unstable particles tend to stay small as we increase their mass.

We emphasize that these models should be distinguishable by measurements of electromagnetic form factors or diffraction scattering[4] involving higher Regge excitations. The charge radius in the oscillator model would increase as $J^{1/2}$ along a trajectory while the lack of centrifugal effects in the coupled-channel model suggest a universal size for a single trajectory.

*Work supported in part by U. S. Air Force Office of Scientific Research Grants Nos. 1282-67 and 68-1524.

[1]T. Regge, Nuovo Cimento 14, 951 (1959), and 18, 947 (1960).

[2]L. Susskind, Phys. Rev. 165, 1535 (1968).

[3]In the real case of meson physics we would probably want to choose P-wave couplings rather than S-wave since the $\pi\pi\rho$ system couples through $l=1$. This can be done by forming a vector from the indices describing the oscillator states and coupling it to the orbital angular momentum.

[4]This type of assumption has proved useful in correlating electromagnetic form factors with high-energy elastic diffraction scattering. For example, see T. T. Chou and C. N. Yang, Phys. Rev. Letters 20, 1213 (1968).

DUALITY AND THE HADRON SPECTRUM*

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A form of exchange degeneracy for mesons and baryons is derived from duality and the absence of resonances in exotic channels. The implications of this structure for the hadron spectrum are discussed.

From crossing, SU(3) symmetry, no resonances in exotic channels, and a weak form of duality,[1] we deduce a form of exchange degeneracy which implies specific patterns in the hadron mass spectrum. Nothing in our approach is in conflict with unitarity. In particular, pole residues factor and trajectories need not be linear. We find exchange-degenerate nonets for mesons, while the "natural" solution for baryons requires the degeneracy of a $\underline{1}\oplus\underline{8}$ of trajectories with one signature and an $\underline{8}\oplus\underline{10}$ of the other.[2] For mesons, we have all representations in $(\underline{3}^*\otimes\underline{3})\oplus(\underline{3}\otimes\underline{3}^*)$, while for baryons all representations in $\underline{3}\otimes\underline{3}\otimes\underline{3}$ appear. The result for mesons is essentially well known[3] and consistent with the quark model ($\underline{36}$, all L).[4] Our result for baryons is new and suggests the following quark model structure for the baryon states: $\underline{56}\oplus\underline{70}$, even L; $\underline{20}\oplus\underline{70}$, odd L. For mesons, we then deal with all representations in $(\underline{6}^*\otimes\underline{6})\oplus(\underline{6}\otimes\underline{6}^*)$; for baryons, all representations in $\underline{6}\otimes\underline{6}\otimes\underline{6}$ are included.

The scattering amplitude for a process is divided into two parts, a resonant amplitude and the background.[5] The background is assumed to be primarily diffractive while the remaining resonant piece may be expressed at high energies in terms of Regge trajectories. In "exotic" channels where we expect no resonances to appear, we assume that the imaginary part of the resonant amplitude is zero. This weak form of duality is satisfied by requiring cancellation of the imaginary parts of the contributing Regge trajectories.[3] This cancellation, together with factorization, can be accomplished only with definite patterns of exchange-degenerate trajectories. We work in the SU(3)-symmetric limit where we assume the nonexotic channels to be $\underline{1}$ and $\underline{8}$ for mesons and $\underline{1}$, $\underline{8}$, and $\underline{10}$ for baryons.

The reactions considered are

$$\begin{aligned} &M+M'\to M''+M''', \\ &M+B\to M'+B', \\ &M+B\to M'+\Delta, \\ &M+\Delta\to M'+\Delta'. \end{aligned} \tag{1}$$

M stands for an octet of mesons, and B and Δ correspond to an octet and decuplet of baryons.[6]

To derive exchange-degeneracy constraints on s-channel trajectories and residues, consider a definite t- or u-channel SU(3) representation a. The imaginary part of the nondiffractive piece of the scattering amplitude for large t or u is given by

$$\begin{aligned} &\mathrm{Im}A_{(t)}{}^{a}=\textstyle\sum_b(X_{ts})^{ab}\,\mathrm{Im}R_{(s)}{}^{b}, \\ &\mathrm{Im}A_{(u)}{}^{a}=\textstyle\sum_b(X_{us})^{ab}\,\mathrm{Im}R_{(s)}{}^{b}, \end{aligned} \tag{2}$$

where X_{ts} is the crossing matrix from the s to the t channel. The imaginary part of the Regge term $R_{(s)}{}^b$ is given by

$$\mathrm{Im}R_{(s)}{}^b=\sum_i\beta_i{}^b(s)t^{\alpha_i{}^b(s)},\quad t\ \text{large};$$
$$\mathrm{Im}R_{(s)}{}^b=\sum_i\tau_i{}^b\beta_i{}^b(s)u^{\alpha_i{}^b(s)},\quad u\ \text{large},\qquad(3)$$

where $\tau_i{}^b$ is the signature.

If a is an exotic SU(3) channel, duality implies

$$\mathrm{Im}A_{(t)}{}^a=0,\quad t\ \text{large};$$
$$\mathrm{Im}A_{(u)}{}^a=0,\quad u\ \text{large}.\qquad(4)$$

This condition requires that trajectories come in degenerate sets and gives for trajectories in a particular set the fundamental equations[7]

$$\sum_{b,i}(X_{ts})^{ab}\beta_i{}^b(s)=0,$$
$$\sum_{b,i}(X_{us})^{ab}\tau_i{}^b\beta_i{}^b(s)=0.\qquad(5)$$

For the scattering of identical mesons, these equations may be satisfied with the exchange-degeneracy patterns $(\underline{1},\underline{8})_1$ plus $(\underline{8})_2$. The subscripts 1 and 2 refer to different signature. Simultaneous consideration of $PP\to PP$ and $VV\to VV$ show, in fact, that the smallest solution is $(\underline{1},\underline{8})_1$ plus $(\underline{1},\underline{8})_2$. P and V denote octets of pseudoscalar and vector mesons. The natural associated quark model is $\underline{36}$, all L.

For baryons we find that these equations cannot be satisfied with fewer than three trajectories.[8] Either $(\underline{8})_1$ plus $(\underline{8},\underline{10})_2$ or $(\underline{8})_1$ plus $(\underline{8},\underline{1})_2$ will work and the octets will have fixed f/d ratios. These are particular cases of the four-trajectory solution $(\underline{1},\underline{8})_1$ plus $(\underline{8},\underline{10})_2$. In this more general solution, one f/d ratio is arbitrary and all other relative couplings are determined in terms of it. Specific choices decouple either the singlet or decuplet. The only other four-trajectory solutions are $(\underline{8},\underline{10})_1$ plus $(\underline{8},\underline{10})_2$ or $(\underline{1},\underline{8})_1$ plus $(\underline{1},\underline{8})_2$. Many solutions exist with more than four exchange-degenerate trajectories.

Starting with $\underline{56}$ $L=0$ and $\underline{70}$ $L=1$ (both of which appear to be present experimentally[9]) the following patterns for the baryon spectrum are suggested: (a) $(\underline{56},\underline{70})$ $L=0,2,4,\cdots$, and $(\underline{20},\underline{70})$ $L=1,3,5,\cdots$; (b) $(\underline{56},\underline{70})$ $L=0,1,2,\cdots$; (c) $(\underline{56},\underline{70})$ $L=0,2,4,\cdots$, and $\underline{70}$ $L=1,3,5,\cdots$.

We note that (1) case (a) is built entirely from one pattern, $(\underline{1},\underline{8})_1$ plus $(\underline{8},\underline{10})_2$, and the baryon spectrum written in the language of the quark model is as shown in Fig. 1. Cases (b) and (c) contain, in addition, the patterns $(\underline{8},\underline{10})_1$ plus $(\underline{8},\underline{10})_2$, $(\underline{1},\underline{8},\underline{8},\underline{10})_1$ plus $(\underline{1},\underline{8},\underline{10})_2$, or $(\underline{8})_1$ plus $(\underline{8},\underline{10})_2$. (2) The pattern $(\underline{1},\underline{8})_1$ plus $(\underline{8},\underline{10})_2$ contains precisely those representations in $\underline{3}\otimes\underline{3}\otimes\underline{3}$. In addition, case (a) contains precisely those representations in $\underline{6}\otimes\underline{6}\otimes\underline{6}$. (3) In all cases, a $\underline{70}$ $L=0$ representation degenerate with the $\underline{56}$ $L=0$ is required. We do not necessarily expect these to be closely degenerate in reality. Indeed, comparable considerations for the mesons result in $\pi(140)$-$X(965)$ degeneracy.

In conclusion, starting from SU(3), with no resonances in exotic channels, a weak form of duality requires that both meson and baryon trajectories appear in degenerate sets with definite allowed patterns of SU(3) representations in each set. One may imbed these degenerate trajectories into a quark model, the most "attractive" possibility being

baryons: $\underline{56},\underline{70}$ even L, $\underline{20},\underline{70}$ odd L;

mesons: $\underline{36}$ all L.

We wish to thank Professor Murray Gell-Mann for his encouragement and insistence on the importance of the problem of exchange-degenerate baryon trajectories.

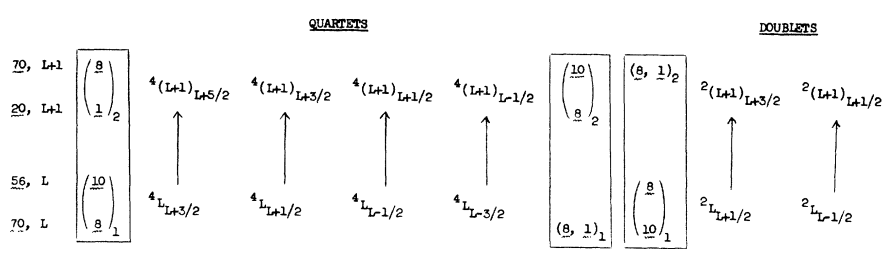

FIG. 1. Quark states contained in $\underline{56},\underline{70}$ even L; $\underline{70},\underline{20}$ odd L. The SU(3) representations contained in the boxes make up the exchange-degenerate pattern. The SU(3) representations written on one line comprise the SU(6) representation indicated on the extrme left. The vertical arrows represent a particular trajectory. Note that there are 16 quartet and 16 doublet trajectories.

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-68 of the San Francisco Operations Office, U. S. Atomic Energy Commission.

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[8]R. H. Capps, Phys. Rev. Letters 22, 215 (1969). His result differs from ours because he considers only one channel.

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ASYMPTOTIC BEHAVIOR OF ELECTROPRODUCTION STRUCTURE FUNCTION*

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The electroproduction function $F_2(-\nu/q^2)$, defined as the limit of the structure function $\nu W_2(q^2,\nu)$ for $\nu\to\infty$, ν/q^2 fixed, is experimentally observed to approach a constant for $-\nu/q^2\to\infty$. We derive this result from an integral representation of the scattering amplitude and the assumption of Regge behavior for the limit $\nu\to\infty$, q^2 fixed.

Bjorken[1] has recently shown that the electroproduction structure functions[2] $W_i(\kappa,\nu)$, $i=1,2$, defined by[3]

$$\frac{p_0}{2\pi}\int d^4x\, e^{iq\cdot x}\langle p|[J_\mu(x),J_\nu(0)]|p\rangle=\left(p_\mu-\frac{\nu q_\mu}{\kappa}\right)\left(p_\nu-\frac{\nu q_\nu}{\kappa}\right)W_2-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{\kappa}\right)W_1, \tag{1}$$

are expected to have simple behavior in the limit[4] $\kappa\to\infty$, $\rho\equiv-\nu/\kappa$ fixed. Bjorken derived

$$\lim_A \nu W_2(\kappa,\nu)=F_2(\rho),\quad \lim_A W_1(\kappa,\nu)=F_1(\rho), \tag{2}$$

where the limits satisfy $0\leq F_i(\rho)<\infty$. Present experiments[5] are in agreement with (2) and, furthermore, indicate the a priori rather surprising property

$$F_2(\rho)\xrightarrow[\rho\to\infty]{}\text{const}\neq 0. \tag{3}$$

In this note we shall present a derivation of (3) and estimate the value of the constant. The essential idea is to relate the A limit to the Pomeranchuk-dominated Regge limit[6] $\nu\to\infty$, κ fixed.[7] The derivation is not rigorous, but is valid in the absence of pathologies.

Current Regge-pole theory[8] implies

$$W_2\xrightarrow[R]{} w_2(\kappa)\nu^{\alpha-2},\quad W_1\xrightarrow[R]{} w_1(\kappa)\nu^{\alpha},$$

where α is the $t=0$ intercept of the leading appropriate Regge trajectory. Assuming that the Pomeranchuk trajectory with $\alpha=1$ dominates, we have

$$W_2\xrightarrow[R]{} w_2(\kappa)\nu^{-1}, \tag{4}$$

$$W_1\xrightarrow[R]{} w_1(\kappa)\nu, \tag{5}$$

where $w_i\neq 0$.

Our essential assumption is that T_2 satisfies the Deser-Gilbert-Sudarshan (DGS) representation[9]

$$W_2(\kappa,\nu)=\int_0^\infty da\int_{-1}^1 db\,\kappa\sigma_2(a,b)\times\delta(\kappa+2b\nu+b^2-a)\epsilon(\nu+b). \tag{6}$$