

***Ab initio* electronic relaxation times and transport in noble metals**Jamal I. Mustafa,<sup>1,2</sup> Marco Bernardi,<sup>1,2,3</sup> Jeffrey B. Neaton,<sup>1,2,4,5</sup> and Steven G. Louie<sup>1,2,\*</sup><sup>1</sup>*Department of Physics, University of California, Berkeley, Berkeley, California 94720-7300, USA*<sup>2</sup>*Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*<sup>3</sup>*Department of Applied Physics and Materials Science, California Institute of Technology, Pasadena, California 91125, USA*<sup>4</sup>*Molecular Foundry, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*<sup>5</sup>*Kavli Energy NanoSciences Institute at Berkeley, Berkeley, California 94720, USA*

(Received 24 December 2015; published 5 October 2016)

Relaxation times employed to study electron transport in metals are typically taken to be constants and obtained empirically. Here, we use fully *ab initio* calculations to compute the electron-phonon relaxation times of Cu, Ag, and Au and find that they vary significantly on the Fermi surface, with values from  $\sim 15$  to 45 fs that are correlated with the Fermi surface topology. We compute room-temperature resistivities in excellent agreement with experiment by combining *GW* quasiparticle band structures, Wannier-interpolated band velocities, and *ab initio* relaxation times. We introduce an importance sampling scheme to speed up the convergence of resistivity and transport calculations.

DOI: [10.1103/PhysRevB.94.155105](https://doi.org/10.1103/PhysRevB.94.155105)

Copper, silver, and gold are noble metals with broad application in electronics, power generation, catalysis, and plasmonics. They have attracted interest since the early days of solid state theory, as their electronic structure deviates from the free-electron model that applies to the alkali metals. The Fermi surface (FS) of noble metals is not spherical as in free-electron theory but is deformed due to the proximity of the *d* bands to the free-electron-like *sp* band [1–4]. Electron scattering processes at the FS are of particular relevance for noble metal applications, as they regulate charge and heat transport [5,6]. At room temperature in relatively pure metals, scattering with phonons [7,8] is dominant, while scattering with defects and impurities is important at low temperatures and in alloys or samples of low purity.

Transport in metals can be understood heuristically with the Drude theory [9], which assumes free electrons with a constant (that is, band- and  $\mathbf{k}$ -independent) relaxation time (RT). Even in noble metals, where important deviations are expected from the Drude theory, resistive losses and optical experiments are routinely interpreted using constant relaxation times [9–11]. More advanced models, such as state-of-the-art *ab initio* calculations of resistivity and other transport properties [12,13], typically employ density functional theory (DFT) band structures combined with constant RTs inferred from experiment or estimated heuristically.

However, the RT of an electron in a Bloch state depends, in general, on the band and crystal momentum  $\mathbf{k}$ , a point that has so far been often neglected. Accurate calculations of electron RTs are computationally costly, as they require fine Brillouin zone (BZ) sampling [14–16], prompting adoption of simplified schemes that employ either a constant RT [17] or an average scattering strength [18]. An exception are recent transport calculations in Al [19] and two-dimensional materials [19–21] that included the band and  $\mathbf{k}$  dependence of the RTs.

As striking new experimental findings on noble metals emerge [22–24], predictive theories are needed to study electron scattering in these materials. For example, recent

experiments by Kim *et al.* [22] show a remarkable resistivity drop when a single crystal of Ag is doped with a small fraction of Cu, contrary to the intuition that the resistivity should increase upon alloying due to enhanced impurity scattering. While the constant-RT approach may be too simplistic, at present it is not known whether more refined theories are necessary to improve our quantitative understanding of transport in the noble metals.

Here we compute electron-phonon (*e-ph*) RTs and *GW* band velocities of Cu, Ag, and Au on fine  $\mathbf{k}$ -point grids. We develop a linear interpolation approach to sample  $\mathbf{k}$  points directly on the FS and use it to study the correlation of the RTs and velocities with FS topology. We find that the *e-ph* RTs are far from constant on the FS. They vary by a factor of 2–3 between their minima and their maxima, located in FS regions with different curvatures. The band velocity maps on the FS exhibit anticorrelation with the *e-ph* RT maps. We investigate approximations to compute the resistivity of Cu, Ag, and Au at room temperature and find excellent agreement (within  $\sim 10\%$ ) with experiment only when *ab initio* RTs and *GW* velocities are used. Our FS sampling approach dramatically speeds up the convergence of resistivity calculations and may be extended to study heat transport and thermoelectric effects in materials.

We carry out DFT calculations at experimental lattice constants (Cu: 3.61 Å, Ag: 4.09 Å, Au: 4.08 Å) using the local density approximation [25] and a plane-wave basis with the QUANTUM ESPRESSO code [26]. To determine the ground-state charge density, we use scalar-relativistic norm-conserving pseudopotentials [27] (including semicore *s* and *p* states), kinetic energy cutoffs of 240 Ry for Cu and 180 Ry for Ag and Au, and shifted  $10^3$ - $\mathbf{k}$ -point grids. Spin-orbit effects are neglected. The quasiparticle energies are computed within the  $G_0W_0$  and generalized plasmon pole approximations [28,29] using the BERKELEYGW package [30]. Our *GW* calculations are well converged using a 50-Ry cutoff for the dielectric matrix, which was evaluated on an  $8^3$ - $\mathbf{k}$ -point grid for interband transitions (finite  $\mathbf{q}$ ) and a  $16^3$ - $\mathbf{k}$ -point grid for intraband transitions ( $\mathbf{q} \rightarrow 0$ ) and using  $\sim 1000$  unoccupied bands, consistent with previous studies [31–33].

\*sglouie@berkeley.edu

The  $e$ -ph matrix elements are computed on fine  $\mathbf{k}$ -point grids with a procedure detailed in Ref. [16]. Briefly, we use density functional perturbation theory [34] to determine the phonon dispersions and displacement patterns, and the  $e$ -ph coupling matrix elements on a coarse  $4 \times 4 \times 4$   $\mathbf{q}$ -point grid. The band structures, phonon dispersions, and  $e$ -ph matrix elements are interpolated using maximally localized Wannier functions [35–37] constructed from Bloch states on a  $12^3$ - $\mathbf{k}$ -point grid. All quantities are interpolated using the EPW code [38,39] with our recently developed approach [14–16]. The velocities are calculated in the Wannier representation [40] using  $GW$  eigenvalues. For  $e$ -ph RT and resistivity calculations, we use the tetrahedron method [41] to obtain the FS as a collection of triangular facets, whose centroids constitute an ultrafine  $\mathbf{k}$ -point grid employed in the calculations to sample the FS.

Following Mahan [8], the  $e$ -ph transport relaxation scattering rates  $\Gamma_{n\mathbf{k}}$  (and their inverse, the RTs  $\tau_{n\mathbf{k}} = \Gamma_{n\mathbf{k}}^{-1}$ ) are computed with perturbation theory, due to the interaction with phonons:

$$\Gamma_{n\mathbf{k}} = \frac{2\pi}{\hbar} \sum_{m\mathbf{q}\nu} |g_{nm,\nu}(\mathbf{k},\mathbf{q})|^2 (1 - \cos \theta_{\mathbf{k},\mathbf{k}+\mathbf{q}}) \cdot [(N_{\nu\mathbf{q}} + f_{m\mathbf{k}+\mathbf{q}}) \delta(\epsilon_{n\mathbf{k}} + \hbar\omega_{\nu\mathbf{q}} - \epsilon_{m\mathbf{k}+\mathbf{q}}) + (N_{\nu\mathbf{q}} + 1 - f_{m\mathbf{k}+\mathbf{q}}) \delta(\epsilon_{n\mathbf{k}} - \hbar\omega_{\nu\mathbf{q}} - \epsilon_{m\mathbf{k}+\mathbf{q}})], \quad (1)$$

where  $g_{nm,\nu}(\mathbf{k},\mathbf{q})$  are  $e$ -ph coupling matrix elements for an electron in Bloch state  $|n\mathbf{k}\rangle$  (with quasiparticle energy  $\epsilon_{n\mathbf{k}}$ ) that scatters into a Bloch state  $|m\mathbf{k}+\mathbf{q}\rangle$  (with quasiparticle energy  $\epsilon_{m\mathbf{k}+\mathbf{q}}$ ) due to a phonon with polarization  $\nu$ , wave vector  $\mathbf{q}$ , and frequency  $\omega_{\nu\mathbf{q}}$ . Here,  $\theta_{\mathbf{k},\mathbf{k}+\mathbf{q}}$  is the scattering angle between  $\mathbf{k}$  and  $\mathbf{k}+\mathbf{q}$ . The two terms in brackets correspond to phonon absorption and emission, respectively, and the temperature dependence stems from the occupation factors  $f_{n\mathbf{k}}$  and  $N_{\nu\mathbf{q}}$ , for electrons and phonons, respectively (in this work, both the electrons and the phonons are at 300 K). The conductivity tensor from Boltzmann transport theory within the relaxation-time approximation is expressed as the FS integral [5,8]

$$\sigma_{\alpha\beta} = \frac{2}{(2\pi)^3} \frac{e^2}{\hbar} \sum_n \int dS_F \frac{v_{n\mathbf{k},\alpha} v_{n\mathbf{k},\beta}}{|\mathbf{v}_{n\mathbf{k}}|} \tau_{n\mathbf{k}}, \quad (2)$$

where  $v_{n\mathbf{k},\alpha}$  are Cartesian components ( $\alpha = x, y, z$ ) of the velocities and  $\tau_{n\mathbf{k}}$   $e$ -ph RTs. The area elements  $dS_F$  correspond to the area of the triangular facets that make up the FS within our interpolation scheme. The resistivity  $\rho$  is obtained from the trace of the inverse of the conductivity tensor, i.e.,  $\rho = \text{Tr}[\sigma^{-1}]/3$  [42].

The computed Fermi surfaces (see Fig. 1) exhibit well-known [1–4] topological features, including open-orbit regions with upward curvature (known as “necks”) near the  $L$  points of the BZ, spherical regions known as the “belly,” as well as flatter regions near the  $K$  points between the necks, and “bulges” where the FS approaches the BZ edge at the  $X$  points. The FS of Ag is noticeably different from those of Cu and Au, with much smaller necks, less pronounced bulges, and an overall more spherical shape due to the  $d$  bands being lower in energy compared to Cu and Au.

Figure 1 shows maps of the  $e$ -ph RTs and  $GW$  velocities calculated at  $\mathbf{k}$  points on the FS. For all three materials, we

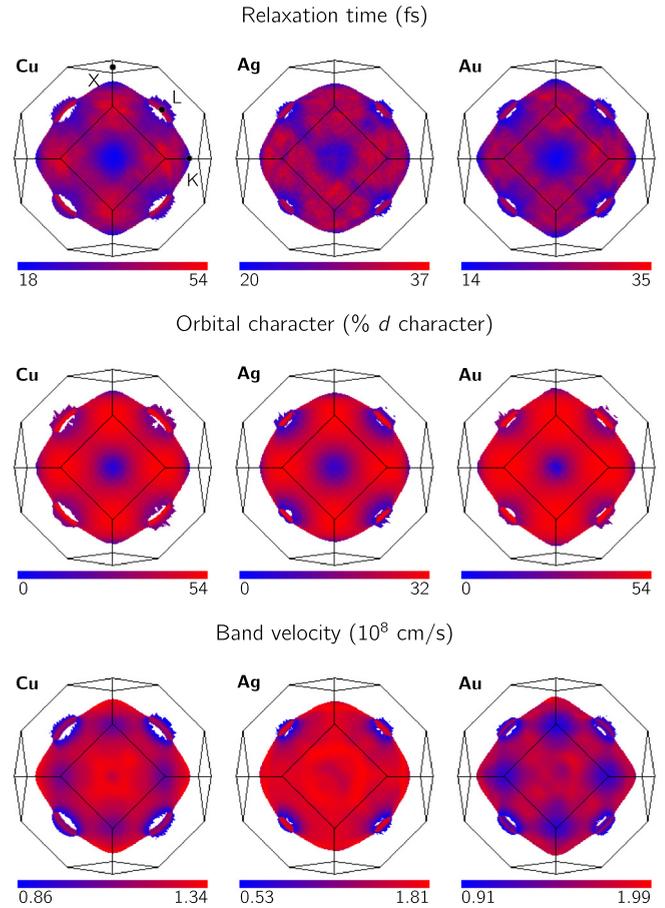


FIG. 1. Electron-phonon relaxation times (top), orbital character (middle), and band velocities (bottom) of states on the Fermi surface of Cu, Ag, and Au.

find large variations in the RTs, with the necks and bulges exhibiting shorter RTs than the belly and flat regions between the necks. The RTs vary the most in Cu and Au, with a factor of  $\sim 3$  difference between the minima (located near the  $L$  and  $X$  points) and the maxima (located near the  $K$  point), while the variation in Ag is smaller, with only a factor of  $\sim 2$  difference between minima and maxima. The velocities show an opposite trend, so that FS regions with longer (shorter) RTs are associated with lower (higher) velocities. Cu and Au exhibit moderate variations in the velocities, while Ag shows a large difference. The difference between the minimal velocities at the necks and the maximal velocities at the belly is a factor of  $\sim 2$  in Cu and Au and a factor of  $\sim 4$  in Ag.

We investigate the physical origin of the variation of the RTs and velocities on the FS. By mapping the orbital character of the electronic states on the FS (see Fig. 1), we find a correlation between the character of the state and its RT. Electronic states with a predominantly  $sp$  character, as found in the necks and bulges of the FS, are associated with shorter RTs, while states with large  $d$  character exhibit longer RTs.

In Fig. 2, we explore quantitatively the origin of the anisotropic RTs for the case of Ag, by choosing initial states with  $sp$  and  $sp + d$  character, respectively, and computing the coupling matrix elements  $|g|^2$  connecting these initial  $\mathbf{k}$  points

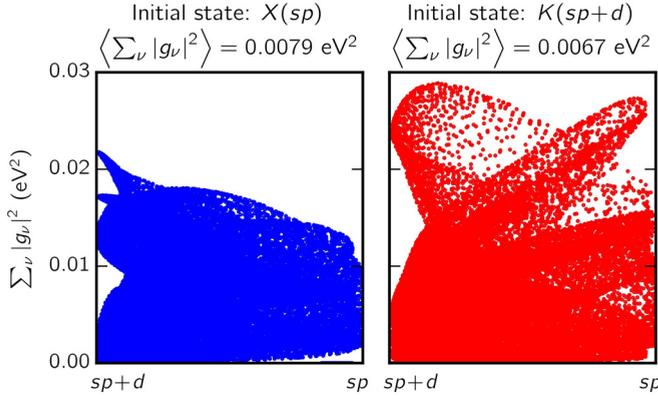


FIG. 2. Electron-phonon coupling matrix elements for an initial electronic state of predominantly  $sp$  character (left) and an initial state with large  $d$  character (right). Shown are the  $e$ -ph coupling matrix elements (summed over phonon polarizations) versus the character of the final electronic state involved in the  $e$ -ph scattering process. For each initial state, the average coupling matrix element is also given.

to all other final  $\mathbf{k}$  points on the FS. These matrix elements are the ones entering the calculation of the RTs for each of the initial states within our approach [Eq. (1)], and we now seek to categorize them based on the character of the initial and final states in the  $e$ -ph scattering processes.

For an initial state near  $X$  with dominant  $sp$  character, the coupling, as measured by the sum over phonon polarizations of the matrix element between the initial and the final states,  $\sum_{\nu} |g_{\nu}|^2$ , is strongest to final states with  $d$  admixture along the  $[110]$  directions. We find that coupling to other final states with some  $d$  character is in general weaker and strongly anisotropic, so that different groups of final  $d$ -like states exhibit  $|g|^2$  values distributed in multiple sets (see Fig. 2). As the fraction of  $d$  character of the final state decreases, the coupling decreases monotonically.

For an initial state near the  $K$  point with large  $d$  character, coupling is strong to one set of  $d$ -like states (along  $[110]$ ) and weak to most other  $d$ -like states. Two distinct sets of  $|g|^2$  values are found for such  $d$ - $d$  coupling (see Fig. 2). As the  $d$  character of the final state decreases, different final states exhibit different trends: for some states the coupling becomes stronger, while for others it becomes weaker. On average, the  $|g|^2$  is greater by a factor of 1.2 for the initial  $sp$  state near  $X$ , thus explaining the higher scattering rate (which is proportional to  $|g|^2$ ) and shorter RT compared to states with large  $d$  character, as discussed above. While further investigation is needed to more completely understand the role of the character of the initial and final state and the perturbation potential, our results demonstrate that the  $e$ -ph matrix elements are highly anisotropic and depend strongly both on the character of the initial and final states and on the wave vector connecting the initial and final states through the perturbation potential induced by the phonons. The combination of these effects results in the anisotropic RTs found here. The variation in the velocities is easier to explain. It can be understood by considering the curvature of the FS: flatter regions have lower velocities compared to more curved regions, given that the latter are associated with a higher gradient of the band at the Fermi energy.

TABLE I. Comparison of relaxation times and velocities computed by averaging on the Fermi surface (headings in angle brackets) versus the same quantities obtained within Drude theory (headings with subscript  $D$ ). The Fermi-surface-averaged relaxation times are computed using  $GW$  band structures, while the band velocities are given for both the DFT and the  $GW$  band structures for comparison. Drude values were obtained using Eq. (3) with resistivity experimental data [22,43] from single-crystal samples.

	$\tau$ (fs)		$v$ ( $10^8$ cm/s)		
	$\langle \tau_{nk} \rangle$	$\tau_D$	$\langle v_{nk}^{DFT} \rangle$	$\langle v_{nk}^{GW} \rangle$	$v_D$
Cu	37	27 <sup>a</sup>	1.13	1.19	1.57
Ag	30	41 <sup>b</sup>	1.45	1.55	1.39
Au	24	30 <sup>c</sup>	1.38	1.52	1.39

<sup>a</sup>From Ref. [43].

<sup>b</sup>From Ref. [22].

<sup>c</sup>Using the estimated SC resistivity.

To highlight the difference between the *ab initio* and the empirical data, we compare in Table I the Fermi-surface-averaged RTs ( $\tau_{nk}$ ) and velocities ( $v_{nk}$ ), obtained by averaging the *ab initio* data in Fig. 1, with the values derived from the free-electron Drude model,

$$\tau_D = \frac{m^*}{ne^2\rho}, \quad v_D = \frac{\hbar(3\pi^2n)^{1/3}}{m^*}, \quad (3)$$

where  $\tau_D$  and  $v_D$  are the empirical Drude RT and free electron velocity, respectively. The *ab initio* average quantities  $\langle X \rangle$  (with  $X = \tau, v$ ) are computed over the triangular facets that compose the FS, using  $\sim 10\,000$   $\mathbf{k}$  points and weighing by the facet area,

$$\langle X \rangle = \frac{\sum_t A_t X_{\mathbf{k}_c}}{\sum_t A_t}, \quad (4)$$

with  $A_t$  the facet area for a triangle  $t$  and  $\mathbf{k}_c$  its centroid. The Drude RTs  $\tau_D$  are calculated using the resistivity  $\rho$  measured on single-crystal (SC) samples [22,43] (see Table II); we further obtain the charge density  $n$  using one free electron per unit cell and take the effective mass  $m^* = m_e$ , consistent with

TABLE II. Resistivity values (in units of  $\mu\Omega$  cm) at 300 K computed using  $GW$  band velocities and different approximations for the relaxation times:  $\tau_{nk}$  are band and  $\mathbf{k}$  dependent relaxation times from *ab initio* data,  $\langle \tau_{nk} \rangle$  are Fermi surface averages of the  $\tau_{nk}$ 's, and  $\tau_D$  are empirical Drude relaxation times. Also listed are the resistivity values computed from DFT velocities and empirical Drude relaxation times. Experimental data for single-crystal (SC) and polycrystalline (PC) samples are listed for comparison.

	Computed $\rho$				Experimental $\rho$	
	$\tau_{nk}$	$\langle \tau_{nk} \rangle$	$\tau_D$	DFT + $\tau_D$	SC	PC
Cu	1.51	1.51	2.05	2.19	1.52 <sup>a</sup>	1.67 <sup>a</sup>
Ag	1.81	1.83	1.34	1.43	1.49 <sup>b</sup>	1.59 <sup>b</sup>
Au	2.24	2.24	1.82	2.01	2.0 <sup>c</sup>	2.26

<sup>a</sup>From Ref. [43].

<sup>b</sup>From Ref. [22].

<sup>c</sup>Estimated from the PC resistivity.

a simple Drude treatment of noble metals [9]. We find that the  $\langle\tau_{n\mathbf{k}}\rangle$  of Cu is 50% larger than the Drude value, while the  $\langle\tau_{n\mathbf{k}}\rangle$ 's of Ag and Au are 15%–25% smaller than their respective Drude RTs. The magnitude of the velocities averaged over the FS show the opposite behavior, with  $\langle v_{n\mathbf{k}}\rangle$  smaller in Cu and larger in Ag and Au than the Drude value. These results challenge the accuracy of the widely used empirical Drude model. In particular, since the tabulated Drude RTs are constants, they miss the complex trends and significant variation on the FS found here. The Drude RTs should thus be regarded as mere model parameters used to interpret transport and optical experiments in the absence of *ab initio* data.

Next, we use our results to compute the resistivity of Cu, Ag, and Au, with a range of approximations for the RTs and velocities. Table II compares the resistivities calculated with the *GW* velocities in combination with (i) full band- and  $\mathbf{k}$ -resolved RTs  $\tau_{n\mathbf{k}}$ , (ii) FS-averaged *ab initio* RTs  $\langle\tau_{n\mathbf{k}}\rangle$ , and (iii) empirical Drude RTs  $\tau_D$ . For comparison, we also compute the resistivities using the common approach combining DFT (as opposed to *GW*) velocities with Drude RTs. We compare our calculated resistivities with experiments at 293 K for SC samples. Polycrystalline (PC) experimental data [44] are also given. Since in pure SC samples at room temperature the source of resistivity is almost exclusively *e*-ph scattering, our approach using *e*-ph RTs is justified.

We first discuss the data in Table II obtained with our best approximation, namely, *ab initio*  $\tau_{n\mathbf{k}}$  and *GW* velocities. In the case of Cu, the agreement with SC experiment is excellent (within 1%). For Ag and Au we find resistivities that are higher by  $\sim 10\%$  than experiment. We attribute this small discrepancy to a number of sources, including the  $\sim 0.1$ -eV accuracy of the quasiparticle energies obtained via the *GW* method and a  $\sim 5\%$  uncertainty in the RTs depending on whether or not the cosine factor in Eq. (1) is employed [8]. Using the average RTs  $\langle\tau_{n\mathbf{k}}\rangle$  with *GW* velocities gives nearly identical resistivities as the band- and  $\mathbf{k}$ -resolved RTs, thus suggesting that approximate schemes using a constant RT should employ FS-averaged *ab initio* data rather than empirical data.

For all three materials, we find that the velocities obtained from the *GW* quasiparticle band structures are  $\sim 10\%$  higher than those obtained via DFT (see Table I), consistent with previous studies in Au [33] showing a large effect of *GW* on the *d*-band energies and *sp* band width. For Ag and Au, the lower DFT velocities, combined with Drude RTs that are larger than the *ab initio* values, yield resistivities in fortuitous agreement with experiment when using the DFT velocity plus  $\tau_D$  approach, a result of compensation of errors. For Cu, on the other hand, the DFT velocity plus  $\tau_D$  approach gives a resistivity almost 50% higher than experiment. Our results highlight the predictive character of calculations combining *GW* band structures and velocities with *ab initio* *e*-ph RTs.

The resistivity calculations in Table II are obtained with a novel approach that employs direct FS sampling instead of a homogeneous  $\mathbf{k}$  grid in the BZ, affording orders of magnitude speed up the convergence of the resistivity with respect to the number of  $\mathbf{k}$ -points. Figure 3 compares the convergence of the resistivity with respect to number of  $\mathbf{k}$ -points for uniform and importance sampling. Since we focus only on the convergence behavior with respect to number of  $\mathbf{k}$ -points, the resistivity, for illustration in Fig. 3, is determined using DFT velocities and

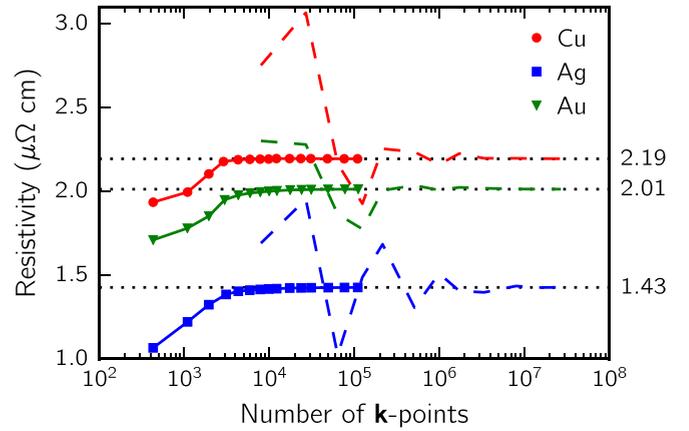


FIG. 3. Convergence of the resistivity computed with a uniform  $\mathbf{k}$ -grid (dashed lines) and using Fermi surface importance sampling (solid line with markers). The dotted lines indicate the converged values.

$\tau_D$ , though the trends are general. We find that the importance sampling scheme reaches convergence for  $\sim 10^4$   $\mathbf{k}$ -points and approaches monotonically the converged value. On the other hand, the commonly employed uniform grid requires  $\sim 10^7$   $\mathbf{k}$ -points to converge and fluctuates considerably with increasing number of  $\mathbf{k}$ -points as it approaches convergence. Since calculations of *e*-ph relaxation times are computationally expensive, the importance sampling scheme provides great advantages when *ab initio* relaxation times are employed. For the noble metal cases discussed here, importance sampling decreases the number of  $\mathbf{k}$ -points in the *e*-ph relaxation time calculation by a factor of  $10^3$ . When higher order terms in the Sommerfeld expansion are needed, e.g. to compute high temperature electrical resistivity or thermal conductivity and thermopower, additional isoenergy surfaces (at  $E_F \pm dE$ ) can be computed with a small computational overhead. Our approach is thus highly promising for computing transport properties beyond the resistivity.

In conclusion, we show that the *e*-ph RTs and velocities vary considerably on the FS of Cu, Ag, and Au. The correlation between RTs, velocities, and FS topology found here may extend to other classes of materials with complex Fermi surfaces. Our work points out the shortcomings of employing empirical RTs, DFT bands, and uniform  $\mathbf{k}$  grids to compute transport properties in materials. It further suggests that even energy-dependent (but not  $\mathbf{k}$ -dependent) RTs, also commonly employed in transport studies, would miss the complex interplay of band structure and scattering processes. These observations may extend to defect and impurity scattering, which will be the subject of future investigations. While complex nonequilibrium approaches are being explored for carrier dynamics, we argue that accurate *ab initio* state-dependent RTs may yield a close agreement with experiment in many cases of practical interest, even within the RT approximation of the Boltzmann transport equation.

This research was supported by the Theory of Materials Program at the Lawrence Berkeley National Laboratory through the Office of Basic Energy Sciences, U.S. Department of Energy, under Contract No. DE-AC02-05CH11231, which

provided the GW calculations and scattering rate simulations, and by the National Science Foundation under Grant No. DMR-1508412, which provided for basic theory and electron-phonon coupling matrix element calculations. Work at the Molecular Foundry was supported by the Office of Science, Office of Basic Energy Sciences, and by the U.S. Department

of Energy under Contract No. DE-AC02-05CH11231. This research used resources of the National Energy Research Scientific Computing Center, a DOE Office of Science User Facility supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231.

J.I.M. and M.B. contributed equally to this work.

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