OBSERVATION OF π-π RESONANCE IN PION PRODUCTION*

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The difficulty of obtaining direct experimental information on pion-pion scattering is well known. Goebel and Chew and Low[1] have suggested a procedure to be applied to the process $\pi+N\rightarrow\pi+\pi+N$ which involves extrapolating the experimental data into the nonphysical region, to a point at which the pion-pion cross section may be extracted with no theoretical uncertainties. If the experiments can be performed with sufficient accuracy, this is certainly the method of choice for learning about the π-π process, and in particular for seeing whether or not the suggested resonance[2] in the $J=1$, $T=1$ state of the π-π system exists. We would here like to suggest another procedure for finding out if this resonance exists; one which unfortunately is much more beset with theoretical uncertainties and ambiguities than the method of reference 1, but which is perhaps more feasible experimentally.

Our suggestion is based on the analysis of the reactions

$$\gamma+N\rightarrow\pi+\pi+N, \tag{1}$$

and

$$\pi+N\rightarrow\pi+\pi+N. \tag{2}$$

Let p and p' denote the four-momenta of the initial and final nucleons, respectively; q_1 and q_2, the four-momenta of the two final pions; and k, the four-momentum of the initial photon [process (1)] or pion [process (2)]. What we have in mind is attempting to identify a resonance in the final-state interaction of the two outgoing pions. In general, separating the final-state interaction of the pions from the total process is extremely difficult due to the strong interaction of each pion with the recoil nucleon. However, if the π-N interaction could be held constant, the variations in the cross sections for (1) and (2) would presumably be due to variations in the π-π interaction. It is of course impossible to keep the π-N interaction precisely constant, but by a suitable choice of the kinematics we may hope to reduce the variations produced by it to a minimum.

Let $S=(q_1+q_2)^2$; $S_1=(q_1+p')^2$; and $S_2=(q_2+p')^2$. Then $S(S_1, S_2)$ is the total center-of-mass energy squared of the π-π(π_1-N, π_2-N) system. If we program experiments to hold S_1 and S_2 constant, and vary S, then at least the total center-of-mass energy of each π-N system is held constant. This does not imply that the π-N interaction is held constant for two reasons. First, as the center-of-mass energy of the two pions is changed, the amplitude that the two pions be in a given relative JT state changes due to the π-π interaction, and this alters the amplitude to find either pion in a given state relative to the nucleon. Secondly, there may still be additional changes in the π-N interactions since we are holding only S_1 and S_2 fixed, and these do not uniquely determine the π-N interaction. However, if we choose S_1 and S_2 to be in an energy region—say beyond the first resonance—where the various π-N phase shifts are more or less comparable, there should be no drastic variations due to this. Hopefully, then, as S is varied through the region of the π-π resonance, there will be a peak in the cross sections for (1) and (2) if and only if the π-π resonance exists.[3]

The specific kinematics corresponding to fixing S_1 and S_2 and varying S is most conveniently taken to be the following: In the center-of-mass system of the complete reaction, choose the symmetrical point where $|\vec{q}_1|=|\vec{q}_2|$, and the angles θ_1 and θ_2 are equal, as shown in Fig. 1. Then we have

$$S=2\mu^2+2\omega_1\omega_2-2q_1q_2\cos\theta_{12}$$
$$=4\omega^2-4q^2\cos^2\theta=4E\omega+M^2-E^2, \tag{3}$$

where $|\vec{q}_1|=|\vec{q}_2|=q$, $\theta_1=\theta_2=\theta$, and $\omega^2=q^2+\mu^2$; and where E is the total center-of-mass energy. Also,

$$S_1=S_2=M^2+\mu^2+2E_{p'}\omega+2p'q\cos\theta$$
$$=M^2+\mu^2+2E\omega-4\omega^2+4q^2\cos^2\theta$$
$$=M^2+\mu^2+2E\omega-S. \tag{4}$$

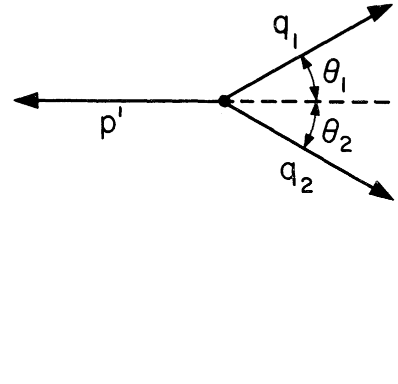

FIG. 1. Angles and momenta for emerging nucleon and pions.

It is clear then, that it is possible to fix S_1 and S_2, and to vary S, provided the total energy E is varied as well.

The energies necessary to cover the interesting region in S, which is roughly[2] $4\mu^2 \leqslant S \leqslant 16\mu^2$, are easily obtained from (3) and (4). We may first express S in terms of S_1 and E by inserting (4) in (3): $S = E^2 + M^2 + 2\mu^2 - 2S_1$. Then if we set $S_1 \approx (M + 3\mu)^2$, corresponding to a π-N center-of-mass energy in the valley beyond the first π-N resonance, we have $S = E^2 - 12M\mu - M^2 - 16\mu^2$. Thus to make S range from $4\mu^2$ to $16\mu^2$, we must make E^2 range from $M^2 + 12M\mu + 20\mu^2$ to $M^2 + 12M\mu + 32\mu^2$. The maximum value of E which is needed is something less than $M + 6\mu$, or about 1175-Mev incident gamma energy in the lab system for process (1), or 1165-Mev incident pion energy in the lab system for process (2).

The size of the peak which should appear in the cross sections for processes (1) and (2) if the π-π resonance does exist is not an easy thing to predict accurately on the basis of present theoretical knowledge and techniques. One thing which will certainly affect the size significantly is the amplitude with which the relative $T=1$, $J=1$ state is present in the outgoing two-pion system. It is even conceivable that the bump would be entirely insignificant even if the resonance were present.

It is possible to make a crude estimate of the effect of a π-π resonance on the photoproduction process (1), in order to reassure oneself that there is a reasonable amplitude for the two pions to be in the appropriate state. This estimate is made by grafting part of the effect of the resonance onto the $\gamma + N \to \pi + \pi + N$ amplitude as calculated in the static model,[4] and corresponds to adding to the photoproduction without π-π scattering, Fig. 2(a), an effect such as shown in Fig. 2(b). Formally, this may be done by extracting from the static theory result[4] that part of the amplitude describing the pions in a relative $T=1$, $J=1$ state, and multiplying it by $F_\pi(S)$, where F_π is the pion electromagnetic form factor. To see the basis for doing this, let M be the pion production amplitude. Then we may expand $M = \sum_{JT} M_{JT} P_J Q_T$, where P_J and Q_T are projection operators, into the indicated two-pion states. Now write $M_{JT} = (M_{JT} D_{JT})/D_{JT}$, where D_{JT} is the determinantal function[5] for π-π scattering in the TJ state. Considered as a function of S, M_{JT} has singularities from $4\mu^2$ to ∞ but $M_{JT} D_{JT}$ has singularities only from $16\mu^2$ to ∞.[6] Thus it is plausible to expand $(M_{JT} D_{JT})$ in

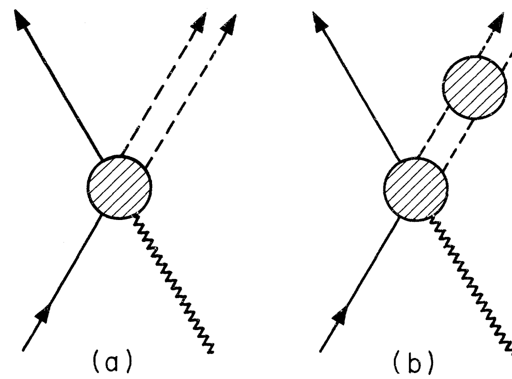

FIG. 2. (a) Diagram for production of two pions (dashed line) by incident gamma ray (wavy line) on nucleon (solid line) without π-π scattering. (b) Modification to include π-π scattering.

powers of the π-π interaction. If we keep just the lowest term in this expansion (corresponding to no π-π interaction) we have $M_{JT} = M_{JT}^{(0)}/D_{JT}$, where $M^{(0)}$ is the production amplitude without π-π scattering. Now, unless $J=1$ and $T=1$, there is presumably no π-π resonance, so we may replace D_{JT} by 1 in these cases. However, if $J=1$ and $T=1$, $1/D_{11} = F_\pi$,[6] so we have $M_{11} = M_{11}^{(0)} F_\pi$, and therefore $M = M^{(0)} + M_{11}^{(0)}(F_\pi - 1)$.

If $M^{(0)}$ is chosen as the expression resulting from the static theory [Eqs. (15) and (16) of reference 4] and the $J=1$ state is picked out, taking into account the difference between the pion-nucleon center-of-mass system and the center-of-mass system of the two pions only in the projection operator, then the resulting expression for the $\gamma + p \to \pi^+ + \pi^- + p$ cross section is the same as Eq. (17) of reference 4 with the replacements

$$(\tfrac{1}{2}\sin^2\theta_+ + \tfrac{1}{3}) \to (\tfrac{1}{2}\sin^2\theta_+ + \tfrac{1}{3})(\tfrac{4}{9}|F_\pi|^2 + \tfrac{4}{9}\mathrm{Re}F_\pi + \tfrac{1}{9}),$$

$$\tfrac{1}{9}(\tfrac{1}{2}\sin^2\theta_- + \tfrac{1}{3}) \to (\tfrac{1}{2}\sin^2\theta_- + \tfrac{1}{3})(\tfrac{4}{9}|F_\pi|^2 - \tfrac{4}{9}\mathrm{Re}F_\pi + \tfrac{1}{9}),$$

$$\tfrac{1}{3}(\tfrac{5}{3}\cos\varphi - \cos\theta_+\cos\theta_-) \to (\tfrac{5}{3}\cos\varphi - \cos\theta_+\cos\theta_-)(\tfrac{4}{9}|F_\pi|^2 - \tfrac{1}{9}). \quad (5)$$

Thus in this approximation the two pions are in the resonant state a large fraction of the time. At the Frazer-Fulco value for the π-π resonance $S=10$ and $|F_\pi|^2 \approx 17$, $\mathrm{Re}F_\pi = 0$. With these parameters we find from (5) that the cross section should be enhanced by a factor of ≈ 14. The half-width of the bump is roughly $5\mu^2$.

This result is not to be interpreted as a reliable calculation of the pair production cross section including the effects of a π-π resonance. First of all, it is based on the static model which is certainly not accurate at the energies of interest to us here. Secondly, even if the static model were itself sensible, the π-π resonance has been included in only the most primitive way. The only virtue of (5) is that it shows that there are no unexpected factors which tend to damp out the effect of the π-π resonance, and, in this most modest prediction, there is no reason that (5) should be wildly wrong.

It is difficult to make a similar rough estimate for the second reaction, $\pi + N \rightarrow \pi + \pi + N$, due to the absence of any plausible starting point such as existed for the photoproduction through the static-theory calculation, which is known to be accurate for low energies.[4] Nevertheless, there is no reason to believe that the effect of the resonance would be completely suppressed in that process either.

There are two remaining comments which can help to isolate the effect of the resonance. The first is that while we cannot calculate much about processes (1) and (2), we can calculate the density of states factor, and the variations produced by it, at least, can be removed from the cross sections. The differential cross section for either process (1) or process (2) will have the form

$$d\sigma = \left(\frac{1}{2\pi}\right)^5 \frac{1}{v_{\mathrm{rel}}} \frac{1}{64 k_0 E_p} \sum_{\substack{\text{spins,}\\ \text{polarizations}}} |M|^2 \frac{q^2}{E - 2\omega + 2\omega \cos^2\theta} d\omega_1 d\Omega_1 d\Omega_2$$

at the symmetrical point defined earlier. Here k_0 and E_p are the initial energies, v_{rel} is the incident relative velocity, M is the invariant Feynman matrix element, and the remaining factors speak for themselves. In terms of the useful variables S, and $S_1(=S_2)$, the density of states factor becomes

$$\frac{q^2}{E - 2\omega + 2\omega \cos^2\theta} = \frac{(S+S_1-M^2-\mu^2)^2 - 4\mu^2E^2}{4E^2} \left[\frac{E^2+M^2+\mu^2-S-S_1}{E} + \frac{S+S_1-M^2-\mu^2}{E} \times \frac{(S+S_1-M^2-\mu^2)^2 - E^2S}{(S+S_1-M^2-\mu^2)^2-4\mu^2E^2}\right]^{-1}$$

The second comment is that in order to assist in the elimination of variations in the cross section due to processes other than the $J=1$, $T=1$ π-π resonance, it would be useful in process (2) to measure the ratio of cross sections, for example,[1]

$$\sigma(\pi^- + p \rightarrow \pi^+ + \pi^- + n)/\sigma(\pi^+ + p \rightarrow \pi^+ + \pi^+ + n),$$

since in the lower process the two final pions always have $T=2$ and are therefore never in the resonant state. In the photoproduction process this taking of ratios cannot be done since there is no way of making the final pions in a pure $T=2$ or $T=0$ state.

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[2]W. Frazer and J. Fulco, Phys. Rev. 117, 1603 (1960).

[3]Similarly a peak at a π-N resonance should be observed by fixing S and S_2, say, and varying S_1 through a resonance energy for the π-N system. A recent discussion of the π-π resonance in π interactions is given by P. Carruthers and H. A. Bethe, Phys. Rev. Letters 4, 536 (1960).

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