Approximations in the quasiparticle random phase evaluation of double beta decay rates

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Within the quasiparticle random phase approximation, we compare two-neutrino double beta decay amplitudes calculated with zero-range and finite-range nucleon-nucleon interactions. We find that if, as is customary, interaction constants are fit to energies of giant Gamow-Teller states and to β^+ strengths, the two interactions lead to very similar two-neutrino double beta amplitudes. We discuss additional approximations related to the inclusion (or neglect) of the Hartree-Fock part of the pairing interaction, and to the treatment of differences between the initial and final nuclear ground states. The numerical values of the amplitudes depend more on the way these approximations are made than on the form of the nucleon-nucleon force.

Double beta ($\beta\beta$) decay is a rare transition between two nuclei of the same mass number involving a change of the nuclear charge by two units. The observation of neutrinoless (0ν) double beta decay would constitute proof that there exist massive Majorana neutrinos. But before the mass can be quantitatively determined from such an experiment, the nuclear amplitudes that govern the decay must be reliably calculated. Two-neutrino (2ν) $\beta\beta$ decay, the rate of which does not depend on the nature of the neutrino, is useful for testing our ability to understand the relevant nuclear structure.

The shell model is in principle the ideal way to evaluate the corresponding nuclear amplitudes. However, a complete treatment for heavier nuclei is not possible, and one has to rely on sophisticated but stringent approximations.[1]

Substantial activity has been recently devoted to the application of another method, the quasiparticle random phase approximation (QRPA) to double beta decay. While also involving dramatic approximations, the QRPA has several advantages: It automatically satisfies the model-independent Gamow-Teller sum rule

$$\sum_m \left|\left\langle m \left|\sum_k \sigma_k t_k^+\right| 0^+\right\rangle\right|^2 - \sum_n \left|\left\langle n \left|\sum_l \sigma_l t_l^-\right| 0^+\right\rangle\right|^2 = 3(N-Z) \quad (1)$$

and allows easy evaluation of β^- and β^+ strengths, the first and second terms in the sum rule (1). These strength functions can be measured; β^- strength has long been studied in (p,n) reactions at forward angles,[2] and β^+ strength has recently become accessible through (n,p) reactions at TRIUMF.[3] The QRPA correctly describes the main features of these strengths, in particular the energies of the giant Gamow-Teller resonances. (However, in the light s,d shell nuclei with small neutron excess QRPA overestimates[4] the β^+ strength by a factor of ≈ 2 compared to the shell model with an identical interaction.) Consistent inclusion in the QRPA of ground-state correlations, particularly those associated with the particle-particle (p-p) component of the neutron-proton interaction, leads to suppression of the 2ν decay rate.

Two issues should be considered in this context. The first involves the suitability of the "collective" QRPA framework for calculating suppressed quantities like double beta decay rates. We discuss this question in previous publications,[5-7] comparing the QRPA and exact results in simplified situations. We conclude that the two solutions are qualitatively similar, both leading to a vanishing of the amplitude $M_{GT}^{2\nu}$ (defined below) for similar values of the interaction parameters. The second issue is more quantitative. Several research groups have by now applied the QRPA to both 2ν and 0ν decay, and their results are not in exact agreement with one another. In this report, we address the sources of the discrepancies between the results of Refs. 5–7 and those of 8–11.

For simplicity we restrict the discussion to the 2ν decay mode governed by the amplitude

$$M_{GT}^{2\nu} = \sum_m \frac{\langle f|\sigma t^+|m\rangle\langle m|\sigma t^+|i\rangle}{E_m - (M_i + M_f)/2} \ . \quad (2)$$

All QRPA calculations predict a change of sign (a zero crossing) of $M_{GT}^{2\nu}$ when plotted vs g_{pp}, the strength of the particle-particle component of the neutron-proton interaction. But the various calculations yield different values of $M_{GT}^{2\nu}$ near $g_{pp}=0$, and different slopes of the $M_{GT}^{2\nu}$ versus g_{pp} curve near the crossing point. These translate into disagreements in predicted 2ν lifetimes. How significant are the discrepant features and what causes them?

One obvious difference between the various calculations is the form of the neutron-proton interaction. References 5–7 use a zero-range δ force, while Refs. 8–11 employ a finite-range, G-matrix based interaction. Neither interaction is parameter free; before comparing the two, we discuss how the strengths are fit. The approach first suggested in Ref. 5 (and now widely accepted) is to adjust the particle-hole (p-h) interaction constant so that the energy of the giant Gamow-Teller state is correctly reproduced, and then fit the particle-particle interaction constant *independently* to the experimental β^+ strength in selected nuclei. Roughly the same p-p interaction is used in the BCS gap equations; the interaction strengths required to obtain the experimental pairing gaps are within $\approx 10\%$ of those determined from the β^+ decay. Following this fitting procedure, one obtains coupling constants that vary between 0.8 and 1.3 of their bare values for the G-matrix based interactions.[8–11] The p-p couplings are usually smaller than their p-h counterparts. With the δ-force based interaction[5–7] this latter tendency is more marked—the effective p-p interaction constant is only $\approx 40\%$ of the p-h constant.

Given this renormalization procedure, we may inquire how significant the choice of nucleon-nucleon interaction really is. In Fig. 1(a) we compare the amplitudes $M_{GT}^{2\nu}$ calculated with the δ-force interaction and with the G-matrix based interaction.[12] (The value $g_{pp}=1$ for the δ-function curves has somewhat arbitrarily been chosen to correspond to the constant α'_1 from Ref. 6 $=-390$ MeV fm^3; we do not mean to imply that the "Pandya relation" is obeyed.) The results obtained with these very different forces are displayed again in Fig. 1(b), where we plot $M_{GT}^{2\nu}$ against the total β^+ strength in the final nucleus ^{76}Se. This figure is a more meaningful comparison than the one above because the adjustment of the interaction constants to reproduce the total β^+ strength is explicitly displayed. The two dashed (or two solid) curves lie quite close to each other. We conclude from this and similar results in other nuclei that the numerical discrepancies between the calculations of Refs. 5 and 6 and Refs. 8 and 10 are due only in small part to the form of the neutron-proton interaction.

There are, however, significant differences between the dashed and solid curves in Figs. 1(a) and 1(b). The dashed curves incorporate the approximations, discussed below, used in Refs. 8–11; the solid lines reflect the versions employed in Refs. 5–7. Part of the difference, most pronounced at unphysically small g_{pp} values, is related to an approximation used in evaluating (2). In the QRPA, the states $|m\rangle$ of the intermediate odd-odd nucleus are treated as "one-phonon" states built on the "phonon vacuum" representing, by necessity, the ground state of either the initial or the final even-even nucleus. The two sets of states $|m\rangle$ constructed this way are not identical. In Refs. 5 and 6, we replaced the state $|f\rangle$ by $|i\rangle$ and vice versa in (2) and averaged the two resulting amplitudes. In Ref. 8–10, by contrast, Eq. (2) was generalized and the term $|m\rangle\langle m|$ replaced by the expression

$$|\bar{m}\rangle\langle\bar{m}|n\rangle\langle n| = |\bar{m}\rangle\left[\sum_{p,n} X_{pn}(\bar{m})X_{pn}(n) - Y_{pn}(\bar{m})Y_{pn}(n)\right]\langle n| ,$$

where X, Y are the QRPA amplitudes and $\bar{m}$ refers to the "phonons" of the final nucleus while n refers to the initial. The summation in (2) must be extended to both $\bar{m}$ and n in this case.

Both approximations are expected to work well far away from closed shells, where the initial and final nuclei have similar structure. In the exactly solvable model treated in Ref. 6 the averaging procedure is more accurate; in other cases, e.g., the two-level model of Ref. 7, the overlap approximation is better. In real nuclei it is unclear which is preferable. Our numerical comparison suggests that while averaging tends to give smaller $M_{GT}^{2\nu}$ at $g_{pp}=0$, the two approximations lead to similar amplitudes near $g_{pp}=1$, the physically interesting region.

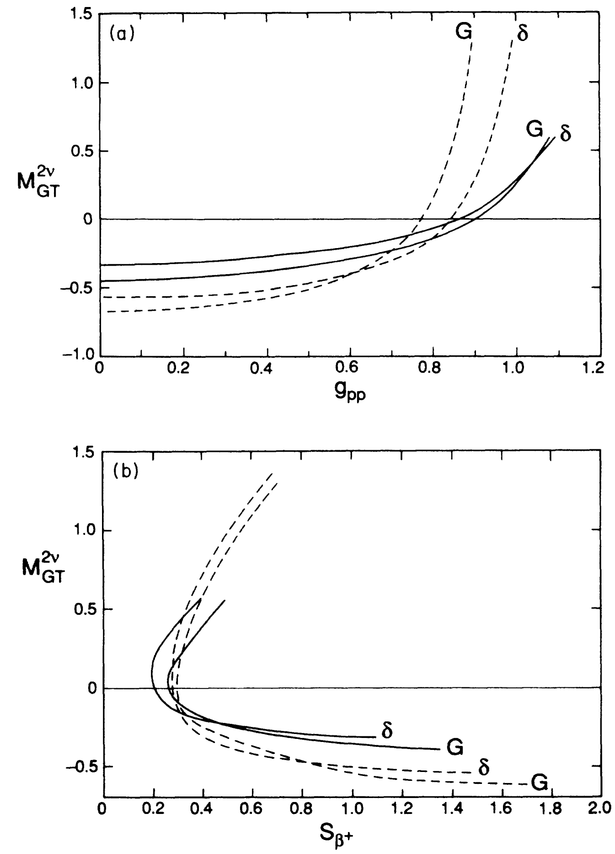

FIG. 1. Amplitudes $M_{GT}^{2\nu}$ for the decay ^{76}Ge→^{76}Se. Solid lines are obtained when, as in Refs. 5–7, the HF term (4) is included in the BCS equations and the amplitudes from initial and final nuclei are averaged. Dashed lines are obtained without the HF term and with the overlap expression (see text) inserted in (2). Within each pair, the two curves correspond to the finite-range G-matrix-based interaction (Ref. 12) and to the contact δ force (Ref. 6). In the latter case $g_{pp}=-\alpha'_1/390$ MeV fm^3, where α'_1 is the coupling constant for $S=1$, $T=0$ component of the δ force. (a) The amplitude is plotted against the coupling constant g_{pp} that renormalizes the interaction matrix elements for $T=0, J=1$. (b) Plotted versus the total β^+ strength.

The slopes of the different curves in Fig. 1(a) near the zero-intersect are important because they determine the sensitivity of the 2ν amplitudes to g_{pp} in the physical region. Here the differences between the two sets of lines can be traced to the treatment of pairing. All QRPA calculations[5–11] adjust the strength of the pairing force so that the experimental odd-even mass differences (pairing gaps) are correctly reproduced. The $T=1$ nucleon-nucleon interaction, however, enters the BCS equations in two ways, through the pairing gap

$$\Delta_a = -(2j_a+1)^{-1/2}\sum_b (2j_b+1)^{1/2} u_b v_b \times \langle (j_a^2)^{J=0} | V | (j_b^2)^{J=0}\rangle , \qquad (3)$$

and through the single-particle (Hartree-Fock) term

$$\mu_a = -(2j_a+1)^{-1/2}\sum_{J,b}(2J+1)v_b^2(1+\delta_{a,b}) \times \langle (j_a j_b)^J | V | (j_a j_b)^J \rangle \ . \quad (4)$$

In an *ab initio* treatment both (3) and (4) would be included. However, when the single-particle energies are based on an empirical potential (e.g., the Woods-Saxon well) the situation is less clear. It is often argued that the HF part has little effect, or that it is to some extent already present in the empirical single-particle potential, making its inclusion in the BCS equations redundant. Here, in any event, it has a noticeable effect. In our work[5,6] the HF term was included in the BCS equations, resulting in relatively gentle slopes in the $M_{GT}^{2\nu}$ vs g_{pp} curves. In Refs. 8–11 the HF term was not included and the resulting curves were steeper. It appears difficult to decide without further investigation which of the treatments is more accurate. Clearly, though, the precise form of these approximations affects the calculated 2ν rates far more than the choice of an effective nucleon-nucleon force.

We have noted the effect of approximations on the slope of $M_{GT}^{2\nu}$ near the crossing. In this connection, it is worth pointing out that the curves become steeper there because the QRPA equations approach an instability. It is generally agreed that the quality of the QRPA diminishes near such points. Although this problem is mitigated by renormalizing g_{pp} to β^+ strengths, an alternative that avoids the instability altogether is desirable. Work in this direction is currently in progress.

[1]W. C. Haxton and G. J. Stephenson, Jr., Prog. Part. Nucl. Phys. **12**, 409 (1984); W. C. Haxton, G. J. Stephenson, Jr., and D. Strottmann, Phys. Rev. D **25**, 2360 (1982).

[2]C. Gaarde *et al.*, Nucl. Phys. **A369**, 258 (1981).

[3]O. Hausser and M. C. Vetterli, Nucl. Phys. **A478**, 559c (1988).

[4]B. Lauritzen, Michigan State University Cyclotron Laboratory Report No. MSUCL-647, 1988.

[5]P. Vogel and M. R. Zirnbauer, Phys. Rev. Lett. **57**, 3148 (1986).

[6]J. Engel, P. Vogel, and M. R. Zirnbauer, Phys. Rev. C **37**, 731 (1988).

[7]J. Engel, P. Vogel, O. Civitarese, and M. R. Zirnbauer, Phys. Lett. B **208**, 187 (1988).

[8]O. Civitarese, A. Faessler, and T. Tomoda, Phys. Lett. B **194**, 11 (1987).

[9]T. Tomoda and A. Faessler, Phys. Lett. B **199**, 475 (1987).

[10]K. Muto and H. V. Klapdor, Phys. Lett. B **201**, 420 (1988).

[11]J. Suhonen *et al.*, University of Tubingen report, 1988.

[12]N. Anantaraman, H. Toki, and G. F. Bertsch, Nucl. Phys. **A398**, 269 (1983).