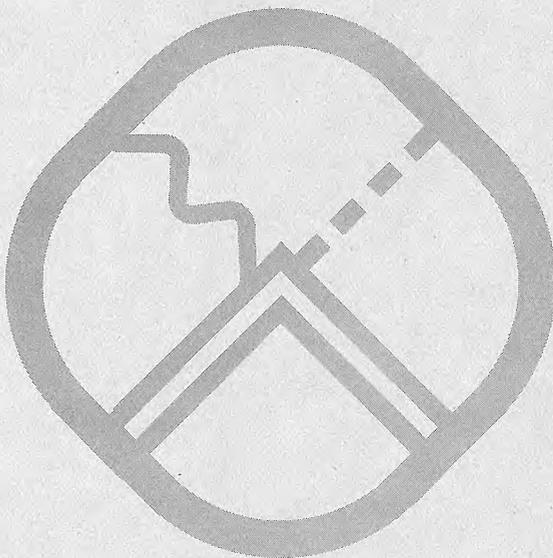


**A RADIO FREQUENCY SYSTEM FOR A
300 GEV PROTON SYNCHROTRON**

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JANUARY 19, 1961



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Abstract

An investigation has been made of a radio-frequency system for a 300-Gev proton synchrotron, using the constant - frequency, phase - shifting method.

I Introduction

In a 300-GeV proton synchrotron, the orbital period is large compared with the time constant of an ultra-high frequency, high Q cavity. It then becomes more economical in terms of radiofrequency power and cavities, to operate the radio-frequency system at a constant frequency, and to shift the phase of the rf voltage during a fraction of an orbital period so as to be correct for the next traversal of the particles through the rf cavity. In this method¹⁾, the harmonic order and the spacing between bunches change as the velocity of the protons increases.

II Accelerating Units

The radiofrequency power requirements are determined by the power needed to replace the energy stored in the rf accelerating structure in the time interval allowed for shifting the phase. For this reason it is desirable to choose the rf frequency to be high, so as to reduce the energy stored in the rf structure. The frequency is chosen to be in the 400 to 500 m.c. range, as power sources exist in this frequency range with high average and peak power capability.

The rf system may be designed to be either a travelling wave or a standing wave structure. The travelling wave structure requires a constant power level from the power source, while the standing wave structure requires a higher power level when the phase is being shifted by a large amount, and a lower power level during the time the particles are traversing the cavity and the phase is remaining constant. The travelling wave structure also has the advantage that the rf power source may always operate into a matched load. For these reasons the rf structure is chosen to be a travelling wave structure.

The form of the rf structure would probably be a disk loaded waveguide similar to the structure used for electron linear accelerators. The structure would be

¹⁾Acceleration of Protons at Constant Frequency by Phase Shifting by Kenneth W. Robinson, CEA-82, (Dec. 25, 1960).

about 20 inches in diameter and the spacing between the loading disks would be about 8 inches.

We choose the time in which the phase is shifted to be 1/3 of the orbital period, so as to allow particles to be accelerated in 2/3 of the total circumference. The orbital period in the proposed 300 Bev synchrotron is about 27 microseconds, so the time in which the phase is shifted is 9 microseconds.

We assume that an accelerating structure, 5.5 meters long, may be fitted into a straight section of the 300 Bev synchrotron. For the energy to travel the length of this rf structure in 9 microseconds, then requires a group velocity of .002 of the velocity of light.

It seems reasonable with this type of cavity structure to operate with maximum accelerating gradients of 1 Mev per meter. This is estimated to require a stored energy of about .4 joules per meter. The power required is given by

$$P = dU/d\ell \times v_g = .4 \times 2 \times 10^{-3} \times 3 \times 10^8 = \underline{240 \text{ kw}}$$

The attenuation length of this structure is given by

$$l_0 = \frac{Qv}{\pi f} g$$

We take the basic Q as the structure to be 25,000, and the frequency to be 400 m.c. Then

$$l_0 = \frac{2.5 \times 10^4 \times 2 \times 10^{-3} \times 3 \times 10^8}{\pi \times 4 \times 10^8} = 12.0 \text{ meter}$$

The total acceleration produced by the structure is given by

$$V = E_0 \int_0^{\ell} e^{-z/l_0} dz = 1 \times \int_0^{5.5} e^{-z/12} dz = 4.4 \text{ Mev}$$

The maximum rate of energy gain in the proposed 300 Bev synchrotron is about 8 Mev per turn. In order to allow a margin for phase stability, it should be sufficient to have three of these units developing a maximum accelerating voltage

per turn of 13.2 Mev.

Injection of particles into the main synchrotron takes place at an energy of 10 Bev. The rate of rise of the magnetic field will be small at that time. The maximum energy acceptance is given by:

$$\frac{\Delta U}{U_0} = \pm \left[\frac{2k}{\pi} \left(\frac{U_0}{mc^2} \right)^2 \frac{V}{U_0} \right]^{1/2}$$

The harmonic order $k = 11,000$, $\left(\frac{U_0}{mc^2} \right) = 11.7$ and $V = 13.2$ Mev.

$$\frac{\Delta U}{U_0} = \pm \left[\frac{2 \times 1.1 \times 10^4}{\pi} \times (11.7)^2 \times \frac{1.32 \times 10^7}{1.09 \times 10^{10}} \right]^{1/2} = \pm .0034$$

Since the fractional energy deviations from the correct value of the injected protons is expected not to exceed .001, most of the injected particles will be accepted into phase stable synchrotron oscillations.

The frequency of the synchrotron oscillations at injection is given by:

$$\begin{aligned} \frac{\omega_s}{\omega_0} &= \left[\frac{k}{2\pi} \left(\frac{mc^2}{U_0} \right)^2 \frac{V}{U_0} \right]^{1/2} \\ &= \left[\frac{1.1 \times 10^4}{2\pi} \frac{1}{(11.7)^2} \times \frac{1.32 \times 10^7}{1.09 \times 10^{10}} \right]^{1/2} = .124 \end{aligned}$$

The frequency of synchrotron oscillations at injection is then about 1/8 of the orbital frequency. This frequency is sufficiently small that the above calculations are valid whether the rf units are distributed around the synchrotron, or located in the same portion of the circumference.

III Phase Control

The control system for the radiofrequency system would use measurements of the phase and radial position of the circulating proton beam to determine the amount by which the phase is shifted between successive revolutions.

A block diagram of a possible control system is shown in Figure 1. The amount of phase shift is determined by the length of the gating pulse applied to the circuits which shift the phase of the oscillator in a forward or backward direction. The information which determines the amount by which the phase is to be shifted every revolution is stored in the control circuit and modified by the radial position of the circulating beam so as to maintain the beam in the center of the aperture. Also, the phase of the circulating beam relative to the phase of the rf fields is measured, and the next phase shift is modified so as to eliminate the phase error on the next traversal of the particles through the rf system. The control system must also determine when the amount by which the phase is shifted reaches 180° , and then change the phase shift to 180° with the opposite sign.

For purposes of analysis, we assume that the rf accelerating structures are all located in the same region of the circumference, and are all controlled by one control system. The operation of the rf system may be described in terms of matrices, which relate the error in the phase of the circulating beam relative to the rf system ($\Delta\phi$), the fractional error in the energy of the particles ($\Delta U/U_0$), and the error in the amount by which the phase is shifted every revolution ($\Delta\phi_0$). The matrix for the traversal of the particles through the rf system is given by:

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{V}{U_0} \cos \phi_s & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\phi \\ \frac{\Delta U}{U_0} \\ \Delta\phi_0 \end{pmatrix}$$

V is the peak acceleration of the rf system, ϕ_s is the synchronous phase angle.

The matrix for the magnet structure is:

$$\begin{pmatrix} 1 & 2\pi k \left(\frac{mc^2}{U_0} \right)^2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The matrix for the control system is chosen to be:

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & K_1 \frac{U_0}{V \cos \phi_s} & 1 \end{pmatrix}$$

This matrix has the effect shifting the phase by the amount of the phase error on the previous revolution, also the phase shift per turn is modified by the measured error in radial position by the amount $K_1 \frac{U_0}{V \cos \phi_s} \cdot \frac{\Delta U}{U_0}$.

The characteristics of the system are determined by multiplying the matrices together in the following order

$$M_M \quad M_C \quad M_{RF}$$

and then determining the characteristic values of the total matrix. It is found that one value is zero which means that the error vector corresponding to that value is eliminated in one revolution. The other two values are given by:

$$\lambda = 1 - \frac{\omega_s^2}{2} \pm \left[\omega_s^4 - 4k_1 \right]^{1/2}$$

$$\omega_s = \left[\frac{k}{2\pi} \left(\frac{mc^2}{U_0} \right)^2 \frac{V}{U_0} \cos \phi_s \right]^{1/2},$$

is the angular frequency of synchrotron oscillations of individual particles.

In order to obtain the maximum damping rates for both values, we choose

$$K_1 = \frac{\omega_s^4}{4}.$$

The other characteristic values are both then

$$\lambda = 1 - \frac{\omega_s^2}{2}$$

At injection into the large ring

$$1 - \frac{\omega_s^2}{2} \approx .8$$

The error vectors corresponding to these characteristic values, are then damped about 20% per revolution. Since all errors are quickly damped, the phase shift in one revolution does not have to be particularly accurate.

The exact frequency of the oscillator is not important and the frequency may be adjusted to obtain the proper phase velocity in the rf structure. The frequency of the oscillator should have a short term stability so as to avoid phase changes between different portions of the orbital period which change from one revolution to the next. Since the harmonic order is about 10^4 , the required frequency stability would probably be one part in 10^5 to 10^6 . The time interval over which this stability is required is a period long compared with the period of synchrotron oscillations, perhaps about one millisecond. The effects of frequency variations in the oscillator could also be eliminated by a more complex control system which adjusts the phase continuously during the time that the particles are traversing the rf structure, according to measurements of phase error made during the previous traversal of the particles through the rf structure.

IV Beam Loading

If the intensity of the circulating beam in the accelerator becomes sufficiently large, it will modify the amplitude and phase of the rf voltage during the traversal of the beam through the rf structure. Small changes in amplitude and phase during one orbital period are not important, as they will be approximately the same on every revolution, and the particles will automatically compensate for the change by a change in the synchronous phase angle. If the beam causes the amplitude to be reduced appreciably, the particles may be lost from phase stability. For an estimate of the allowable intensity, we choose that the beam should reduce the total amplitude of the accelerating voltage by 2 Mev. The voltage induced in a single rf structure by the circulating beam is given by:

$$V = I_o R_s \int_0^{\ell} \left[1 - e^{-z/\ell_o} \right] dz = 1.08 I_o R_s$$

The shunt impedance of the structure is given by:

$$R_s = \frac{E_o^2}{\frac{dU}{d\ell}} \cdot \frac{Q}{2\pi f} = \frac{10^{12} \times 2.5 \times 10^4}{.4 \times 2\pi \times 4 \times 10^8} = 2.5 \times 10^7 \Omega/\text{meter}$$

Since the synchronous phase angle of the particles is about 45° , the allowable induced voltage in one rf structure is $\frac{2}{3} \times \sqrt{2} = .94$ Mev.

The allowable current is then

$$I_o = \frac{.94 \times 10^6}{1.08 \times 2.5 \times 10^7} = .035 \text{ amperes}$$

This corresponds to an intensity of about 4×10^{12} particles per pulse. In order to attain higher intensities, it would be necessary to modify the rf system. This could be done by increasing the group velocity of the rf structure and increasing the rf power to each structure so as to maintain the same accelerating voltage. If

the group velocity of the structure were increased by a factor of 5, the rf power required for each structure, to maintain the same accelerating voltage, is increased by a factor of about 3.65 to 880 kw maximum. The voltage induced in one structure is reduced by a factor of 4.5. The allowable intensity would then be increased to about 2×10^{13} particles per pulse.

V Bunching at High Energy

The very high frequency of the rf system could possibly be useful, as the longitudinal spread of the bunches of particles will be small, and will thus result in the production of secondary particles at targets in very short pulses. By the use of fast counters and coincidence circuits, the secondary particles may then be identified by time of flight.

We assume that the rf voltage at injection is reduced, so that the maximum phase stable fractional energy variation is $\pm .001$. Most of the injected particles will then be accepted into phase stable synchrotron oscillations. The phase spread at 300 Bev is estimated by assuming that the integral $\oint \Delta U d\phi$ is conserved throughout the acceleration cycle.

$$\Delta\phi \approx \pm \left[\frac{128 k \alpha (\Delta U_1)^2}{\pi U_0 V \cos \phi_s} \right]^{1/4}$$

The momentum compaction factor $\alpha \approx 1/1500$

$$\Delta\phi \approx \pm \left[\frac{128 \times 1.1 \times 10^4 \times 6.7 \times 10^{-4} \times 10^{14}}{\pi \times 10^{11} \times 1.32 \times 10^7 \times .707} \right]^{1/4} = \pm .42$$

At a frequency of 400 m.c., this corresponds to a time spread of $\pm .17$ millimicroseconds. It would then be possible to measure time of flight differences in the range .17 to 2.33 millimicroseconds, to an accuracy of $\pm .17$ millimicroseconds.

VI Cost

The cost of the radiofrequency system would be determined principally by the cost of the major components, the rf power units and loaded waveguide structures. The rf power units would probably be designed using an RCA superpower tube, similar to the A-2346. The cost of the complete rf power units should not be more than \$1000 per kilowatt. The disk loaded waveguide structure may cost about \$5000 per foot. The total cost of the rf system for low intensity operation would then be about one-million dollars.

VII Possible Application to Booster Ring

It would also be possible to use the constant frequency, phase shifting method to accelerate particles in the booster ring. Since the velocity of the particles changes by about a factor of 3 during acceleration, the particles which are to be accelerated could only occupy one-third of the circumference at injection into the small ring. Also, the phase velocity of the rf structure must vary by a factor of 3 during the acceleration cycle. Since the rf structure has a very high dispersion, the phase velocity may be varied by a small change in frequency during the acceleration cycle. In order to have a minimum phase velocity of $1/3$ of the velocity of light, the spacing of the loading disks must be less than $1/6$ of a wavelength.

The minimum orbital period in the booster ring is about 1.5 microseconds. We choose to shift the phase in a time interval of 0.5 microseconds, and assume that a cavity 1.5 meters long may be put into one straight section. The group velocity in the structure must be then .01 of the velocity of light. In order to change the phase velocity by a factor of 3 should require a fractional frequency shift of about .03.

If the booster ring is operated at 10 cycles per second, with a biased sinusoidal waveform, the maximum rate of acceleration is about 210 kV per turn. We assume a maximum accelerating voltage of 300 kV per turn. The accelerating fields in the structure are then about 200 kV per meter. The energy stored in the structure is about

$$(.4) \times (.2)^2 = .016 \text{ joules/meter}$$

The rf power required is given by:

$$P = dU/d\ell \times v_g = .016 \times 10^{-2} \times 3 \times 10^8 = \underline{48 \text{ kw}}$$

The control system required for the rf system of the booster ring would be somewhat more difficult due to the much shorter interval of time in which the phase must be shifted.

FIGURE 1

BLOCK DIAGRAM OF CONTROL SYSTEM

