Baryon-antibaryon annihilation in the Skyrme model

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The dynamics of Skyrmion–anti-Skyrmion annihilations in 3+1 dimensions is examined by the numerical integration of the classical Hamilton equations of motion. The baryon number is found to disappear extremely rapidly, close to the causal limit, while the energy distribution still remains concentrated in the annihilation region. The emission of pion waves emitted by the annihilation process is investigated.

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Within the class of nucleon models inspired by quantum chromodynamics (QCD), the Skyrme model [1] has a unique advantage in describing $N\bar{N}$ annihilation: this effective theory provides a self-contained dynamics that encompasses nonlinear processes such as meson production, baryon excitation, and annihilation. The Skyrme model requires no additional dynamical assumption, such as the *ad hoc* dynamical behavior of the color-confinement wall that must be assumed in bag models. Our investigation here, therefore, involves no approximation other than numerical discretization. In particular, we have no need for the product ansatz commonly used in *SS* calculations, which might be especially unsatisfactory for the annihilation process. An exploratory calculation was previously carried out [2], but the present work is more comprehensive and includes new findings.

As in our previous work on Skyrmion-Skyrmion (*SS*) scattering in 3+1 dimensions [3], we calculate the time evolution of the Skyrmion–anti-Skyrmion ($S\bar{S}$) system by numerically integrating the classical equations of motion in a hybrid Hamiltonian-Lagrangian scheme. The standard Skyrme Lagrangian in Cartesian coordinates is given by

$$\mathcal{L}=-\tfrac{1}{8}F_\pi^2(\partial_\mu\Phi_k)^2-\frac{1}{4e^2}(\partial_\mu\Phi_k)^2(\partial_\nu\Phi_l)^2 +\frac{1}{4e^2}(\partial_\mu\Phi_k\partial_\mu\Phi_l)^2-\tfrac{1}{8}m_\pi^2F_\pi^2\Phi_k^2 , \tag{1}$$

where the chiral fields, $\Phi_k (k=0,1,2,3)$, form a SU(2) matrix $U=\Phi_0+i\boldsymbol{\tau}\cdot\boldsymbol{\Phi}$ with the boundary condition $U=1$ at infinity. The results in this paper were obtained using the parameter values [4] $F_\pi=129$ MeV, $e=5.45$, and the pion mass $m_\pi=138$ MeV.

The initial $S\bar{S}$ field configurations are constructed by applying Lorentz boosts to well-separated static Skyrmion and anti-Skyrmion solutions. The Skyrmion is taken to be the standard spherical "defensive" hedgehog. The anti-Skyrmion is obtained by reversing the direction of the pion field of the Skyrmion at each point, followed by a rotation of the pion field through 180° about the direction of motion. In this way, the anti-Skyrmion is properly related to the Skyrmion via charge conjugation of the chiral fields. The symmetries of the system allow us to restrict the computation to one quadrant of the total space; boundary conditions at the interface introduce an image Skyrmion that we need not treat explicitly.

Our uniform Cartesian spatial lattice for the computational quadrant is a grid with $21\times61\times41$ mesh points, having spacing $\Delta x=0.084$ fm. The resulting physical dimensions of the complete collision plane are 6.72 fm $\times 5.04$ fm, and the dimension of the total box perpendicular to the collision plane is 3.36 fm. Note that the width (5.04 fm) of the collision plane is 50% greater than the height (3.36 fm) of the box so that processes with relatively large impact parameters can be studied reliably. Up to 2250 time steps are needed to describe one complete scattering or annihilation event, with Δt ranging from 0.006 to 0.0015 fm/c, depending on the projectile velocity. Typical runs of our vectorized code on the NMFECC Cray-2 or on the SDSC Cray X-MP/48 and Cray Y-MP8/864 took on the order of 60 min of CPU time.

A measure of the numerical stability of the finite difference equations is the Courant condition $c\Delta t/\Delta x \ll 1$; values of this parameter in our calculation, which adopt the staggered leapfrog method, range from 0.067 to 0.017 [5]. In our previous work on *SS* scattering [3], no artificial viscosity terms were needed to stabilize the code. Unfortunately, however, in the course of the $S\bar{S}$ annihilation it becomes necessary to introduce dissipative terms to control unphysical fluctuations of the fields. We include in the discretized differential equations viscosity terms of the form

$$\alpha\frac{\Delta x^4}{6}(\partial_x^2+\partial_y^2+\partial_z^2)^2\Phi_k , \tag{2a}$$

$$\beta\frac{\Delta x^4}{6}(\partial_x^2+\partial_y^2+\partial_z^2)^2\Pi_k , \tag{2b}$$

where Π_k are the canonical momenta, defined as $\delta\mathcal{L}/\delta(\partial_t\Phi_k)$. These terms damp the unphysical short-wavelength fluctuations (on the order of the lattice spac-

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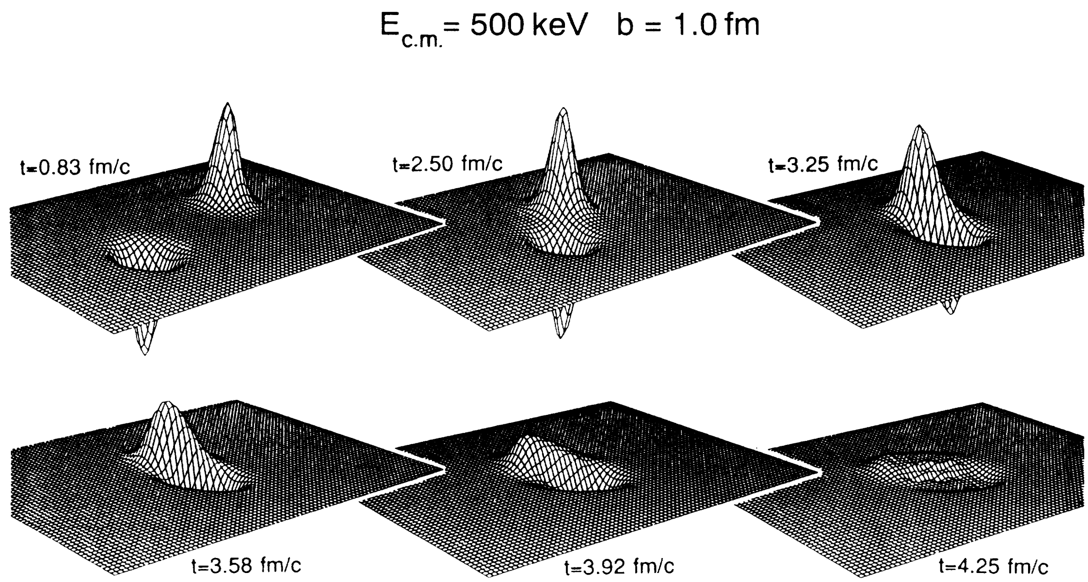

FIG. 1. Time evolution of the baryon density distribution in the course of the annihilation process for $E_{\rm c.m.}=500$ MeV and $b=1.0$ fm. The distribution shown is obtained by integrating the baryon-number density in the direction perpendicular to the collision plane. The viscosity terms are switched on at $t=3.75$ fm.

ing), while leaving the long-wavelength components basically unaffected. The coefficients α and β range from 0.1 to 1.0 in our calculations.

The initial energy and baryon number of the Skyrmion and anti-Skyrmion on our three-dimensional lattice are numerically within 4% of their exact values. The total energy is conserved to within 2% until the viscosity terms are switched on in the course of the annihilation process. Energy dissipation becomes appreciable only in the final stage of the annihilation when short-wavelength fluctuations become significant. Energy losses of typically 15% are observed. We expect baryon-number dissipation due to the viscosity terms to be small, based on a numerical test on a freely translating Skyrmion that showed the baryon-number dissipation to be less than 0.25% in 250 time steps. The total baryon number of the system is maintained precisely zero by symmetry.

We have calculated $S\bar{S}$ processes for center-of-mass kinetic energies $E_{\rm c.m.}$ of 100, 250, and 500 MeV, with impact parameters b of 0, 0.56, 1.12, and 1.68 fm. In Figs. 1 and 2, we illustrate results for $E_{\rm c.m.}=500$ MeV and $b=1.0$ fm.

In Fig. 1 the time evolution of the baryon-number density (integrated over the direction perpendicular to the collision plane) is shown. The solitons propagate virtually unchanged with velocity $v=0.63c$ for a time $t\approx 3$ fm/c, until the tails of the baryon-number-density distributions begin to overlap. At that point, the annihilation of the baryon number sets in and proceeds rapidly until the baryon-antibaryon system has "dissolved." The annihilation time is ≈ 1 fm/c, which is roughly the causal limit (i.e., the time that it takes for light signals to traverse a soliton). The time evolution of the energy density distribution for this event is illustrated in Fig. 2. A

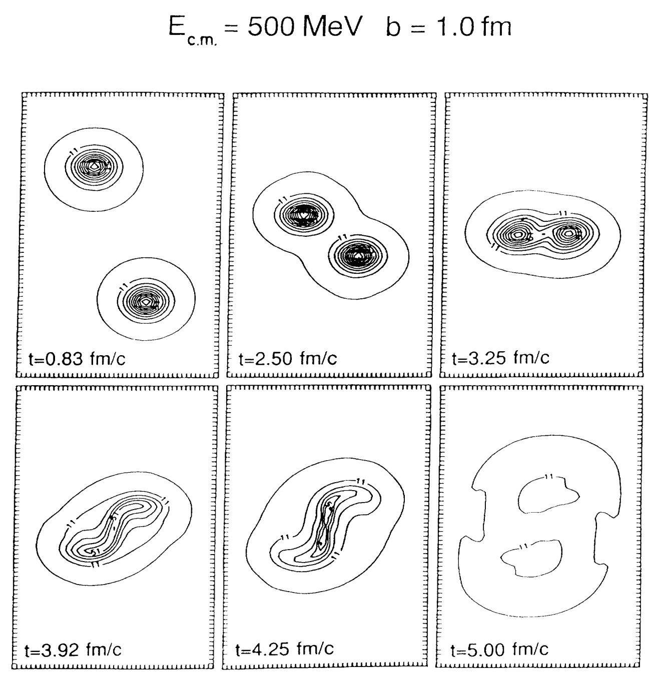

FIG. 2. Time evolution of the energy density distribution taken from the same run as for Fig. 1. The energy density distribution is integrated over the direction perpendicular to the collision plane. The contours are equally spaced, and the value of the density on the first contour is 10% of this contour spacing. The area shown in each frame corresponds to about half of the total computational area. Note that the energy density distribution is shown here for the time span longer than the baryon density distribution in Fig. 1 because the former disperses more slowly.

comparison of Figs. 1 and 2 shows that the time scale for the disappearance of the baryon number is much shorter than that for the dispersion of the energy density.

The total baryon number of the $S\bar{S}$ system is, of course, exactly zero at all times. In order to monitor the annihilation process, we therefore define a baryon number $B(t)$ equal to one-half of the integral, over all space, of the absolute value of the baryon-number density. The time dependence of $B(t)$ is shown in Fig. 3 for several runs in two different energy groups with various impact parameters. The plot reveals that the baryon-number annihilation time τ_a depends on the energy and impact parameter only slightly, except for the case of the largest energy and impact-parameter case in which we successfully observed the annihilation. τ_a is approximately $2.5/eF_\pi$, or also roughly $1/2m_\pi$, and is doubled in the largest energy and impact-parameter case. Note that our numerical results are changed little by removing the pion-mass term in the Lagrangian. The rapid annihilation in the $S\bar{S}$ system is thus a dynamical consequence of the Skyrme model, independent of the pion-mass scale in the theory.

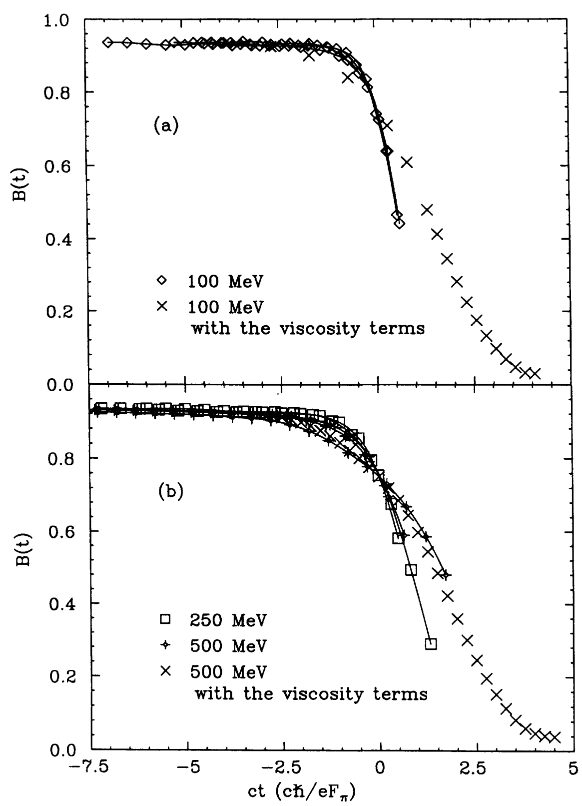

FIG. 3. Time dependence of the baryon number (half of the absolute value of the baryon-number density integrated over the entire space). The collision time is defined to be zero, in each case, when the baryon number has the value 0.75. ×'s are the baryon numbers calculated with the viscosity terms. (a) $E_{\text{c.m.}}=100$ MeV, with $b=0$, 0.56, 1.12, and 1.68 fm without the viscosity terms and with $b=1.5$ fm with the viscosity terms. (b) $E_{\text{c.m.}}=250$ MeV with $b=0.56$ and 1.12 fm, and $E_{\text{c.m.}}=500$ MeV with $b=0$, 0.56, and 1.12 fm without the viscosity terms; and $E_{\text{c.m.}}=500$ MeV with $b=1.0$ fm with the viscosity terms.

Since $m_\pi=O(1)$ in the large-N_c limit, τ_a is thus $=O(1)$. This large-N_c order of τ_a may be argued as anticipated, since the massless Skyrmion Lagrangian can be expressed in terms of the dimensionless four-vector coordinate with a factor of $eF_\pi=O(1)$ and the nucleon size is $=O(1)$. Such a large-N_c argument is, however, too qualitative to reveal important underlying dynamics. We emphasize that the time scale of the accompanying energy dispersion is much longer than τ_a, being larger than the time limit of the present computation. The difference in these time scales is beyond what one would expect based on the large-N_c argument. We believe that our finding of $\tau_a \approx 1/2m_\pi$ is a nontrivial consequence of the annihilation dynamics.

The value of τ_a also corresponds to a dimensional estimate of the pion formation time in the bag-model description through pairing of a quark and an antiquark. This suggests that the common neglect of dynamical participation of the confinement wall in the bag description [6] might be justifiable for length scales of $\gtrsim 1/m_\pi$ (i.e., the nucleon size).

In Fig. 3 we also show effects of the viscosity terms. The viscosity terms slow down the annihilation of the baryon number. The value of τ_a is now nearly doubled, though its energy dependence appears to remain slight, similar to the energy dependence without the viscosity terms.

Another dynamical feature present in our calculations is the emission of pion waves as the annihilation proceeds. We have Fourier analyzed the pion waves emitted by the annihilation process. Asymptotically, these pion waves diminish to nonsolitonic perturbations that obey the linearized wave equation obtained from the Skyrme Lagrangian, Eq. (1). Unfortunately, the finite size of our spatial lattice and the energy losses due to the viscosity terms do not allow us to study the actual asymptotic state. Instead, we analyze the momentum structure of the pion fields at the end of the numerical time evolution, when $B(t)$ falls by 97%.

Figure 4 illustrates the number of the pions emitted per unit energy:

$$\frac{dN_\pi}{dE_\pi}=\frac{k}{(2\pi)^2}\left|\int d^3x\,[E_\pi(k)\mathbf{\Phi}(\mathbf{x})\exp(-i\mathbf{k}\cdot\mathbf{x}) + i\dot{\mathbf{\Phi}}(\mathbf{x})\exp(+i\mathbf{k}\cdot\mathbf{x})]\right|^2, \quad (3)$$

where $E_\pi(k)=(k^2+m_\pi^2)^{1/2}$ and k is the pion momentum. It shows that dN_π/dE_π is peaked at a pion total c.m. energy ≈ 500 MeV. Since the total energy of the system is about 2.2 GeV, we estimate the total number of the emitted pions to be about 4. Note that the proton-antiproton annihilation is known to produce mostly the pions with the multiplicity of a little more than 5 [7].

The integration of dN_π/dE_π in Fig. 4 yields the total number of the emitted pions to be only about 1.6. We should, however, double this number, and our result is roughly consistent. We double it by taking account of the energy loss due to the viscosity terms and of the contribution from the nonlinear parts of the Hamiltonian,

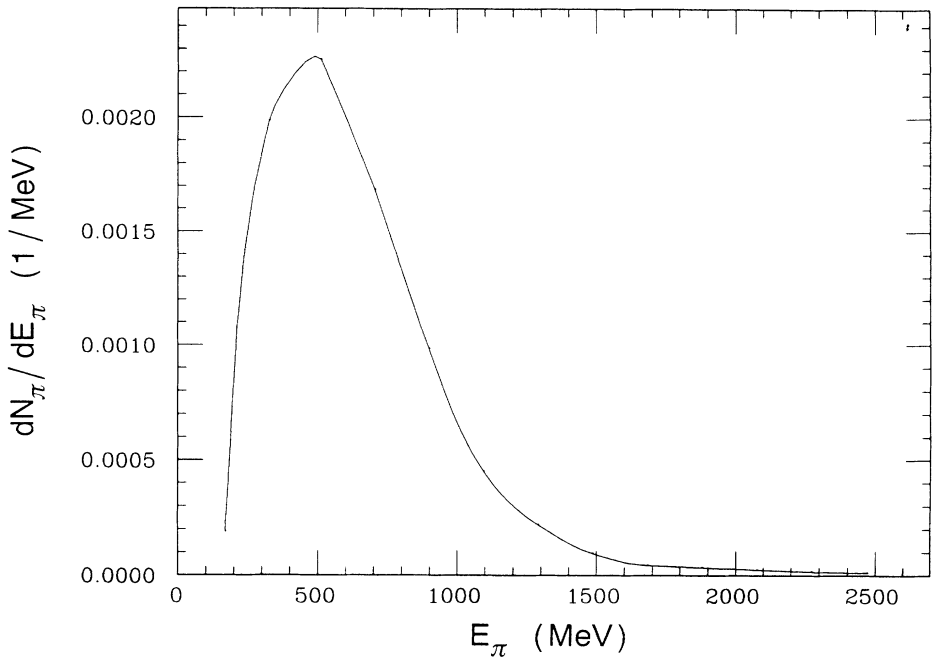

FIG. 4. The number of the pions emitted per unit energy as a function of the total pion energy E_π. The curve is smoothed by averaging over neighboring momentum lattice points.

each of which we find to be about 25% of the initial energy. The finite nonlinear contribution corresponds to the fact that some pions have not yet completely dispersed from the interaction region. As we proceed with the computation, the viscous energy loss increases and the nonlinear contribution decreases; we consider the present termination of the computation to be optimal. Note that when we artificially analyze the initial (noninteracting) Skyrmions in the same way, we find that the nonlinear contribution amounts to about $\frac{2}{3}$, so that the above 25% nonlinear contribution at the end of our computation in fact shows a significant dispersion of the pions.

As noted above, the massless Skyrmion Lagrangian can be expressed in terms of the dimensionless four-vector coordinate $eF_\pi x_\mu$. The Lagrangian then becomes F_π/e times a dimensionless expression. As a consequence, the values of the dimensionful quantities in our calculation scale as appropriate combinations of e and F_π. For example, lengths and times (such as the annihilation time) scale as eF_π, while energies (such as $1/[dN_\pi/dE_\pi]$) scale as F_π/e. Dimensionless quantities remain, however, unchanged by a different choice of the parameter values. Our major finding of the rapid baryon-number annihilation, $\tau_a \approx 2.5/eF_\pi$, and of the number of the emitted pions is thus independent of the choice of the parameter values.

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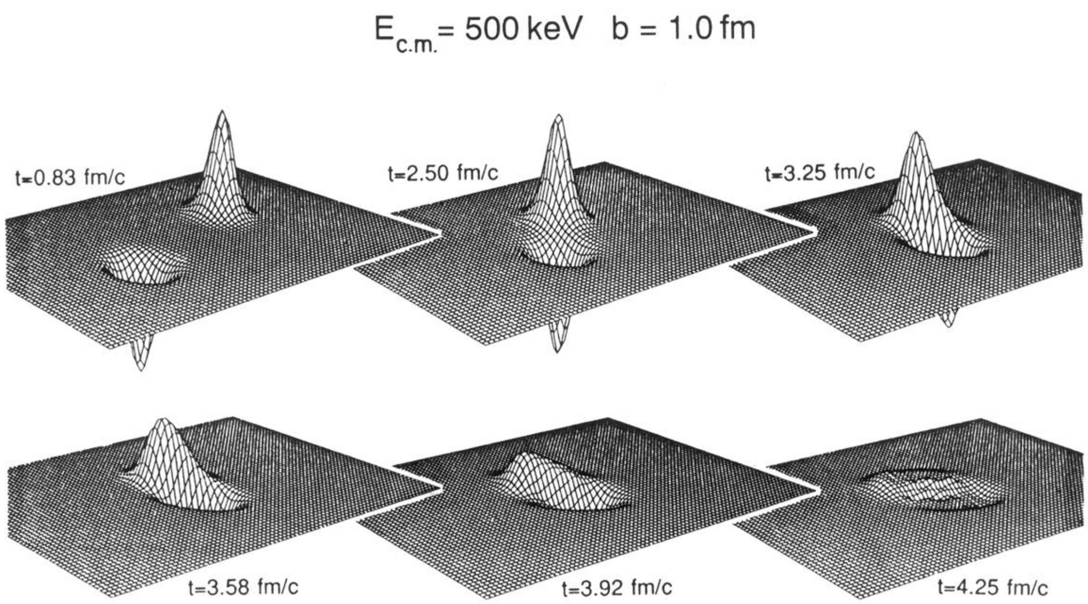

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