Radiative Effects in Meson-Nucleon Scattering*

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An integral equation is derived which sums the contributions of a certain (infinite) set of radiative corrections to lowest-order meson-nucleon scattering. This equation for the S matrix is examined and an approximate treatment given. The resulting scattering amplitude is exhibited in the Thomson and high-energy limits.

INTRODUCTION

ONE of the shortcomings of perturbation theory has been that only finite subsets of the terms relevant to a given process can be evaluated in practice. On occasion it is possible to include an infinite subset and thus partially avoid the perturbation approach.

In the present account, such an infinite subset of the contributing terms to lowest-order meson-nucleon scattering will be summed to give an integral equation for the S matrix for the process. The terms in question will be seen to consist of all radiative corrections in which a number of mesons are emitted by the nucleon, the lowest-order scattering then taking place, followed by the reabsorption of the virtual quanta in inverse order to that of emission.

DERIVATION OF THE EQUATION

In order to derive the equation in question, we start with a quantity closely related to S, the Green's function of the meson-nucleon system. Its symbolic solution in terms of one-particle quantities has been previously shown to be, in the symmetric $ps(ps)$ theory[1] (which we shall use),

$$G_{MN}-\Delta_+G_+=-ig^2G_+\Delta_+[\gamma_5G_+\gamma_5'+\gamma_5'G_+\gamma_5+\delta\Gamma_5/\delta g\phi']\Delta_+G_+, \quad (1)$$

where G_+, Δ_+ are the modified nucleon and meson propagation functions, respectively, and Γ_5 is the vertex operator and is an isotopic spin vector.

More explicitly, Eq. (1) reads

$$(\xi_1|G_{MN}-\Delta_+G_+|\xi_2)=-ig^2G_+\int(d\eta)(d\eta')\Delta_+(\xi_1,\eta)\times[\Gamma_5(\eta)G_+\Gamma_5(\eta')+\Gamma_5(\eta')G_+\Gamma_5(\eta)+\delta\Gamma_5(\eta)/\delta g\phi(\eta')]\Delta_+(\eta',\xi_2)G_+. \quad (2)$$

The quantity inside the brackets is essentially the S matrix minus the unit matrix, $S-1$, which we shall denote by T.[2]

We now make our first approximation in taking the vertex operator, Γ_5, to satisfy[3]

$$\Gamma_5(\eta)=\gamma_5-ig^2\int(d\eta')(d\eta'')\gamma_5(\eta')\times G_+\Gamma_5(\eta)G_+\gamma_5(\eta'')\Delta_+(\eta'',\eta'), \quad (3)$$

from which it follows that

$$\delta\Gamma_5(\eta)/\delta g\phi(\eta')=-ig^2\int(d\xi)(d\xi')\gamma_5(\xi)G_+\times[\Gamma_5(\eta')G_+\Gamma_5(\eta)+\Gamma_5(\eta)G_+\Gamma_5(\eta')+\delta\Gamma_5(\eta)/\delta g\phi(\eta')]G_+\gamma_5(\xi')\Delta_+(\xi',\xi). \quad (4)$$

We see that $\delta\Gamma_5/\delta g\phi$ reproduces the structure of T, and, therefore,

$$T=\Gamma_5G_+\Gamma_5'+\Gamma_5'G_+\Gamma_5-ig^2\int(d\xi)(d\xi')\times\gamma_5(\xi)G_+TG_+\gamma_5(\xi')\Delta_+(\xi',\xi). \quad (5)$$

This is the promised integral equation for the scattering. Its inclusion of the processes mentioned earlier may be verified by iteration, for example.

A further simplification will be made in replacing Γ_5 by γ_5. In coordinate space, Eq. (5) now reads

$$T_{ij}(x\xi,x'\xi')=\tau_i\tau_j\gamma_5G_+(x,x')\gamma_5\delta(x-\xi)\delta(x'-\xi')+\tau_j\tau_i\gamma_5G_+(x,x')\gamma_5\delta(x-\xi')\delta(x'-\xi)-ig^2\tau_k\gamma_5\int(dx'')(dx''')G_+(x,x'')T_{ij}(x''\xi;x'''\xi')\times G_+(x''',x')\gamma_5\tau_k\Delta_+(x,x'), \quad (6)$$

and its Fourier transform is

$$(2\pi)^{-4}T_{ij}(p_1p_2;p_1'p_2')=[\tau_i\tau_j\gamma_5G_+(p_1+p_2)\gamma_5+\tau_j\tau_i\gamma_5G_+(p_1-p_2')\gamma_5]\delta(p_1+p_2-p_1'p_2')-ig^2(2\pi)^{-4}\tau_k\gamma_5\int(dk)G_+(p_1-k)(2\pi)^{-4}\times T_{ij}(p_1-k,\,p_2;\,p_1'-k,\,p_2')G_+(p_1'-k)\gamma_5\tau_k\Delta_+(k). \quad (7)$$

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[1] S. Deser and P. Martin, Phys. Rev. **90**, 1075 (1953). The same notation will be used here.

[2] The connection between the Green's function and the S matrix in the meson-nucleon situation is worked out in detail by Karplus, Kivelson, and Martin, Phys. Rev. **90**, 1072 (1953).

[3] The assumption of Eq. (3) is equivalent to the inclusion of only those graphs for the vertex operator in which virtual mesons are emitted and reabsorbed by the nucleon in a "nesting" pattern about the vertex point. The equation has in fact been employed by S. F. Edwards, Phys. Rev. **90**, 284 (1953), in an attempt to determine Γ_5.

In the above, free-field G_+ and Δ_+ are used. In center-of-mass coordinates, $P=p_1+p_2$ and $p=\frac{1}{2}(p_1-p_2)$, the equation simplifies. Since T is a function of $x-x'$ only, in its center-of-mass dependence, or, alternately, since T is defined on the energy shell, we employ the fact that

$$T(p_1p_2;\,p_1'p_2')=T(pP;\,p'P')=T(p,p';\,P)(2\pi)^4\delta(P-P') \quad (8)$$

to write our equation as

$$T_{ij}(p,p';\,P)=\tau_i\tau_j\gamma_5G_+(P)\gamma_5+\tau_j\tau_i\gamma_5G_+(p+p')\gamma_5 -ig^2(2\pi)^{-4}\gamma_5\tau_k\int G_+(\tfrac{1}{2}P+p-k) \times T_{ij}(p-\tfrac{1}{2}k,\,p'-\tfrac{1}{2}k;\,P-k)G_+(\tfrac{1}{2}P+p'-k) \times\gamma_5\tau_k\Delta_+(k)(dk). \quad (9)$$

In this form, the equation couples various spin and isotopic spin components of T_{ij}. The isotopic dependence may be separated out by writing

$$T_{ij}=\delta_{ij}T_1+\tfrac{1}{2}(\tau_i\tau_j-\tau_j\tau_i)T_2\equiv\delta_{ij}T_1+\tau_{ij}T_2. \quad (10)$$

Then, since

$$\tau_k\delta_{ij}\tau_k=3\delta_{ij};\quad \tau_k\tau_{ij}\tau_k=-\tau_{ij}, \quad (11)$$

the equation separates with regards to the isotopic spin

$$T_1=\gamma_5G_+(P)\gamma_5+\gamma_5G_+(p+p')\gamma_5 -3ig^2(2\pi)^{-4}\int\gamma_5G_+T_1G_+\gamma_5\Delta_+, \quad (12a)$$

$$T_2=\gamma_5G_+(P)\gamma_5-\gamma_5G_+(p+p')\gamma_5 +ig^2(2\pi)^{-4}\int\gamma_5G_+T_2G_+\gamma_5\Delta_+. \quad (12b)$$

A similar treatment might be employed for the spin parts; there, one obtains coupled equations for the various coefficients A, B, $\cdots$ of T in an expansion in the elements of the spinor algebra: $T=A\cdot 1+B_\mu\gamma^\mu+\cdots$. We shall make use of this breakup at a later stage.[4]

We now suppose that the effect of the radiative corrections can be approximated in the following way, as to structure: the scattering amplitudes T_i will become modulating, or scale, functions times the ordinary Born approximation amplitudes. The overall effects of the radiative corrections will then be contained in these scale functions. We assume then, for each T_i, that

$$T_i(p,p';\,P)=\lambda_i(p_2,p_2')T_i^{(0)}(p,p';\,P), \quad (13)$$

where

$$T_{1,2}^{(0)}=\gamma_5G_+(P)\gamma_5\pm\gamma_5G_+(2p)\gamma_5.$$

[4] A possible treatment to be noted in this connection assumes that $T=\gamma_5f(p,p';\,P)$. In this case, some progress may be made in actually solving the integral equation. This assumption seems unphysical, however.

When this form for T is substituted under the integral in Eq. (12), the λ_i will not be functions of k so that the result will be[5]

$$T=\lambda T^{(0)}=T^{(0)}+\lambda\int KT^{(0)}. \quad (14)$$

K is the kernel as derived from Eq. (12). Thus,

$$\lambda=\left(1-\int KT^{(0)}/T^{(0)}\right)^{-1}, \quad (15)$$

which is the usual form obtained when damping effects are included.

The actual integrals occurring in Eq. (14) are not feasible in closed form in the general case.[6] We shall therefore restrict ourselves to two limiting cases in which they can be performed: the Thomson and the high-energy regions.

THE THOMSON LIMIT

Here the calculations needed have been performed, and the results given, by Ashkin, Simon, and Marshak.[7] Using these, we find that:

$$\int K_1T_1^{(0)}=1.2(2\pi)^{-2}(2m)^{-1}\alpha';$$

$$\int K_2T_2^{(0)}=-0.22(2\pi)^{-2}(2m)^{-1}\alpha' \quad (16)$$

$$T_1^{(0)}=-4m(4m^2-\mu^2)^{-1};\quad T_2^{(0)}=2\mu(4m^2-\mu^2)^{-1},$$

where $\alpha'\equiv g^2/4\pi$. Taking a conventional value for α' of 10, we find that

$$\lambda_1=0.87,\quad \lambda_2=0.73. \quad (17)$$

Thus, in the Thomson limit, the damping of the Born approximation result is relatively small, as might be expected in this nonrelativistic approximation in which all motion becomes negligible.

THE HIGH-ENERGY REGION

We now turn to the opposite extreme, that of high-energy incoming mesons. In this case we neglect the meson mass μ in comparison with its energy ω; such an approximation might be expected to hold above, say, $\omega=350$ Mev. In addition, we shall restrict ourselves to forward scattering; although this is not a necessary restriction,[8] it greatly simplifies the work. The quali-

[5] This form and that of Eq. (15) are to be understood symbolically, as the ordering of the Dirac matrices in the integral must be taken into account; in general, the λ_i will include such matrices as well.

[6] They are precisely those arising in the calculation of lowest-order radiative corrections to scattering, as is obvious from the fact that $\int KT^{(0)}$ is essentially the T for that process.

[7] Ashkin, Simon, and Marshak, Progr. Theoret. Phys. Japan **5**, 634 (1950). The relevant quantities are C_{I} and C_{II}, which are given in Table IV, p. 658.

[8] The integrals occurring in the more complicated nonforward situation can also be done in closed form; they may be obtained, for example, from the work of L. M. Brown and R. P. Feynman,

tative features of the results will emerge in this case as well. At this stage, our equations reduce to:

$$
\begin{aligned}
T_1(p,P)&=\gamma_5G_+(p)\gamma_5+\gamma_5G_+(2p)\gamma_5\\
&\quad-3ig^2(2\pi)^{-4}\int(dk)k^{-2}\gamma_5G_+(k+p_1)\\
&\qquad\times T_1(p+\tfrac{1}{2}k;\,P+k)G_+(k+p_1)\gamma_5,\\
T_2(p,P)&=\gamma_5G_+(P)\gamma_5-\gamma_5G_+(2p)\gamma_5\\
&\quad+ig^2(2\pi)^{-4}\int(dk)k^{-2}\gamma_5G_+(k+p_1)\\
&\qquad\times T_2(p+\tfrac{1}{2}k;\,P+k)G_+(k+p_1)\gamma_5.
\end{aligned}
\tag{18}
$$

Use will also be made of the facts that, for real nucleons and mesons, $\gamma p_1+m=0$, $p_1^2+m^2=0$, $p_2^2+\mu^2=0$.

In making the assumption (13) here, we must realize that the λ_i are, in general, Dirac matrices and take this requirement approximately into account[9] by writing

$$T_i=(\lambda_i+\gamma p_2\mu_i/m)T_i^{(0)}. \tag{19}$$

Taking the trace and then $tr\times\gamma p_2$ of the equation resulting when Eq. (19) is inserted into (18), we obtain two equations for λ and μ in each case, which will express these in terms of our integrals. These, in turn, can be written as functions of the single variable $\delta=2\omega/m$ in the lab system.

The integrals to be performed have the structure

$$\int\frac{(k_\mu;\,k_\mu k_\nu)(dk)}{k^2(k^2+2k\cdot p_1)^2(k^2+2k\cdot q+\Delta)}\quad\text{or}\quad\int\frac{(1;\,k_\mu)(dk)}{(k^2+2k\cdot p_1)^2(k^2+2k\cdot q+\Delta)}$$

and are straightforward. We merely quote the results:

$$
\begin{aligned}
\lambda_1^{-1}=1-\frac{3\alpha'}{2\pi}\bigg\{&\frac{2\delta-4\delta^3}{(1-\delta^2)^2}\ln\delta-\frac{\delta}{1-\delta^2}-\frac{\delta}{2}+L(1-\delta)\\
&-\bar{L}(1+\delta)+\frac{6\alpha'}{\pi(1-3\alpha'\delta/4\pi)}\bigg[\frac{\delta^2}{1-\delta^2}\\
&-\frac{\delta^2-3\delta^4}{(1-\delta^2)^2}\ln\delta+\bar{L}(1+\delta)+L(1-\delta)-2L(1)\bigg]\\
&-i\pi\bigg[\ln(1+\delta)+\frac{2}{1+\delta}-\frac{1}{2(1+\delta)^2}-\frac{3}{2}\bigg]\\
&\times\bigg[1-\frac{6\alpha'}{\pi(1-3\alpha'\delta/4\pi)}\bigg]\bigg\},
\end{aligned}
\tag{20}
$$

$$\mu_1=2\alpha'/\pi(1-3\alpha'\delta/4\pi)^{-1}\lambda_1,$$

and similar expressions for λ_2, μ_2. The L and $\bar{L}$ are Spence functions, as defined by Brown and Feynman (reference 8).

We note that the λ's and μ's are complex functions, as might be expected.[10] In the usual treatments of scattering, the imaginary part of the forward scattering amplitude gives the total cross section $[\sigma=4\pi k^{-1}f(0)]$ for scattering out of the initial state resulting from all possible transitions. In our approximation, and by the very nature of the ansatz made, only certain states contribute to the decay out of the initial state, rather than all of those into which our state can really decay, as would be the case in the rigorous treatment. In such a treatment, all higher order processes by means of which decay can take place would contribute to the imaginary part; here, however, only insofar as the ansatz simulates the rigorous solution is its imaginary part approximately valid.

We also remark that $T_i=(\lambda_i+\gamma p_2/m\mu_i)T_i^{(0)}\to\lambda_iT_i^{(0)}$ to within terms in μ^2, which have been neglected throughout. However, the form of λ is strongly influenced by the assumption (19). Had we merely taken $T=\lambda T^{(0)}$, an entirely different form of λ would follow.

An approximate estimate of the damping at $\delta=\frac{3}{4}$, corresponding to ω about 350 Mev,[11] again with $\alpha'=10$, yields

$$
\begin{aligned}
\lambda_1&\approx0.01(1-0.1i),\\
\lambda_2&\approx1-0.16i.
\end{aligned}
\tag{21}
$$

Thus, although the T_2 part is relatively unaffected, the T_1 is considerably damped at this energy. It may be mentioned that if μ_i had not been introduced, λ_2 would have been as strongly damped as the λ_1 quoted, whereas λ_1 would become an order of magnitude larger at the same energy.

DISCUSSION

We have seen that at low energies damping tends to be small, whereas at higher ones at least part of the amplitude tends to be very strongly decreased. It is obvious that the quantitative statements are not too trustworthy; the strong rise of the effect with energy, however, is suggestive of what may be expected of field reaction effects. Graphs in which many mesons are simultaneously present would seem to play an important role at higher energies. It should be of interest to compare our treatment with one including a different

Phys. Rev. **85**, 231 (1952). As far as the integrations are concerned, our calculation is equivalent to the electrodynamic case treated there once the approximation $\mu=0$ is made.

[9] More rigorously, one would have to write λ_i as the sum of all 16 elements of the spinor algebra.

[10] The imaginary parts arise only from the parts of the ansatz (19) involving $G_+(P)$, corresponding to the reducibility of radiatively corrected "uncrossed" terms as against irreducibility of the "crossed" terms. Thus only the reducible diagrams have imaginary parts since there is no mode of decay in the irreducible case.

[11] Of course, both T_1 and T_2 give scattering in a mixture of isotopic spin $\frac{1}{2}$ and $\frac{3}{2}$ states; if the effects in the respective pure states are to be obtained, the proper linear combination of the two must be employed. T_1 and T_2 describe ordinary and charge exchange scattering, respectively.

(infinite) set of graphs: the ladder approximation arising from use of the meson-nucleon equation.[1] Such a calculation represents another extreme in approximation schemes: it drops terms with many mesons in the field in favor of higher order (iterated) scattering graphs. However, any method which arbitrarily neglects an infinite number of relevant terms is, of course, open to the objection that inclusion of these other graphs could vitiate all its conclusions.

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The Theory of Quantized Fields. V

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The Dirac field, as perturbed by a time-dependent external electromagnetic field that reduces to zero on the boundary surfaces, is the object of discussion. Apart from the modification of the Green's function, the transformation function differs in form from that of the field-free case only by the occurrence of a field-dependent numerical factor, which is expressed as an infinite determinant. It is shown that, for the class of fields characterized by finite space-time integrated energy densities, a modification of this determinant is an integral function of the parameter measuring the strength of the field and can therefore be expressed as a power series with an infinite radius of convergence. The Green's function is derived therefrom as the ratio of two such power series. The transformation function is used as a generating function for the elements of the occupation number labelled scattering matrix S and, in particular, we derive formulas for the probabilities of creating n pairs, for a system initially in the vacuum state. The general matrix element of S is presented, in terms of the classification that employs a time-reversed description for the negative frequency modes, with the aid of a related matrix Σ, which can be viewed as describing the development of the system in proper time. The latter is characterized as indefinite unitary, in contrast with the unitary property of S, which is verified directly. Two appendices are devoted to determinantal properties.

TIME-DEPENDENT ELECTROMAGNETIC FIELDS

THE previous paper in this series[1] dealt with the Dirac field, as coupled to a second prescribed Dirac field. We shall now discuss the effect of coupling with a prescribed Bose-Einstein field, using the example of the electromagnetic field. The Lagrange function and field equations of this system are presented in Eqs. (IV. 1, 2).

The simplest extension of the work of IV is obtained by supposing that the external electromagnetic field vanishes in the vicinity of the boundary surfaces σ_1 and σ_2, while assuming arbitrary values in the interior of this region. We shall retain the gauge $A_\mu=0$ to describe a zero field. The decomposition of the Dirac field into positive and negative frequency components on σ_1 and σ_2 can be performed as in IV, and the history of the system between σ_1 and σ_2 will be described by the transformation function $(\chi^{(-)\prime}\sigma_1|\chi^{(+)\prime}\sigma_2)$. The source substitution (IV. 33) produces the latter from the transformation function referring to zero eigenvalues,

$$(0\sigma_1|0\sigma_2)=\exp(i\mathcal{W}_0). \tag{1}$$

The dependence of $\mathcal{W}_0$ upon the source is expressed by [(IV. 37)]

$$\delta\mathcal{W}_0=\int_{-\infty}^{\infty}(dx)[\delta\bar{\eta}\langle\psi\rangle+\langle\bar{\psi}\rangle\delta\eta], \tag{2}$$

where now

$$\begin{aligned}\gamma_\mu[-i\partial_\mu-eA_\mu(x)]\langle\psi(x)\rangle+m\langle\psi(x)\rangle&=\eta(x),\\ [i\partial_\mu-eA_\mu(x)]\langle\bar{\psi}(x)\rangle\gamma_\mu+m\langle\bar{\psi}(x)\rangle&=\bar{\eta}(x),\end{aligned} \tag{3}$$

are the equations to be solved in conjunction with the boundary conditions (IV. 40, 41). The associated Green's function [Eqs. (IV. 42, 43)] thus obeys the differential equations

$$\begin{aligned}&\gamma_\mu[-i\partial_\mu-eA_\mu(x)]G_+(x,x')+mG_+(x,x')\\ &=[i\partial_\mu'-eA_\mu(x')]G_+(x,x')\gamma_\mu+mG_+(x,x')=\delta(x-x'),\end{aligned} \tag{4}$$

and the boundary condition that G_+ as a function of x, shall contain only positive frequencies for $x_0>x_0'$, A, and only negative frequencies for $x_0<x_0'$, A. We have indicated by $x_0>A$ and $x_0<A$ that the domain of nonvanishing field is confined, respectively, to earlier or later times than x_0. The same statements apply with x and x' interchanged.

The compatibility of these two forms of the boundary condition, for arbitrary A_μ, can be ascribed to the charge symmetry of the theory. If the second differ-

[1] J. Schwinger, Phys. Rev. 92, 1283 (1953).