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A 12 On the Theoretical Status of Perturbative QCD

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Features of the long suspected but long elusive all orders justification of the impulse picture for inclusive high energy processes from QCD are briefly discussed.

Over the last year, innumerable calculations of radiative corrections to the parton model have been presented. The specific subject matter ranges over general features of spin-averaged inclusive cross sections, jet cross sections, features of jets, lepton-pair production, hadron production at large $P_\perp$, heavy hadron production, direct photons, angular correlations, polarization effects, ... and the list goes on. With these predictions of QCD being imminently testable, it behooves us to determine whether they are in fact consequences of QCD. Is the computational algorithm, developed to handle the inevitable infrared divergences and large logarithms of a naive application of perturbation theory, self-consistent? And what can and what cannot be computed reliably from a purely perturbative approach?

I think the most important theoretical advance of late is the all orders, all logarithms proofs of the quark–gluon–parton picture (involving distribution and decay functions, jets, and systematic radiative corrections.) Thus, a wide class of QCD predictions have been put on a par with the orthodox analyses of inclusive leptoproduction and e^+e^- annihilation, *i.e.*, the calculations have been justified order by order in perturbation theory.

Several groups [1-6] have arrived at similar conclusions using similar theoretical tools. I will focus on inclusive processes with observed initial and final state hadrons. (Jets are trivially related or can be studied in their own right.[2]) I will use the language of my own work on the subject,[6] but I emphasize that the analyses are quite parallel. I will mention some of the differences.

The parton model is an impulse picture in which high energy interactions are viewed as being characterized by two time scales. A short time scale is set by the large momentum transfers between constituents in hard scattering or rapid production while a long time scale is set by the soft forces of binding. The soft distribution and recombination of bound constituents cannot effect the short or hard processes. Thus we are led to a convolution form for inclusive hadron cross sections $d\sigma(P_i)$ in terms of a hard (parton) cross section, $d\sigma^{\text{parton}}$, and functions that summarize the soft effects, f_i. (The difference in time scales implies the lack of interference.) Schematically,

$$d\sigma(P_i)=\int\prod_i d\beta_i f_i(\beta_i)d\sigma^{\text{parton}}(\beta_i P_i)+O(P_i^2). \tag{1}$$

(The inclusion of real and vitual photons and W bosons is straightforward.)

Is the clean separation of time scales true in QCD at high energies, with functions f_i that are process independent, but depend only on the type of observed hadron? With only pertubation theory at our disposal, we cannot see bound states, so the long time scale associated with binding will appear as infinitely long. For convenience, we work in momentum space and take the light quarks as massless.[7] Infrared divergences (now inlcuding mass singularities) occur for processes that, in position space, can take place over arbitrarily long times. Hence, we are looking for a factorization of all infrared divergences. To put it more precisely, if we now interpret $d\sigma(P_i)$ in eq. (1) as the inclusive cross section for "observed" quarks and gluons (henceforth generically called partons) computed as the sum of all Feynman graphs, is $d\sigma(P_i)$ of the convolution form where $d\sigma^{\text{parton}}$ is free of all infrared divergences (and so are its integrals)? The answer is yes for all infrared divergences, at least in perturbation theory.

The application to hadrons involves the assumption that the perturbative, infrared divergent f_i's are replaced by finite functions for the hadrons in computing $d\sigma^{\text{hadron}}$. For incoming hadrons and leptons this same assumption is made in leptoproduction and inclusive e^+e^-. For outgoing hadrons and currents there are analogous (but perhaps logically distinct) assumptions that must be made regarding non-perturvative effects. (Reference 1 contains interesting relevant comments on μ-pair production.) No systematic characterization of these assumption exists, but it is probably simply premature; we should first determine that QCD predicts the existence of hadrons.

I will give here only a flavor of the proof. It essentially has two parts. Write the total parton inclusive cross section in terms of two-particle irreducible (2*PI*) parts (irreducible in the individual observed particle channels of the analogous forward amplitude). For notational simplicity, concentrate on a single such channel. See Fig. 1. $I(k')$ is the 2*PI* scattering; it actually depends on as many momenta as observed partons; $K(k, k')$ is the 2*PI* kernel. (I will ruthlessly suppress all inessential—and some essential—complications. For an honest effort see ref. 6. For example, at present I ignore type, color, spin, and flavor indices as well as the fact that $K(k, k')$ includes self-energy corrections on the k' legs; this is an inessential complication for the features I will discuss explicitly below.) So that we can discuss the divergences with some precision, we take the $P_i^2 \neq 0$ to regulate them. To distinguish the various f_i for different i and establish their process independence, we take $P_i^2 \neq P_j^2$ for $i \neq j$. Our first task is to prove that I and K are infrared finite, *i.e.*, that $I(k)|_{k^2=0}$ and $K(k, k')|_{k^2=0}$ exist. We can relate $d\sigma$, I and K in an integral equation (see, again, Fig. 1) where I use a matrix notation: $d\sigma$, I and K are matrices, and the contraction of indices on adjoining matrices stands for an integral, *e.g.*, d^4k', over the momentum linking the parts:

$$d\sigma = I + K\, d\sigma. \tag{2}$$

The solution is formally

$$d\sigma = \left(\frac{1}{1-K}\right) I. \tag{3}$$

Our second task is to use the infrared finiteness of I and K to derive the form of eq. (1). In particular, we must reduce the four dimensional d^4k' to a one dimensional $d\beta$ and show that the remainder is $O(P^2)$.

The underlying reason that all this works is that all infrared divergences come from "physical" processes, *i.e.*, processes in which the external lines can be connected by internal lines that are actually real, *i.e.*, on their mass shells. This is Landau's classic analysis.[8] The infrared finiteness of 2*PI* parts only holds in "physical" gauges, such as axial gauge, which have only the two physical polarizations for lightlike gluons. The 2*PI* parts are then well-behaved because intermediate states with more than two particles have too small phase space to give divergences. The reduction of the d^4k' to a $d\beta$ reflects the fact that divergences only occur for $k'^2 \simeq 0$ and moreover for $k' \simeq \beta p$ as $p^2 \to 0$.

Before giving just a few of the particulars of theoretical interest, I would like to emphasize some of the important physics consequences.

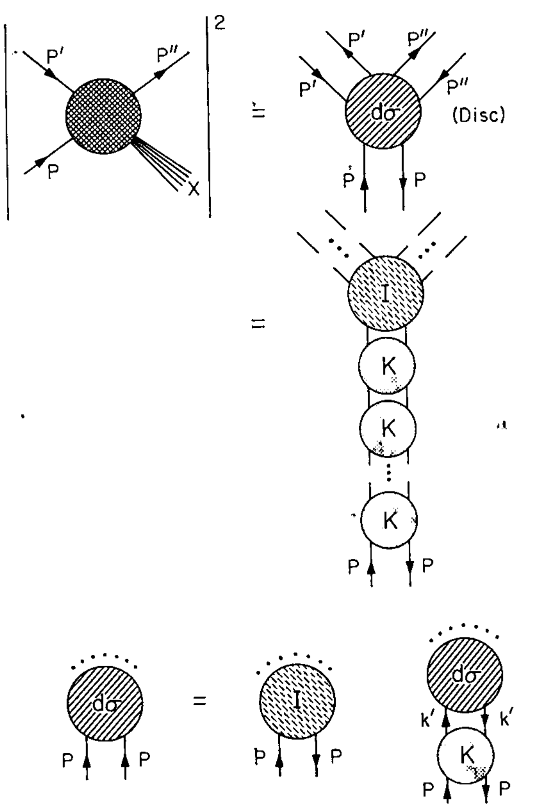

Fig. 1.

Although axial gauge is essential in organizing a simple proof, factorization of large or infinite logarithms, an observable phenomenon, must be true in any gauge. (The $2PI$ decomposition will become useless in covarient gauges.) Hence one can compute $d\sigma$, the sum of all relevant graphs, in any gauge, unfold and amputate the universally occuring infrared sensitive factors, thus leaving an infrared finite $d\sigma^{\text{parton}}$.

In no sense do infrared finite parts of the calculation factor. They must be different in different processes.

The f_i's and $d\hat{\sigma}^{\text{parton}}$'s necessarily have a convention-dependent ambiguity related to how much of the finite parts are factored out with the divergences into the f_i's and how much are left in $d\sigma^{\text{parton}}$. This causes no real problems, but care must be exercised in going from one process to another to factor out the same f_i's.

The correction terms of $O(P_i^2)$ are more precisely $O(P_i^2/P_i\cdot P_j)$, *i.e.*, vanish like inverse powers of the large invariants. These terms are most definitely process-dependent. Hence, they cannot be expressed simply as an elaboration of the process-independent f_i, *e.g.*, as a primordial parton transverse momentum dependence introduced into the f_i. In other words, while parton transverse momenta are certainly non-zero, there are process dependent corrections (not presently calculable) that have similar net effects. Forcing these effects into the form of effective parton transverse momenta is futile because it will be inconsistent from process to process.

The implementation of the renormalization group improvement using the scale-depedent coupling constant is essential (but straightforward) if $\log s/s_0$ is large, where s is the scale of a typical large invariant and s_0 is the scale initially used to measure the f_i's, or if $\log s/M^2$ is large, where M^2 is the scale used to define the original coupling constant, $g^2(M^2)$. To get it all correctly we must remember that the factoring off of the f_i requires the introduction of an arbitrary scale, M'^2, used to define an interval of small k'^2, *e.g.*, $0\leq|k'^2|\leq M'^2$. For simplicity we may choose $M'^2=M^2$. Now

$$d\rho(P_i)=\int\prod_i d\beta_i f_i(\beta_i, M^2)$$
$$\times d\hat{\sigma}^{\text{parton}}(\beta_i P_i, g^2(M^2), M^2)$$
$$=\int\prod_i d\beta_i f_i(\beta_i, s)$$
$$\times d\sigma^{\text{parton}}(\beta_i P_i, g^2(s), s) \qquad (4)$$

where s is anything of the order of the $P_i\cdot P_j$. The s evolution of the $f_i(\beta_i, s)$ is determined by the same anomalous dimensions and same integral-differential equation as is familiar for the effective quark distributions in leptoproduction.

The so-called "leading log" terms are not necessarily more important than non-leading logs or even constant terms. This is because large logs that appear before factorization of the form $\log s/m^2$ where m^2 is a light quark or gluon mass get massgaged into terms like $\log s/s_0$ in $d\sigma^{\text{parton}}$, which then turn into $\log s/s$ when renormalization group improved. That is to say all the large logs of perturbation theory (if all the $P_i\cdot P_j$ are comparable) are put into the scale dependence of the $f_i(\beta, s)$ and of $g^2(s)$. The residues of the logs that wind up in $d\sigma^{\text{parton}}$ are not distinguishable from the non-logarithmic terms that were in $d\hat{\sigma}$ all along.

There may be logs of ratios of large invariants in $d\sigma^{\text{parton}}$ which become large for certain momentum configurations. These also pose a threat to the validity of the thus-far-massgaged evaluation of $d\sigma^{\text{parton}}$. Yet further particle summations may be necessary. A sample problem might be in μ-pair production: if q is the pair momentum, $d\sigma^{\text{parton}}$ will contain $\log q_\perp^2/q^2$. The first serious effort at taming such effects (for $q^2\gg q_\perp^2$) is given in ref. 3. (See also ref. 9.)

Calculations to a given order of g^2 in $d\hat{\sigma}^{\text{parton}}$ are not necessarily self-consistent in the following sense: the hadronic cross section involves a convolution of $d\sigma^{\text{parton}}$ with real distribution and decay functions. These functions have radically different magnitudes and β dependence. So a parton process that is $O(g^4)$ may successfully compete with one of $O(g^2)$ if the appropriate f_i functions are significantly larger.

There are so many f_i functions (for all pairings of hadron with parton) that it appears to me to be essential to try to fit all relevant data at once. That is to say, in fitting a single experiment and ignoring other data, there is enormous flexibility in the several f_i's.

At the risk of insulting very many of my good friends, I don't think that careful phenomenology along these lines has been done yet.

I wish to mention an intriguing discussion in one of the works on factorization. In ref. 4 there is an analogous discussion for exclusive processes for color singlets. However, I must confess I do not at present understand the arguments.

I wish to return now to the central ideas of the basic proof. The potential problem in theories with massless particles that couple to each other (or in the limit of energies enormous compared to the masses) is that energy and momentum conservation do not prohibit a massless particle from becoming any number of massless particles as long as they all remain colinear. This is a potential disaster for Feynman diagrams because arbitrarily many of the internal particles may simultaneously go on shell.[8] A colinear phase space power counting argument suffices to show that only integrations over two particle intermediate states give rise to infrared divergences. More particles have insufficient phase space. Hence the $2PI$ parts are infrared finite. The heart of the argument goes as follows:

Let us consider the most dangerous part of phase space, *i.e.*, some subgraph with E external lines all roughly colinear with a lightlike momentum P ($P^2=0$) and I internal lines in the region of phase space where they, too, are approximately proportional to P. Let N_4 be the number of 4-point vertices and N_3 be the number of 3-point vertices. Introduce Sudakov or lightcone variables for each of the lines' momenta k_i:

$$k_i=\alpha_i m+\beta_i P+k_{i\perp} \tag{5}$$

where m is another lightlike vector such that $m^2=0$ and $m\cdot P\neq 0$, and $k_{i\perp}$ is defined by requiring $m\cdot k_{i\perp}=0=P\cdot k_{i\perp}$. We will consider the phase space as all α_i and $k_{i\perp}^2$ go to zero proportional to a common scale α. (At the end of the argument, the reader may wish to reconstruct why this is the right scaling to study.) Each internal line corresponds to a factor d^4k_i/k^2:

$$\frac{d^4k_i}{k^2}\sim\frac{m\cdot P\,d\alpha_i d\beta_i d^2\boldsymbol{k}_{i\perp}}{2\alpha_i\beta_i m\cdot P-\boldsymbol{k}_{i\perp}^2}\sim\frac{\alpha^2}{\alpha}\sim\alpha \tag{6}$$

for fixed β_i. (We will see later that the β_i must indeed be bounded away from zero.)

Each vertex has a δ-function for conservation of four-momentum. Hence, if all lines entering the vertex are roughly colinear to P, we lose one $d\alpha$ and one $d^2\boldsymbol{k}_{i\perp}$. Indeed, as a consequence, the 4-vertex scales like α^{-2}. With malice of forethought, let us leave open how the 3-vertex scales (*i.e.*, $\sim\alpha^{-?}$) allowing for further α dependence from spin factors.

The subgraph then scales like

$$\alpha^{I-2N_4-?N_3}. \tag{7}$$

"Conservation of ends" implies

$$E+2I=4N_4+3N_3. \tag{8}$$

Hence an all colinear subgraph with E external lines scales like

$$\alpha^{1/2(-E+(3-2?)N_3)}. \tag{9}$$

If $?>3/2$ we are doomed. Higher and higher order graphs are more and more singular as $\alpha\to 0$. In massless ϕ^3 theory, $?=2$; the consequent infrared inferno is related to how the particle actually acquires a mass from $\langle 0|\phi|0\rangle$. In covariant gauges of gauge theories, $?=2$ also, but cancellations given by the Ward identities ensure that all infrared divergences are logarithmic,[1] but the graphical structure is arbitrarily complicated. In axial gauge (*e.g.*, $A_\mu\cdot n^\mu=0$, $n^2>0$) $?=3/2$, so the index of α depends only on E. I will omit here the rest of the argument[6] which uses this fact (upon studying how the E lines can fit into the rest of the graph) and leads to the conclusion that the only divergences have index α^0 (*i.e.*, logarithmic) and occur only in the integral over d^4k' as indicated in Fig. 2, coming from the region $k'^2\approx 0$ and $k'\approx\beta$P.

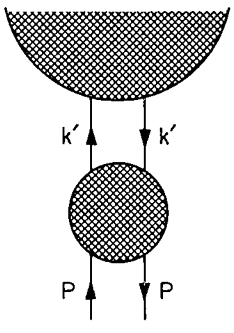

Fig. 2.

I would like to suggest how ?=3/2 in axial gauge. If the axial gauge gluon propagator is written as $\Delta_{\mu\nu}(k)/k^2$, then $g^{\mu\nu}\Delta_{\mu\nu}(k)\to 2$ as $k^2\to 0$, *i.e.*, there are only two polarizations that propagate as $k^2\to 0$. For physical polarizations, the dangerous physical processes vanish identically: One gluon cannot decay into two colinear gluons because of conservation of angular momentum. Likewise the quark–gluon coupling vanishes for colinear lightlike momentum because the vector coupling is helicity conserving on the quark line while overall helicity conservation would require a quark helicity flip. Careful evaluation for slightly off-shell momenta yields an $\alpha^{1/2}$ for each 3-vertex to soften the α^{-2} coming from the $\delta^4(\sum k_i)$.

The reduction of the d^4k' in eq. (3) to the $d\beta_i$ in eq. (1) uses the infrared finiteness of the $2PI$ parts I and K. The finiteness means we can define on-shell quantities

$$\tilde{I}(k')\equiv I(k')|_{k'^2=P^2=0}=I(\beta P) \qquad (10)$$

and

$$\tilde{K}(k,k')\equiv K(k,k')|_{k^2=P^2=0}=K(\beta P,k') \qquad (11)$$

The idea is to rewrite eq. (3) as

$$d\sigma=\left(\frac{1}{1-K}\right)\tilde{I}+\cdots \qquad (12)$$

where an integral d^4k' links $(1-K)^{-1}$ to $\tilde{I}$. $\tilde{I}$ depends only on β when k' is written as $\alpha m+\beta P+\boldsymbol{k}_\perp$. Instead of α, β and $\boldsymbol{k}_\perp$, choose β, $\boldsymbol{k}_\perp$ and k^2 as independent variables, then

$$\alpha=\frac{k^2+k_\perp^2+\beta^2P^2}{2\beta m\cdot P}. \qquad (13)$$

So as long as β is kept away from zero by the hadron kinematics (*i.e.*, no "wee" partons) we can do the integrals $dk^2d^2\boldsymbol{k}_\perp$ and define

$$\Gamma(\beta,P^2)=\int dk^2d^2\boldsymbol{k}_\perp(1-K)^{-1}. \qquad (14)$$

Γ is infrared divergent as $P^2\to 0$, but that is no matter; only an integral $d\beta$ links the process independent $\Gamma(\beta)$ to $\tilde{I}$. To complete the ... in eq. (12), define

$$\Delta I=I-\tilde{I} \qquad (15)$$

$$\Delta K=K-\tilde{K}. \qquad (16)$$

Then

$$d\sigma=\frac{1}{1-K}\left[\tilde{I}+\tilde{K}\left(\frac{1}{1-\Delta K}\right)\Delta I\right]+\left(\frac{1}{1-\Delta K}\right)\Delta I. \qquad (17)$$

The potential danger in the integral d^4k' linking $\tilde{K}$ to $(1/(1-\Delta K))\Delta I$ is eliminated by the fact that $(1/(1-\Delta K))\Delta I$ vanishes proportinal to k'^2 as $k'^2\to 0$, where k' is its left most matrix index. (When $1/(1-\Delta K)$ is expanded in pertubation theory—all we really ever do—the leftmost matrix is always a Δ.) Since the $\tilde{K}$ depends only on a β linking it to $1/(1-K)$, we again get an integral $d\beta$ of the universal Γ with a finite, calculable function. The last term $(1/(1-\Delta K))\Delta I$ is $O(P^2)$ because upon expanding $1/(1-\Delta K)$ the leftmost matrix is always a Δ. The leftmost matrix index is the external P.

My conclusions were presented in the middle of this talk. Let me close with the hope that the over-simplifications made here, while indeed barefaced lies, are of some pedagogical value and the serious reader will consult ref. 1–6.

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