μ -p SCATTERING AND THE GLUE FRACTION OF THE PROTON *

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The QCD version of the energy-momentum sum rule is tested against SLAC electroproduction and FNAL muon-proton scattering data. While providing only a single number for each Q^2 , the energy momentum sum rule is significant because the scaling violations due to gluons can be well determined. In particular, gluons carry $30 \pm 8\%$ of the proton's energy-momentum at $Q^2 = 3.5 \text{ GeV}^2$ (which means 22% at 1 GeV^2). The largest single corrections of previous estimates of the glue fraction is the inclusion of gluon bremsstrahlung effects. Agreement with experiment is within the uncertainties in the μ -p data. However, the sum rule does not give a very sensitive test of the value of the quark-gluon coupling constant. The prediction for charm production agrees with the observed rates for dimuon and trimuon production in inelastic μ scattering.

1. Introduction

The inclusive μ -p cross sections reported recently [1] are exciting from the standpoint of QCD. They reveal the qualitative scaling violations as predicted on the basis of colored-quark-gluon-gauge theory [2]. With increasing Q^2 , νW_2 rises at low x and falls at higher x with a crossover at $x \sim 0.15$. (See fig. 1.) Furthermore, this behavior cannot be reproduced by an astute choice of scaling variable [†]. Encouraged by this qualitative success, we must press on to ask: Does the detailed shape in x of the scaling violations agree with QCD? And does their magnitude as a function of Q^2 agree with the expectation based on the quark-gluon coupling constant, determined from other considerations?

These quantitative questions are stymied by one fact: the presence of gluons. We believe there exist electrically neutral quanta in the proton, and in QCD these are

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[†] It is often said that scaling at a fixed point is also compatible with this qualitative behavior. However, without the ability to compute any of the fixed point parameters, we must admit that scaling at a fixed point is compatible with virtually any behavior.

Fig. 1. Muon-proton inclusive cross sections *versus* x , binned in Q^2 , presented in ref. [1], except for $15 \leq Q^2 \leq 30$ which is preliminary. The triangles are selected SLAC points [4]. The lines are hand-drawn fits, for the purpose of x integration.

gluons. Gluons enter the scaling violations as follows: There is gluon “bremsstrahlung” off the struck quark. This is of order $\alpha_s(Q^2)$, the QCD “fine structure” constant, and is proportional to the quark distribution in the proton. There are also scaling violations from gluons making virtual transitions to quark pairs. This is the same order in α_s and is proportional to the gluon distribution. However the gluon distribution cannot be measured directly by lepton scattering (or by any other means). So there is an unknown component in the scaling violations.

In a recent detailed analysis [3] of SLAC electroproduction data [4] the initial gluon distribution was neglected for $x \gtrsim \frac{1}{3}$. With this assumption the predictions were unambiguous, and furthermore they agreed with the data. The justification for the claim that there are no “hard” gluons in the proton is that it follows from a picture of the proton being mostly three quarks confined by an indefinite number of gluons. It is a corollary of the apparent absence of hard anti-quarks; it is also consistent with detailed (but not too relevant to this issue) perturbative calculations [5].

But the gluons must be somewhere because they are known to carry a sizeable fraction of the proton’s energy momentum. In the present analysis I extend the no-gluon assumption to $x \geq \frac{1}{4}$. The above arguments remain valid; it is likely still a good approximation for $x \geq \frac{1}{5}$, but these are just guesses. The region of present interest is $0.01 \lesssim x \lesssim 0.25$ which is what I will hence forth refer to as “small x ”*. For small x , one cannot ignore the primary gluon distribution in the study of scaling violations.

* The limiting behavior as $x \rightarrow 0$ is a Regge problem, which cannot be addressed by the present methods [6].

This paper is devoted to a modest piece of phenomenology which uses the one feature of the gluon distribution that can be deduced immediately, its total energy momentum. The energy momentum carried by the quarks can be measured by inclusive lepton scattering. The total energy momentum of the proton can be measured by a mass spectrometer. The energy momentum carried by gluons is the difference. I use the fundamental tool of subtraction again, in a second way: the energy-momentum sum rule [7] predicts an integral over all x . For $x \geq \frac{1}{4}$ we have relatively precise electroproduction data [4] and a successful QCD analysis [3]. Subtracting the consequences of the $x \geq \frac{1}{4}$ analysis from the energy-momentum sum rule, we are left with predictions for scaling violations in integrals over $0 \leq x \leq \frac{1}{4}$.

The result of integrating over "small x " is rather flat in Q^2 . From the standpoint of QCD, this is a cancellation between several rapidly varying factors. (See fig. 2, which will be explained in detail only later.) QCD survives this confrontation with experiment, in which the necessary theoretical assumptions are not only plausible but have each received some phenomenological support in other analyses. But the most interesting lesson of this exercise is not that QCD is still a viable theory, but rather that the proton is somewhat simpler than we had thought. The zeroth order parton analysis gave a glue fraction of $49 \pm 8\%$ [8]. Taking into account a variety of kinematic and dynamical effects (most significantly, gluon bremsstrahlung), I arrive at $30 \pm 8\%$ at $Q^2 = 3.5 \text{ GeV}^2$ *. At $Q^2 = 1 \text{ GeV}^2$, still a short distance relative to the proton diameter, the glue fraction is only $22 \pm 9\%$. So, looked at crudely, the proton really is mostly three quarks **, with a small fraction of glue (order α_s) — perhaps someday computable in some perturbative scheme. (QCD predicts that the glue fraction rises as $Q^2 \rightarrow \infty$ to a value determined by group theory, independent of the particular target state. For four quark flavors this number is 57%, but the approach is measured on a log log scale [2,5], and quantum gravity will be important long before the asymptotic value is approached.)

2. Game plan

Before going into numerical details, I would like to list the important elements of the analysis and outline the various assumptions and procedures:

I use the language of the operator product expansion [11–13] because it is precise. For example, it offers an unambiguous prescription for separating primary distributions and interaction corrections.

The ξ -scaling analysis [5,14–16], tested in electroproduction [3], is of numerical importance here, too, even though ξ differs from x by only order $x^2 m^2 / Q^2$. Its direct effect on the μ -p data is small but non-negligible. In particular, for the lowest three points in fig. 2, the raw data (crosses) moves to the dots when ξ -massaged; the

* See note added in proof regarding this estimate.

** Evidence for the number three comes not only from the absence of prominent anti-quarks but from the shape of the νW_2 itself [9,10].

Fig. 2. Various factors in the energy-momentum sum rule versus Q^2 . The shaded regions reflect the uncertainty due to relative quark abundances, having normalized to $Q_0^2 = 3.5 \text{ GeV}^2$. The crosses are integrals from 0 to $\frac{1}{2}$ of the raw νW_2 from fig. 1. The dots are corrected for $R \neq 0.18$ and include a ξ scaling factor. The error bars assume the errors in fig. 1 are purely statistical.

highest Q^2 point moves down when the data is corrected for σ_L/σ_T . An even larger ξ -scaling effect occurs in the energy-momentum sum rule, which samples all x . The factor $Z(Q^2)$ in fig. 2 reflects such an effect. While small in magnitude, $Z(Q^2)$ has a non-negligible slope for $Q^2 \lesssim 10 \text{ GeV}^2$.

Fig. 3a.

Fig. 3b.

Fig. 3c.

Fig. 3. R versus Q^2 , theory and experiment, reproduced from ref. [3]. The data is from ref. [4].

Fig. 4. $\bar{R}$, averaged over small x , versus Q^2 . The data is from ref. [4].

As in the electroproduction analysis [3], I will ignore effects of operators of twist [17] greater than two. Based on that analysis, I expect that all higher twist and any other non-calculable effects are of order $0.12 \text{ GeV}^2/Q^2$.

Since different quarks have different electrical charges, one must know the relative quark abundances before determining the glue fraction. In the language of the operator product expansion, there are three operators of twist-2 and spin-2: $\bar{\psi}\gamma D\psi$, $\bar{\psi}\lambda\gamma D\psi$ and FF , where λ is the quark flavor matrix and F is the gluon field strength. We must use neutrino and electroproduction data to estimate the ratio of the quark singlet and non-singlet operators. For numerical work, I choose a nominal plausible value [8,18]. To estimate the consequent uncertainties, I investigate the following extreme possibilities: I let the relevant numbers, i.e. s-quarks, anti-quarks, and the u-d difference, run from zero to twice their nominal values. I then report as uncertainty, e.g. the $\pm 8\%$ in the glue fraction, the largest variation in the final result obtainable by varying the inputs over the described range. This seems to be a safe overestimate.

The calculations presented are based on four quarks, where three are treated as massless $\star$ and the fourth has a mass of 1.5 GeV . While an exact treatment of the charmed quark mass is possible [5], for numerical work it suffices to make the following approximations: where the charmed quark mass, m_c enters the final result in a direct linear fashion, I use an interpolating function of the form $(1 + am_c^2/Q^2)^{-1}$, where a is chosen to agree with the exact result to a couple percent over the entire

$\star$ Their masses are inextricably connected to the higher twist operators.

range of Q^2 . When m_c enters only in the logarithmic derivative of the final result (effectively in the powers of $\log Q^2$), I use a step function, $\theta(Q^2 - Q_0^2)$, with $Q^2 = 3.5 \text{ GeV}^2$. If there are yet heavier quarks, they would not effect the energy-momentum sum rule until much higher Q^2 . For example, a 5 GeV quark would not have noticeable effects until $Q^2 \gtrsim 50 \text{ GeV}^2$.

I use the QCD theory for $R = \sigma_L/\sigma_T$ [3]. This involves correcting the raw μ -p data, given assuming $R \equiv 0.18$, which is straightforward because it is mostly for a single incident energy. Only the highest Q^2 point moves appreciably. Also, implicit in eq. (1) is the theory for R , in that the QCD energy-momentum sum rule is not for νW_2 but for a specific linear combination of W_1 and W_2 . This is corrected for by the $Z(Q^2)$ term. In fig. 3, the QCD prediction ignoring hard gluons is shown along with data from ref. [4]. In fig. 4, the effects of 30% glue at $Q^2 = 3.5 \text{ GeV}^2$ are included on the assumption that they are confined to $\xi \leq \frac{1}{4}$. (Details are given later.)

3. The formula

Eq. (2.13) of ref. [3] gives νW_2 in terms of a function whose moments are the proton matrix elements of the twist-2 operators. Minor juggling and integrating yields the QCD energy-momentum sum rule

$$\int_0^1 \left(1 + 4x^2 \frac{m^2}{Q^2}\right)^{3/2} \frac{\xi^2}{x^2} \nu W_2(\xi, Q^2) d\xi = \int_0^1 F_2^S(\xi, Q^2) \xi^2 d\xi + Z(Q^2), \quad (1)$$

$$Z(Q^2) = 6 \frac{m^2}{Q^2} \int_0^1 d\xi \left(1 + 4x^2 \frac{m^2}{Q^2}\right)^{-1/2} \xi^2 x \\ \times \int_{\xi}^1 d\xi' \left[1 + 2 \frac{m^2}{Q^2} \frac{x(\xi' - \xi)}{(1 + 4x^2 m^2/Q^2)^{1/2}}\right] F_2^S(\xi', Q^2), \quad (2)$$

where

$$\int_0^1 F_2^S(\xi, Q^2) \xi^n d\xi = \left(c_0^n + \frac{\alpha_s(Q^2)}{3\pi} c_1^n\right) \left[\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right]^{-d_n} A_n. \quad (3)$$

In this explicit form, I have neglected everything of order $\alpha_s^2(Q^2)$ and twists ≥ 4 . The coefficients c_i^n and proton matrix elements A_n are vectors, and the anomalous dimensions d_n are matrices in the space of the twist-2, spin- n operators. Q_0^2 is a reference momentum-squared at which the A_n are measured. Λ is determined by the

strength of the quark gluon coupling *via*

$$\frac{\alpha_s(Q^2)}{\pi} \approx \frac{12}{3(11 - \sum_i \frac{2}{3})} \frac{1}{\log(Q^2/\Lambda^2)}, \quad (4)$$

where i runs over effective quark flavors. In ref. [3] Λ was determined to be 500 ± 200 MeV from the logarithmic scaling violations in electroproduction.

The μ - p data can be tested against eq. (1) in the following form

$$\begin{aligned} & \int_0^{0.25} \left(1 + 4x^2 \frac{m^2}{Q^2}\right)^{3/2} \frac{\xi^2}{x^2} \nu W_2 d\xi \\ &= \int_0^1 F_2^s \xi^2 d\xi - \int_{0.25}^1 \left(1 + 4x^2 \frac{m^2}{Q^2}\right)^{3/2} \frac{\xi^2}{x^2} \nu W_2 d\xi + Z(Q^2), \end{aligned} \quad (5)$$

where the integral from 0.25 to 1.0 is determined by the analysis of electroproduction. For the important case of $n = 2$,

$$c_0^2 = (0, Q_1^2, Q_2^2, \dots), \quad c_1^2 = (-\frac{5}{8} \sum_i Q_i^2 f_i, -\frac{19}{9} Q_1^2, \dots), \quad (6)$$

where

$$f_i \approx \frac{1}{1 + \frac{5}{2} m_i^2 / Q^2}. \quad (7)$$

The index i runs over quark flavors; the first entry is the gluon coefficient, as computed by Witten [19]; and Q_i is the quark electric charge squared. The sum of all the components of A_2 is a constant, i.e. the proton's total energy momentum. Following the standard normalization of states and νW_2 , I will take the sum to be 1. In the gluon-quark-quark... basis, the anomalous dimension matrix is [5]

$$d_2 = \frac{4}{33 - 2 \sum_i 1} \begin{pmatrix} -\sum_i 1 & -\frac{8}{3} & \dots & -\frac{8}{3} \\ 1 & \frac{8}{3} & & 0 \\ \vdots & & \ddots & \\ 1 & 0 & & \frac{8}{3} \end{pmatrix}. \quad (8)$$

This has a zero eigenvalue, corresponding to the constant, total energy momentum.

4. The analysis

The solid lines running through the data in fig. 1 are handdrawn fits. These were integrated by counting squares on graph paper to give a raw $\int_0^{0.25} \nu W_2 dx$. The re-

sults are displayed in fig. 2 as crosses. These were subjected to two small corrections (described below) to get the dots, the final experimental determinations of $\int_0^{0.25} (1 + 4x^2 m^2/Q^2)^{3/2} \xi^2/x^2 \nu W_2 d\xi$. The error bars on the integrals assume that the only errors on the original data (fig. 1) are statistical.

The two small corrections are for $R = \sigma_L/\sigma_T$ and $(1 + 4x^2 m^2/Q^2)^{3/2} \xi^2 d\xi/(x^2 dx)$. The latter (including the implicit change in limit of integration) is straightforward and self-explanatory. To correct for $R \neq 0.18$, I assumed that the incident μ energy was 150 GeV and re-separated the data using an average, effective $\bar{R}$ defined by

$$\bar{R} = \frac{\int_0^{0.25} \sigma_L d\xi}{\int_0^{0.25} \sigma_T d\xi}. \quad (9)$$

Since $\bar{R}$ and its effects are small, crude approximations to σ_L and σ_T will suffice. I assume here that σ_T scales and that σ_L gets order m^2/Q^2 and order $\alpha_s(Q^2)$ contributions from the zeroth-order quark distribution [3,20]. σ_L gets an order $\alpha_s(Q^2)$ contribution from the gluon distribution [20]. I assume simply that

$$\int_0^{0.25} \sigma_L^{\text{glue}} d\xi = \int_0^1 \sigma_L^{\text{glue}} d\xi. \quad (10)$$

For a 30% glue fraction (and $\Lambda = 500$ MeV) $\bar{R}$ versus Q^2 is plotted in fig. 4.

Fig. 5. $L(Q^2)$, an integral over the fits of ref. [3] to electroproduction data [4]. The points with error bars are *not* data points but rather indicate only the size of the experimental errors in $\nu W_2(x, Q^2)$ entering $L(Q^2)$.

Fig. 6. $Z(Q^2)$, an integral over fits to all data for νW_2 , which contains the effects of spin > 2 operators on the energy-momentum sum rule. The points with error bars are *not* data points but rather indicate only the size of the experimental errors in $\nu W_2(x, Q^2)$ entering $Z(Q^2)$.

For any given Q^2 only some part of the x interval has been measured. Hence the integrals on the right-hand side of eq. (5) from 0.25 to 1.0 (and 0 to 1) will reflect some theoretical prejudice in piecing together the available data. To get the integral over $0.25 \leq \xi \leq 1.0$ (call it $L(Q^2)$) I used the fits of ref. [3] (improved a bit over $0.25 \leq \xi \leq 0.4$) which ignore the virtual quark pairs coming from primary gluons. To give some idea of the resulting uncertainties, I plot in fig. 5 the results for $L(Q^2)$ using the three fits for $\Lambda = 200, 500$ and 700 MeV. In the regions of good electroproduction data, experiment favors $\Lambda = 500$ MeV but only barely. The points with error bars are not real data, but are placed there to indicate the typical experimental errors in νW_2 , pointwise in ξ , that enter in $L(Q^2)$. $L(Q^2)$ for $\Lambda = 500$ MeV is reproduced again in fig. 2.

For $Q^2 \gg m^2$, the ξ -scaling factor $Z(Q^2)$ of eq. (2) is roughly $6m^2/Q^2 \int_0^1 x^2 \nu W_2 dx$. So even though it is an integral over all ξ , it reflects the behavior of moments with $n \geq 4$. The basic operating assumption is that for these moments the primary gluon contribution is negligible. So what I did was extend the $\xi \geq 1/4$ fit used for $L(Q^2)$ down to $\xi = 0$ so that it went through the μ -p data at $Q_0^2 = 3.5 \text{ GeV}^2$. I then let it evolve in Q^2 using the formulas of ref. [3] to compute $Z(Q^2)$ as defined in eq. (2). While this is not the correct way to get the Q^2 dependence of νW_2 for $0 \geq \xi \geq 0.25$, the explicit $x^2 m^2/Q^2$ makes the error in $Z(Q^2)$ negligible*. The resulting $Z(Q^2)$ for the three fits are shown in fig. 6. Again the dots

* In fact, this procedure is even better than would first appear because the $n = 2$ behavior of the "no gluon" formulas is in fact a fair approximation (in the relevant Q^2 range) to the resulting $n = 2$ behavior coming from the derived linear combination of the three $n = 2$ anomalous dimensions.

with error bars indicate only the pointwise uncertainties in the measurement of νW_2 as they would enter $Z(Q^2)$. $Z(Q^2)$ for $\Lambda = 500$ MeV appears again in fig. 2.

I choose $Q_0^2 = 3.5 \text{ GeV}^2$. The $\int F_2^S \xi^2 d\xi$ at Q_0^2 is determined by $L(Q^2)$, $Z(Q^2)$ and the integrated μ -p data for Q_0^2 . The subsequent Q^2 evolution of $\int F_2^S \xi^2 d\xi$ depends also on the relative quark abundances, which I will now consider. Let the energy momentum carried by the various quark (anti-quark) flavors at Q_0^2 be generically denoted by Q_i ($\bar{Q}_i$), and specifically by capital letters $U, \bar{U}, D, \bar{D}, S, \bar{S}, C$ and $\bar{C}$.

I assume the following:

$$\bar{U} = \bar{D}, \quad S = \bar{S}, \quad C = \bar{C}. \quad (11)$$

The "sea" is certainly not SU(4) symmetric and probably not SU(3) symmetric, simply because of the suppression of the materialization of heavier pairs. A guess that is consistent with perturbative estimates of the pairs created from gluons is

$$\bar{U} = \bar{D} = (2 \pm 1)S. \quad (12)$$

I use $\bar{U} = 2S$ as a nominal value but will explore the rather generous range.

The abundance of C in the proton can be reliably estimated perturbatively [5,19]. Based on earlier computations, I take

$$\frac{C}{\frac{1}{2}(U + D)} = 0.003 \pm 0.002. \quad (13)$$

This is so small at $Q_0^2 = 3.5 \text{ GeV}^2$, that its effects, despite the large uncertainty, are negligible.

Neutrino scattering tells us about anti-quarks. For the sake of the present calculation I take

$$\sum_i \bar{Q}_i / \sum_i Q_i = 0.12 \pm 0.12. \quad (14)$$

The quoted range certainly spans all reasonable guesses, while the nominal value gives a reasonable description of the "preanomalous" data [18].

$U - D$ can be determined from the proton-neutron difference in electroproduction. Since that difference has large fractional uncertainties but their effect is small on the final result here, it suffices to estimate $U - D$ in the traditional parton model fashion [8]:

$$\begin{aligned} \frac{P - N}{\frac{1}{2}(P + N)} &= \frac{\frac{3}{9}(U - D)}{\frac{1}{2}[\frac{5}{9}(U + D + \bar{U} + \bar{D}) + \frac{2}{9}(S + \bar{S}) + \frac{8}{9}(D + \bar{C})]} \\ &= 0.29 \pm 0.20, \end{aligned} \quad (15)$$

where $P(N)$ stands for $\int_0^1 \nu W_2 dx$ for the proton (neutron).

The above considerations determine the relative quark abundances *via* eq. (3) or

more explicitly

$$\int_0^1 F_2^s \xi^2 d\xi = \sum_i \left(1 - \frac{19}{27} \frac{\alpha_s}{\pi} \right) Q_i^2 (Q_i + \bar{Q}_i) - \frac{5}{24} \frac{\alpha_s}{\pi} \left[1 - \sum_i (Q_i + \bar{Q}_i) \right] \sum_i Q_i^2 \frac{1}{1 + \frac{5}{2} m^2 / Q^2}. \quad (16)$$

Taking now the numerical value of $\int \xi^2 F_2 d\xi$ at $Q_0^2 = 3.5 \text{ GeV}^2$ and eq. (16), I find that the glue fraction at Q_0^2 is

$$\theta_{\text{glue}} = 1 - \sum_i (Q_i + \bar{Q}_i) = 0.30, \quad (17)$$

using $\Lambda = 500 \text{ MeV}$ and the nominal relative quark abundances. If we take $S = 0 = \bar{U}$ and the maximum value of $U - D$ allowed by eq. (15), then $\theta_{\text{glue}} = 0.38$, which is the largest value obtainable within the quoted ranges. To minimize θ_{glue} , take $U - D$ as small as possible and $\bar{U} = S$ with $\Sigma \bar{Q} / \Sigma Q$ as large as possible; then $\theta_{\text{glue}} = 0.21$. Hence I quote θ_{glue} as $30 \pm 8\%$.

The explicit prediction for the Q^2 evolution of $\int F_2^s \xi^2 d\xi$ is

$$\int_0^1 F_2^s \xi^2 d\xi = \left[1 - \frac{19}{27} \frac{\alpha_s(Q^2)}{\pi} \right] \left\{ \frac{5}{18} \Sigma + \frac{3}{18} \Delta \right\} - \frac{5}{24} \frac{\alpha_s(Q^2)}{\pi} \{ 1 - \Sigma \} \sum_i Q_i^2 \frac{1}{1 + \frac{5}{2} m^2 / Q^2}, \quad (18)$$

where

$$\Sigma = A + (\Sigma_0 - A) \left[\frac{\log(Q^2 / \Lambda^2)}{\log(Q_0^2 / \Lambda^2)} \right]^{-a},$$

$$\Delta = \Delta_0 \left[\frac{\log(Q^2 / \Lambda^2)}{\log(Q_0^2 / \Lambda^2)} \right]^{-b},$$

$$\Sigma_0 = \sum_i (Q_i + \bar{Q}_i),$$

$$\Delta_0 = U + \bar{U} - D - \bar{D} - S - \bar{S} + C + \bar{C}, \quad (19)$$

and, for four (three) quarks, i.e. above (below) $Q_0^2 = 3.5 \text{ GeV}^2$, $A = \frac{3}{7} (\frac{9}{25})$, $a = \frac{56}{75} (\frac{50}{81})$, and $b = \frac{32}{75} (\frac{32}{81})$. Here $1, \Sigma$ and Δ are the eigenvectors of the anomalous dimension matrix. $\int F_2^S \xi^2 d\xi$ is plotted at the top of fig. 2 for $\Lambda = 500 \text{ MeV}$ and having been normalized to the nominal value of the data at $Q_0^2 = 3.5 \text{ GeV}^2$. The solid line corresponds to $\theta_{\text{glue}} = 0.30$ at Q_0^2 . The flatter edge of the shaded region is for $\theta_{\text{glue}} = 0.38$. The steeper edge is for $\theta_{\text{glue}} = 0.21$.

$L(Q^2)$ and $Z(Q^2)$ can now be subtracted from and added to $\int F_2^S \xi^2 d\xi$ to obtain a prediction for the Q^2 evolution of the integrated μ - p data. This prediction is shown in fig. 2. Again, the shaded region reflects the uncertainty in the relative quark abundances.

Fig. 7 shows the effect on the prediction of taking slightly different numerical values for $\int F_2^S \xi^2 d\xi$ at Q_0^2 . Fig. 7 uses $\Lambda = 500 \text{ MeV}$ and the nominal relative abundances. The lines are not parallel because a larger $\int F_2^S \xi^2 d\xi$ implies a smaller glue fraction (all other things being equal).

Fig. 8 shows the effect of varying Λ . Here again I have used the nominal relative abundances but used the different $L(Q^2)$'s and $Z(Q^2)$'s as well as different Λ 's in eq. (3).

At the bottom of fig. 2 the dashed line indicates the charm contribution to νW_2 integrated over all x or, more precisely, the charm contribution to $\int F_2^S \xi^2 d\xi$. Since the charmed quarks here come from gluons, this piece of νW_2 is virtually all at $\xi < \frac{1}{4}$. So, for example, at $Q^2 = 20 \text{ GeV}^2$, at least 5% of the μ - p cross section integrated over $0 \leq \xi \leq \frac{1}{4}$ will have charmed particles in the final state. There is another, distinct mechanism for charm production, which cannot yet be calculated in detail but

Fig. 7. Prediction for the integrated μ - p data versus Q^2 for $\Lambda = 500 \text{ MeV}$ and the nominal relative quark abundances. The different curves correspond to normalizing at different values at $Q_0^2 = 3.5 \text{ GeV}^2$.

Fig. 8. Prediction for the integrated μ -p data versus Q^2 for $\Lambda = 200, 500$ and 700 MeV using the nominal relative quark abundances, normalized to agree at $Q_0^2 = 3.5 \text{ GeV}^2$.

is likely to be much less important. The above estimate, e.g. 5% at $Q^2 = 20 \text{ GeV}^2$, is for events in which the struck quark is charmed. In addition charm may be produced by the final state interactions.

A preliminary report of multiple muons in the final state in μ scattering gives di-

Fig. 9. The energy-momentum sum rule versus Q^2 for $\Lambda = 500$ MeV, normalized to experiment at $Q_0^2 = 3.5 \text{ GeV}^2$. The experimental points are integrals $0-0.25$ from μ -p data and $0.25-1$ from the fit [3] to e-p data.

muons at about 10^{-3} per inelastic μ -induced event and trimuons at slightly above 10^{-4} [21]. Assuming that the extra muons come from leptonic decays of charmed particles, this would suggest charmed particles are in about 1% of the observed inelastic events. The mean x of the events with extra μ 's was $\langle x \rangle \approx 0.05$ and the mean Q^2 was $\langle Q^2 \rangle \lesssim 10 \text{ GeV}^2$. At 10 GeV^2 , QCD predicts that charmed particles should be in 2% of all the μ -induced events (see fig. 2). Given the existing uncertainties in both theory and experiment, this should be regarded as preliminary agreement. It is a straightforward task to improve both numbers, and it should be done in the coming months.

It is often suggested that the scaling violations observed for $x \gtrsim 0.2$ are to be explained by use of an improved scaling variable (i.e. with Λ then equal to zero). However, no choice of scaling variable can account for the dramatic rise at small x . Consequently, this rise is often ascribed to the production of new quantum numbers. From this point of view, at $Q^2 \approx 10 \text{ GeV}^2$ about 14% of the total cross section must be heavy particle production. Hence, I believe this point of view is untenable.

5. Afterthoughts

The conclusions to be drawn from the present effort are that the energy-momentum sum rule of QCD works for $\Lambda = 500 \text{ MeV}$ and that gluons carry a smaller fraction of the proton's energy momentum ($\sim 22\%$ at $Q^2 = 1 \text{ GeV}^2$) than previously imagined. What can be done in the way of more ambitious tests of the scaling predictions of QCD?

The analysis of the proton-neutron difference [22] is free of any uncertainty due to gluon matrix elements — for all ξ . Furthermore, it has been observed that the scaling violations for the neutron (or deuteron) are distinctly smaller than those of the proton. Qualitatively, this is what is expected because the proton-neutron difference is positive, and it decreases where the proton itself decreases and increases where the proton increases [22]. It is an important, practicable test of QCD whether this can be accounted for in detail by the standard theory, e.g. with the same values of the fundamental parameters.

Fox has recently analyzed all available lepton-hadron inclusive cross sections in the framework of QCD [23]. Allowing for plausible gluon and sea distributions, he finds agreement of theory and experiment for all x .

An earlier analysis of Johnson and Tung [24] of the deuteron data of refs. [25,26] was only able to fit experiment with $\Lambda \sim 30 \text{ MeV}$. This apparent inconsistency of QCD arose only because they assumed a particular gluon distribution. In terms of eigenvectors of the anomalous dimension matrix, they assumed that only the eigenvector with the smallest anomalous dimension was present. Hence their energy-momentum sum rule was a constant with $\sim 57\%$ glue. Furthermore they assumed that the perturbative eigenvalues were valid when continued to $n < 2$. For a given Λ , the expected rise with Q^2 at small x is much larger under these assumptions than allowing 30% of the energy momentum to be carried by gluons.

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Note added in proof

It has been suggested that the gluon coefficient functions computed in ref. [19] may be incorrect in sign and magnitude [27]. In question is the $-\frac{5}{8}$ in c_1^2 of eq. (6). This enters significantly in the determination of the glue fraction as the $-\frac{5}{24}$ in eq. (16). If this factor were positive, θ_{glue} would be larger than 0.3, and the prediction for $\int_0^{0.25} \nu W_2 d\xi$ would increase more rapidly with Q^2 . Eq. (16) also would give a non-trivial lower bound on $\int F_2^g \xi^2 d\xi$. A method for computing c_1^2 is outlined in ref. [3].

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