $E_{b_2,1}$. One easily verifies that $E_{b_2,1}$ and $E_{b_2,2}$ satisfy the free-field commutation relations (3).

The quadrature phases of the extracted light are uncorrelated and have spectral densities $S_{b_2,1}(\epsilon) = \frac{1}{2}R_{b_2,1}(\epsilon)$ and $S_{b_2,2}(\epsilon) = \frac{1}{2}R_{b_2,2}(\epsilon)$. These spectra can be evaluated as

$$\begin{aligned} R_{b_2,1}(\epsilon) &= F^2(1-\chi) + \frac{[1-F(1-\chi)]^2}{\chi} R_{a_2,1}(\epsilon) + (1-F)^2 \frac{1-\chi}{\chi}\frac{1-\eta_1}{\eta_1} \\ &= 1 + (1-F)^2\frac{1-\chi}{\chi\eta_1} + \frac{[1-F(1-\chi)]^2}{\chi}[R_{a_2,1}(\epsilon)-1], \end{aligned} \qquad (11a)$$

$$R_{b_2,2}(\epsilon) = 1 - \chi + \chi R_{a_2,2}(\epsilon). \qquad (11b)$$

In the first form of $R_{b_2,1}$ each term has an obvious source in Eq. (10a). One readily checks that $S_{b_2,1}(\epsilon)S_{b_2,2}(\epsilon) \geqq \frac{1}{4}$, as required by the free-field commutator. Notice that, for perfect feedback ($F = 0$), $R_{b_2,1} \geqq R_{a_2,1}/\chi$ so that the extracted light is less squeezed than the a_2 light; if, in addition, $\eta_1 = 1$, then $R_{b_2,1} = R_{a_2,1}/\chi$, in agreement with the heuristic argument given previously.

Suppose now that the extracted light is detected by a photodetector with quantum efficiency η_2. The resulting photocurrent $I_2(t)$ has a mean value $\langle I_2(t)\rangle = \eta_2\mathscr{P}_2$ and a spectrum

$$S_{I_2}(\epsilon) = 2\eta_2\mathscr{P}_2[\eta_2 R_{b_2,1}(\epsilon) + 1 - \eta_2]. \qquad (12)$$

The factor $2\eta_2\mathscr{P}_2$ is the shot-noise limit. In the case of no squeezing ($R_{a_2,1} = R_{a_2,2} = 1$) Eqs. (11) and (12) agree with the results of Shapiro *et al.*[8]

Specialize henceforth to the case $\eta_1 = \eta_2 \equiv \eta$. For perfect feedback ($F = 0$) the photocurrent spectrum becomes

$$S_{I_2}(\epsilon) = 2\eta\mathscr{P}_2\left\{\frac{1+\eta[R_{a_2,1}(\epsilon)-1]}{\chi}\right\} \xrightarrow[\eta=1]{} 2\eta\mathscr{P}_2\frac{R_{a_2,1}(\epsilon)}{\chi}. \qquad (13)$$

To beat the shot-noise limit at rf frequency ϵ requires that the factor in braces be less than unity, i.e., $R_{a_2,1}(\epsilon) < 1 - (1-\chi)\eta^{-1}$; this condition can be satisfied only if $\eta + \chi > 1$.

The extracted light also has sub-shot-noise statistics without any feedback ($F = 1$) as long as there is some squeezing: $S_{I_2} = 2\eta\mathscr{P}_2[1 + \eta\chi(R_{a_2,1} - 1)] < 2\eta\mathscr{P}_2$ if $R_{a_2,1} < 1$. Without feedback, however, one cannot improve the laser's signal-to-noise ratio. Thus a better figure of merit is provided by a dimensionless function of rf frequency ϵ,

$$Q(\epsilon) \equiv \frac{S_{I_2}(\epsilon)/\langle I_2\rangle^2}{2\eta\mathscr{P}_L/\eta^2\mathscr{P}_L^{\,2}} = \frac{1}{(1-\chi)(1-\xi)T_0}\frac{S_{I_2}(\epsilon)}{2\eta\mathscr{P}_2}. \qquad (14)$$

The numerator of $Q(\epsilon)$ is the photocurrent noise-to-signal spectrum for the extracted light; the denominator would be the noise-to-signal spectrum if the same detector were illuminated by the shot-noise-limited laser. Reducing $Q(\epsilon)$ below unity is more difficult than beating the shot-noise limit: without feedback ($F = 1$), $Q(\epsilon) \geqq 1$; for perfect feedback ($F = 0$), $Q(\epsilon) < 1$ requires $R_{a_2,1}(\epsilon) < 1 - [1 - \chi(1-\chi)(1-\xi)T_0]\eta^{-1}$, a condition that can be satisfied only if $\eta + \chi(1-\chi)(1-\xi)T_0 > 1$.

Another way to describe the technique proposed here is that it transforms low-power squeezed light into high-power squeezed light. A similar technique can be used to stabilize the phase of a laser.

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References

1. R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).
2. R. M. Shelby, M. D. Levenson, S. H. Perlmutter, R. G. DeVoe, and D. F. Walls, Phys. Rev. Lett. **57**, 691 (1986).
3. L.-A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).
4. M. W. Maeda, P. Kumar, and J. H. Shapiro, Opt. Lett. **12**, 161 (1987).
5. S. Machida and Y. Yamamoto, Opt. Commun. **57**, 290 (1986).
6. Y. Yamamoto, N. Imoto, and S. Machida, Phys. Rev. A **33**, 3243 (1986).
7. H. A. Haus and Y. Yamamoto, Phys. Rev. A **34**, 270 (1986).
8. J. H. Shapiro, G. Saplakoglu, S.-T. Ho, P. Kumar, B. E. A. Saleh, and M. C. Teich, J. Opt. Soc. Am. B **4**, 1604 (1987).
9. J. H. Shapiro, M. C. Teich, B. E. A. Saleh, P. Kumar, and G. Saplakoglu, Phys. Rev. Lett. **56**, 1136 (1986).
10. N. Imoto, H. A. Haus, and Y. Yamamoto, Phys. Rev. A **32**, 2287 (1985).