Laser interferometers as dark matter detectors

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While the global cosmological and local galactic abundance of dark matter is well established, its identity, physical size, and composition remain a mystery. In this paper, we analyze an important question of dark matter detectability through its gravitational interaction, using current and next generation gravitational-wave observatories to look for macroscopic (kilogram-scale or larger) objects. Keeping the size of the dark matter objects to be smaller than the physical dimensions of the detectors, and keeping their mass as a free parameter, we derive the expected event rates. For favorable choice of mass, we find that dark matter interactions could be detected in space-based detectors such as LISA at a rate of one per ten years. We then assume the existence of an additional Yukawa force between dark matter and regular matter. By choosing the range of the force to be comparable to the size of the detectors, we derive the levels of sensitivity to such a new force, which exceeds the sensitivity of other probes in a wide range of parameters. For sufficiently large Yukawa coupling strength, the rate of dark matter events can then exceed 10 per year for both ground- and space-based detectors. Thus, gravitational-wave observatories can make an important contribution to a global effort of searching for nongravitational interactions of dark matter.

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I. INTRODUCTION

There is overwhelming evidence that the Universe is dominated by dark energy (DE) and dark matter (DM), which together comprise about 95% of the cosmological critical energy density $\rho_c \times c^2 \simeq 5\ \mathrm{keV/cm^3}$ [1]. Thus far, all the evidence comes from the gravitational influences of DE and DM on regular matter built from the standard model (SM) particles and fields. The concentration of DM is enhanced around collapsed cosmic structures, such as galaxies and clusters of galaxies, where it exceeds its cosmological average by several orders of magnitude. In particular, the energy density of dark matter in the Milky Way close to the location of the solar system has been determined to be about $0.39\ \mathrm{GeV/cm^3}$ [2]. The observed DM behavior is consistent with its being "cold," which implies a certain Maxwellian-type velocity distribution, with an rms velocity of about 270 km/s inside the Milky Way. This random motion is superimposed on the ∼220 km/s constant velocity of the Sun relative to galactic center, so that there is a significant asymmetry in the flux of dark matter for an observer on earth.

Since all information on DM comes from its gravitational interactions, its composition and properties remain unknown. Among the most important questions that do not have any direct observational answers are the following:

(i) What is the relation of DM to the visible matter of the SM? Is there any new interaction that supplements gravity and acts between DM and regular atoms?
(ii) Is DM elementary or composite?
(iii) What is the physical size and mass of the DM objects?

In many particle physics models, DM is elementary and can be represented either by massive particles (e.g., related to the lightest supersymmetric partners of SM particles), or by light fields (e.g., QCD axions). Extensive research aimed at the direct detection of DM has advanced the sensitivity to elementary DM interacting with atoms, nuclei and electromagnetic fields. It has produced bounds on, e.g., weak-scale DM interacting with nuclei [3], but so far has not led to any answers to the above questions. While the next generation of such experimental efforts may bring positive results, it is important to *widen* the DM search program using the multiprobe approach with sensitive instruments.

In this work, we investigate the use of gravitational-wave observatories as detectors of dark matter via gravitational interaction of DM objects with the detectors' test masses. The gravitational interaction is the only guaranteed interaction between DM and SM, and therefore it is important to investigate the prospects of a detection based only on gravitational interaction. Moreover, we will study detection based on possible additional interactions—modeled as a Yukawa potential—between dark matter and the particles of the standard model.

II. THE MODEL OF MACROSCOPIC DM

The discussion of macroscopic-size dark matter was traditionally oriented towards the massive compact halo objets (MACHOs) and primordial black holes. The range of suggested masses for these candidates starts from rather large values, $M > 10^{14}$ g [4,5]. This mass range influenced early discussions on a possible use of space-based gravitational-wave inteferometers in search for dark matter [6,7]. For primoridal black holes, the range below 10^{14} g is disfavored due to Hawking evaporation [8] shortening the lifetime below the age of the Universe. Going away from the black hole candidates, one faces a much broader spectrum of macroscopically sized DM candidates [9–12]. In particular, if sufficiently complex, dark sectors can possess stable topological monopoles [13,14], or non-topological defects, such as Q-balls [15]. Given the unknown properties of the dark sector, the mass range for such DM objects can be almost arbitrary, and their required cosmological abundance can be achieved via the so-called Kibble–Zurek mechanisms [16]. Microscopic particle-type DM can form objects much smaller than galactic size, also known as clumps. The size and mass density of such objects may widely differ depending on DM properties, and the cosmological history.

For the purpose of this study, we will assume that DM consists of macroscopic objects of a certain transverse radius $r_{\rm DM}$ and mass $M_{\rm DM}$. The mass $M_{\rm DM}$ determines the average distance between the DM objects, and the frequency of encounters. Introducing the number density of galactic DM objects, $n_{\rm DM} \equiv L^{-3}$, we obtain the following relation between the mass and the characteristic distance between the DM objects,

$$\rho_{\rm DM} = M_{\rm DM} n_{\rm DM} \Rightarrow \frac{L}{10^4\ {\rm km}} \simeq 1.2 \times \left(\frac{M_{\rm DM}}{1\ {\rm kg}}\right)^{1/3}, \quad (1)$$

where $\rho_{\rm DM}$ is DM mass density, $\rho_{\rm DM} \times c^2 \simeq 0.39\,{\rm GeV/cm^3}$.

This distance can be directly related to the effective flux of DM, and the frequency of close encounters. For a fiducial choice of $M_{\rm DM}$ of 1 kg, the effective flux of DM is $\Phi_{\rm DM} \sim n_{\rm DM} v_{\rm DM} \sim 3 \times 10^{-10}\ {\rm km^{-2}\,s^{-1}}$, and one can expect one DM object per year to pass the detector with an impact parameter of 10 km. This is commensurate with the actual physical size of the interferometer arms of existing graviational-wave detectors such as LIGO [17], and if the interaction between the DM objects and atoms, which the gravitating masses of LIGO are made of, is strong enough, such passage could in principle be detected. The generalization to other types of defects (strings and/or domain walls) is also possible [9,18].

What kind of interaction could one expect to have between the DM and SM? Besides purely gravitational interaction, the number of possibilities is quite large [10]. In this work we will consider additional Yukawa interaction introduced by the exchange of a light scalar, vector or tensor particle with mass $m_\phi \equiv \lambda^{-1} \times (\hbar/c)$. Combining Yukawa and gravitational interactions, we write the nonrelativistic potential between the two compact objects, separated at distance r ($r > r_{\rm DM}$), as follows:

$$V_{i-j} = -M_i M_j \frac{G_{\rm N}}{r} (1 + (-1)^s \delta_i \delta_j \exp[-r/\lambda])$$
$$\text{where } i, j = {\rm SM}, {\rm DM}. \quad (2)$$

This equation assumes that the potential scales with the mass of the object (e.g., ϕT^μ_μ coupling in the scalar case), and the corresponding couplings are parametrized in units of the standard gravitational coupling by the dimensionless numbers $\delta_{\rm SM}$ and $\delta_{\rm DM}$. $(-1)^s$ is equal to $+1$ for scalar and tensor exchange, and -1 for vector exchange. Moreover, we shall assume that the range of the force and the physical size of the detectors (LIGO) are much larger than the size of the DM objects, but smaller than the average distance between them,

$$r_{\rm DM} \ll l_{\rm LIGO}, \qquad \lambda \ll L, \quad (3)$$

which significantly simplifies the analysis.

Extensive tests of the gravitational force, $V_{\rm SM\text{-}SM}$, have set stringent constraints on $\delta_{\rm SM}$ as a function of λ [19]. Thus, for $\lambda \sim 1$ km, $|\delta_{\rm SM}| < 10^{-3}$. At the same time, the coupling of this Yukawa force to DM can be many orders of magnitude stronger. The main constraint on $\delta_{\rm DM}$ comes from the influence of DM self-interaction on structure formation [20] and on the dynamics of cluster collisions [21]. Since the range of the force is assumed to be less than L, only pairwise collisions are important. The momentum-exchange cross section can be easily calculated with the use of the inequalities in Eq. (3). To logarithmic accuracy it is given by

$$\sigma_{\text{DM-DM}} = 16\pi \times \frac{G_N^2 M_{\text{DM}}^2 \delta_{\text{DM}}^4}{v_{\text{DM}}^4} \times \log\left[\frac{\lambda}{r_{\text{DM}}}\right]. \quad (4)$$

At $v_{\text{DM}} \sim 10^{-3}c$, there is a typical constraint on the cross section, $\sigma_{\text{DM-DM}}/M_{\text{DM}} \lesssim 1\ \text{cm}^2/\text{g}$, which translates to the following limit on the value of the DM Yukawa coupling:

$$|\delta_{\text{DM}}| \lesssim 5 \times 10^9 \times \left(\frac{1\ \text{kg}}{M_{\text{DM}}}\right)^{1/4}. \quad (5)$$

In deriving this limit, we set the value of the logarithm to 5.

It is important to emphasize that *saturating* this bound may alleviate some problems of cold DM scenario that emerge when observations are compared to numerical simulations. Self-interaction helps to cure the problem of cold DM overly-dense central regions of dwarf galaxies predicted in simulations [22], as DM self-scattering reduces the DM densities in the central regions relative to non-interacting case (see, e.g., [23]). Therefore, $|\delta_{\text{DM}}| \gg 1$ represents a phenomenologically motivated choice. Taking two limits on δ_i together, one can conclude that at $r < \lambda$ the strength of DM-SM interaction, $|\delta_{\text{DM}}\delta_{\text{SM}}|$, can exceed gravity by up to seven orders of magnitude. One microscopic realization of $|\delta_{\text{DM}}| \gg |\delta_{\text{SM}}|$ possibility would be a new scalar force with reasonably strong coupling to DM, and reduced coupling to the SM mediated e.g., via the Higgs portal [24].

III. MACROSCOPIC DM DETECTION

We perform several Monte Carlo simulations in order to characterize the rate of discrete DM interaction events with laser interferometers. We first consider the case of a single Advanced LIGO detector [25] operating at full sensitivity. Advanced LIGO is part of a worldwide network of kilometer-scale laser interferometers that are already operational or will become operational in the next several years [25–27]. Future terrestrial [28,29] and space-based detectors [30] have also been planned. We therefore also consider the case of a single LISA-type detector [31].

We model the distribution of DM in the galaxy as objects of mass M, with a uniform density in the solar system of $\rho_{\text{DM}} = (0.39\ \text{GeV}/c^2)/\text{cm}^3$, and a randomly directed velocity $\mathbf{v}$ whose magnitude is distributed according to a combination of the galaxy-frame DM velocity (270 km/s rms, normally distributed in each directional component) and the speed of the solar system through the galaxy (220 km/s). As the DM object (or undisrupted clump of DM) passes by the detector, it produces an acceleration $\mathbf{a}^{(k)}(t)$ of the detector's kth test mass (four in the case of LIGO, conventionally labeled as IX, IY, EX, and EY). The acceleration is determined by the gradient of Eq. (2) with $i = \text{SM}$ and $j = \text{DM}$. The detector's GW channel reads out the differential acceleration $a(t) = [a_x^{(\text{EX})}(t) - a_x^{(\text{IX})}(t)] - [a_y^{(\text{EY})}(t) - a_y^{(\text{IY})}(t)]$ [32]. We assume that the signal of this

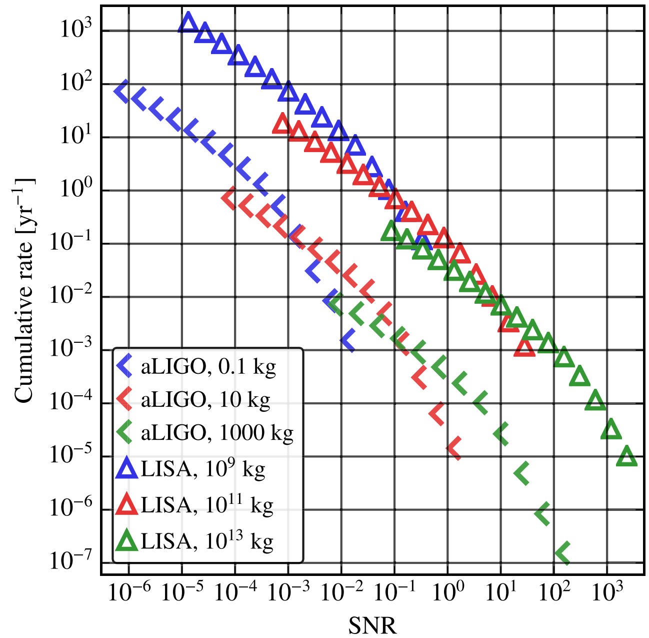

FIG. 1. Cumulative event rate for minimal (pure gravitational) interactions in a single Advanced LIGO detector and in a single LISA detector. SNR > 8 correspond to very infrequent events, with rates below 10^{-3} yr^{-1} for aLIGO and 10^{-1} yr^{-1} for LISA.

event can be optimally recovered from the detector's time stream using matched filtering; i.e., the signal-to-noise ratio (SNR) is $\varrho = [4\int_0^\infty \mathrm{d}f |a(f)|^2/S_{nn}(f)]^{1/2}$, where $a(f)$ is the Fourier transform of $a(t)$ and $S_{nn}(f)$ is the power spectral density (PSD) of the detector's acceleration noise $n(t)$ [33].

In addition to simulating several DM masses for each detector, we also vary the coupling $g = \delta_{\text{SM}}\delta_{\text{DM}}$ and the screening λ, as defined in Eq. (2). The Newtonian case ($g = 0$) has already been analyzed analytically in the context of primordial black hole detection with LISA [6], in the limits $b \ll \ell$ (the "close-approach" limit) and $b \gg \ell$ (the "tidal" limit), where b is the distance of closest approach and ℓ is the detector arm length. In both cases a flat detector noise PSD and a normal incidence of the masses to the detector plane is assumed.

We then compute the cumulative rate function $\dot{\eta}(\varrho)$, which gives the number of events per year with SNR above ϱ. In Fig. 1 we plot the detector interaction rates assuming a Newtonian coupling. One can observe that the parameters leading to SNR > 8 (a typical detection threshold for LIGO) correspond to very infrequent events, with rates below 10^{-3} yr^{-1} for aLIGO and 10^{-1} yr^{-1} for LISA. Therefore, detecting a gravitational strength interaction will be extremely challenging.

Nevertheless, Fig. 1 shows that the current and future instruments are just a few orders of magnitude short of being sensitive to the most minimal model of DM-SM interaction, for an optimal DM mass. This is in contrast to the searches of dark matter in form of elementary particles,

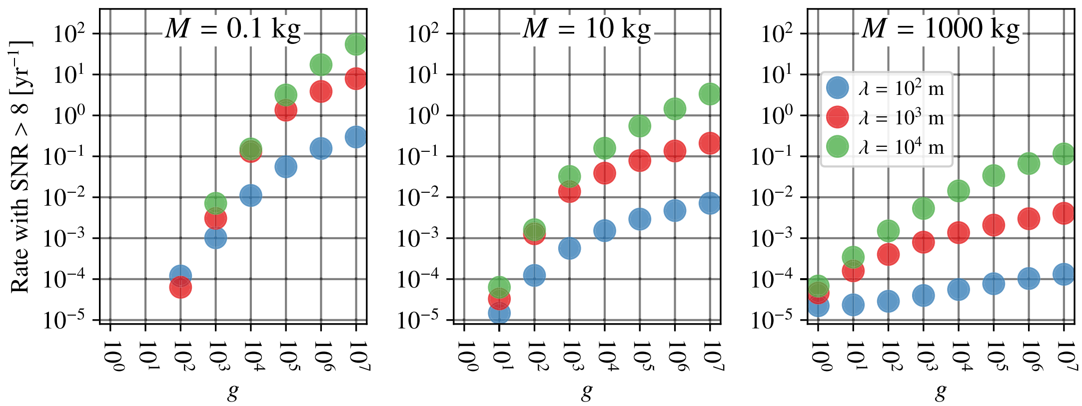

FIG. 2. Event rate $\dot{\eta}(8)$ for non-SM interactions in a single Advanced LIGO detector, as a function of coupling $g = \delta_{\rm SM}\delta_{\rm DM}$ and screening length λ. For a long range force the rate can reach $O(100)$ events per year when g is taken to a maximum value.

where the most sensitive experiments [3] will reach the level of sensitivity to the nucleon-DM elastic cross section $\sigma_p \sim 10^{-48}$ cm^2 for $m_{\rm DM} \sim 100$ GeV$/c^2$. This sensitivity is to be compared to the gravitational cross section that scales as $\propto G_N^2 m_p^2/v_{\rm DM}^4$ (where m_p is the nucleon mass) and does not exceed 10^{-90} cm^2, which is over 40 orders of magnitude below the experimental capabilities. On the other hand, gravitational wave interferometry is insensitive to the microscopic mass of the elementary particle DM, and thus these two methods (gravitational wave detectors and nuclear recoil in underground experiments) are completely complementary, probing different types of DM.

In Figs. 2 and 3 we show how $\dot{\eta}$ is enhanced if the SM–DM interaction follows a Yukawa force law. The ability of LIGO and LISA to place constraints on g and λ depends on the mass of DM object; in both cases, the smallest masses considered (0.1 kg for LIGO, 10^9 kg for LISA) allow for the most sensitivity to $\{g, \lambda\}$ parameter space. If we choose $\delta_{\rm SM}$ close to the existing bounds, and $\delta_{\rm DM}$ to saturate (5), then the rate of loud encounters can exceed $\mathcal{O}(10)$ per year. For LISA, the event rate can become very large, and indeed exceed 100 events per year, when the product of $\delta_{\rm SM}\delta_{\rm DM}$ is taken to its maximum.

To confidently claim detection, a DM signal must be distinguished from glitches and other detector artifacts. One strategy is to look for DM signals using two or three colocated detectors. The current rate of glitches that are uncorrelated between the LIGO detectors is sufficiently low to allow detection of the broadband signals with SNR $\gtrsim 8$ in coincidence between the Hanford and Livingston detectors. The environmental disturbances such as acoustic, seismic, or electromagnetic, can potentially produce glitches that are coincident between colocated detectors. This background can be effectively vetoed by the environment monitoring sensors and in case of the three colocated interferometers by the null stream combination of the interferometer outputs that does not contain the signal. The Advanced LIGO detectors as currently built are not colocated, though the Hanford facility did house two colocated initial LIGO detectors. Some of the plans for LISA-like space missions [34] and for ground based observatories [28,29] involve three colocated detectors.

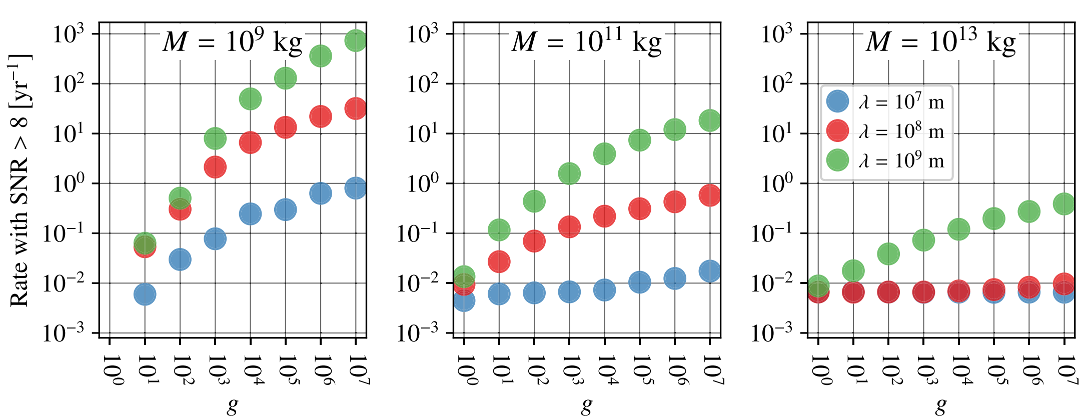

FIG. 3. Event rate $\dot{\eta}(8)$ for non-SM interactions in a single LISA detector, as a function of coupling $g = \delta_{\rm SM}\delta_{\rm DM}$ and screening length λ. The event rate exceeds $O(10)$ events per year at $g \sim 10^4$ and will exceed $O(100)$ at large λ and $g \sim 10^7$.

We assume that the glitch rate of the future detectors will not exceed that of the current generation detectors.

To illustrate the possibility of the null stream in a LISA-like configuration let us consider the flyby trajectories in the vicinity of one of the test masses (note that the signal from the flyby through the center of the triangle and normal to the plane vanishes due to symmetry). Let the forces on the near test masses along two of arms be F_x and F_y and neglect the forces on the other test masses. Then the three interferometer outputs are: $S_1 = F_x - F_y$, $S_2 = F_y$, and $S_3 = -F_x$. Thus the combinations $S_1 - S_2 - S_3$, etc., give the null stream for events near the test masses. More generally one should develop an algorithm to reconstruct the flyby trajectory from the data. The search for dark matter flyby events will be done similar to the 'Coherent Wave Burst' analysis [35,36] which was used to search for gravitational waves from weakly modeled sources.

Also, as Fig. 3 shows, for $g > 10^4$ the rate may approach hundreds of events per year. Such large rates would eventually allow a statistical discrimination of the DM encounters from noise sources. One handle that can be used is the ∼10% annual modulation of the DM event rate, with a very well known phase (maximum at the end of June), when the Earth's velocity vector is constructively added to the velocity of the Solar system resulting in a larger effective flux of DM. When the number of events is large, one can build another statistical discriminator using correlation between the duration and amplitude of the events (close encounters with DM lead to higher amplitude but occur in a smaller time window).

IV. STOCHASTIC DM DETECTION

In addition to single, loud DM events, we alternatively consider the case of a stochastic DM background due to a population of lighter, individually unresolvable DM objects. Cross-correlating the outputs of GW detectors placed at remote points on the earth reduces vastly the event rate. In order to place best-case limits on our ability to detect such a signal, we consider only the case of two identical, colocated, and coaligned detectors whose noise is stationary, Gaussian, and independent.

Assuming the DM background $a(t)$ is independent of, and much weaker than, the detector noises $n_1(t)$ and $n_2(t)$, the optimal SNR is $[2T \int_0^\infty \mathrm{d}f S_{aa}(f)^2/S_{nn}(f)^2]^{1/2}$, where $S_{aa}(f)$ is the PSD of a, T is the observing time, and we assume $S_{n_1n_1} = S_{n_2n_2} \equiv S_{nn}$. We find that a Newtonian DM background is undetectable after $T = 5$ years for the DM masses considered: for LIGO, masses of 10^{-9}-10^{-7} kg result in optimal SNRs of $0.3 - 5 \times 10^{-17}$; for LISA, masses of 10^6, 10^7, and 10^8 kg result in optimal SNRs of 9×10^{-7}, 4×10^{-6}, and 1.4×10^{-4}, respectively. However, for $g \gg 1$, we have $S_{aa} \propto |g|^2$, and hence the SNR increases with $|g|^2$. Therefore, LISA could detect a stochastic background from Yukawa interaction of DM clumps with mass 10^8 kg provided $|g| \gtrsim 10^2$, or clumps with mass 10^6 kg provided $|g| \gtrsim 10^3$.

While our consideration in this paper is primarily about pointlike DM objects, it can be easily extended to other types of defects, including cosmic strings, and domain walls. The latter provide a much cleaner signature, as the passage of the domain walls is guaranteed to happen through all detectors. On the other hand, the case for the DM composed of domain walls is much weaker, but they can exist as a subdominant component to the dark sector energy density, and therefore can be searched for with the existing networks of the gravitational wave detectors.

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[1] K. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).
[2] R. Catena and P. Ullio, J. Cosmol. Astropart. Phys. 05 (2012) 005.
[3] P. Cushman, C. Galbiati, D. McKinsey, H. Robertson, T. Tait *et al.*, arXiv:1310.8327.
[4] J. Yoo, J. Chaname, and A. Gould, Astrophys. J. **601**, 311 (2004).
[5] F. Capela, M. Pshirkov, and P. Tinyakov, Phys. Rev. D **87**, 123524 (2013).
[6] N. Seto and A. Cooray, Phys. Rev. D **70**, 063512 (2004).
[7] A. Adams and J. Bloom, arXiv:astro-ph/0405266.
[8] S. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[9] M. Pospelov, S. Pustelny, M. Ledbetter, D. Jackson Kimball, W. Gawlik, and D. Budker, Phys. Rev. Lett. **110**, 021803 (2013).
[10] A. Derevianko and M. Pospelov, Nat. Phys. **10**, 933 (2014).
[11] Y. Stadnik and V. Flambaum, Phys. Rev. Lett. **114**, 161301 (2015).
[12] P. W. Graham, D. E. Kaplan, J. Mardon, S. Rajendran, and W. A. Terrano, Phys. Rev. D **93**, 075029 (2016).

[13] G. 't Hooft, Nucl. Phys. **B79**, 276 (1974).
[14] A. M. Polyakov, JETP Lett. **20**, 194 (1974).
[15] S. R. Coleman, Nucl. Phys. **B262**, 263 (1985).
[16] W. Zurek, Nature (London) **317**, 505 (1985).
[17] The LIGO Scientific Collaboration and The Virgo Collaboration, Phys. Rev. Lett. **116**, 131103 (2016).
[18] J. Jaeckel, V. V. Khoze, and M. Spannowsky, Phys. Rev. D **94**, 103519 (2016).
[19] S. Schlamminger, K.-Y. Choi, T. Wagner, J. Gundlach, and E. Adelberger, Phys. Rev. Lett. **100**, 041101 (2008).
[20] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000).
[21] D. Harvey, R. Massey, T. Kitching, A. Taylor, and E. Tittley, Science **347**, 1462 (2015).
[22] M. Boylan-Kolchin, J. S. Bullock, and M. Kaplinghat, Mon. Not. R. Astron. Soc. **415**, L40 (2011).
[23] M. Vogelsberger, J. Zavala, and A. Loeb, Mon. Not. R. Astron. Soc. **423**, 3740 (2012).
[24] F. Piazza and M. Pospelov, Phys. Rev. D **82**, 043533 (2010).
[25] J. Aasi, B. P. Abbott, R. Abbott, T. Abbott, M. R. Abernathy, K. Ackley, C. Adams, T. Adams, P. Addesso *et al.* (The LIGO Scientific Collaboration), Classical Quantum Gravity **32**, 074001 (2015).
[26] T. Accadia, F. Acernese, M. Alshourbagy, P. Amico, F. Antonucci, S. Aoudia, N. Arnaud, C. Arnault, K. G. Arun, P. Astone *et al.*, J. Instrum. **7**, P03012 (2012).
[27] T. Akutsu (K. collaboration), J. Phys. Conf. Ser. **610**, 012016 (2015).
[28] M. Punturo, H. Lück, and M. Beker, in *Astrophysics and Space Science Library* Vol. 404, edited by M. Bassan (Springer, Cham, 2014), p. 333.
[29] S. Dwyer, D. Sigg, S. W. Ballmer, L. Barsotti, N. Mavalvala, and M. Evans, Phys. Rev. D **91**, 082001 (2015).
[30] A. Sesana *et al.*, Gen. Relativ. Gravit. **46**, 1793 (2014).
[31] P. Bender and K. Danzmann (the LISA Study Team), Laser Interferometer Space Antenna for the detection and observation of gravitational waves: Pre-phase A report, 2nd Edition (Max-Plank Institut für Quantenoptik, Technical Report No. MPQ 233, 1998).
[32] P. Saulson, *Fundamentals of Interferometric Gravitational Wave Detectors* (World Scientific, Singapore, 1994).
[33] M. Maggiore, *Gravitational Waves: Volume 1: Theory and Experiments* (Oxford University Press, New York, 2007).
[34] NGO, Yellow Book (NGO, Technical Report, 2013), http://sci.esa.int/ngo/49838-ngo-yellow-book/.
[35] J. Abadie, B. P. Abbott, R. Abbott, T. Accadia, F. Acernese, R. Adhikari, P. Ajith, B. Allen, G. Allen, E. Amador Ceron *et al.*, Phys. Rev. D **81**, 102001 (2010).
[36] J. Abadie, B. P. Abbott, R. Abbott, T. D. Abbott, M. Abernathy, T. Accadia, F. Acernese, C. Adams, R. Adhikari, C. Affeldt *et al.*, Phys. Rev. D **85**, 102004 (2012).