UCB-PTH-98/35, LBNL-41948, NSF-ITP-98-070
hep-th/9806171

Glueballs and Their Kaluza-Klein Cousins

Hirosi Ooguri, Harlan Robins and Jonathan Tannenhauser

Department of Physics, University of California at Berkeley,
Berkeley, CA 94720

Theoretical Physics Group, Mail Stop 50A-5101,
Lawrence Berkeley National Laboratory,
Berkeley, CA 94720

Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106

Abstract

Spectra of glueball masses in non-supersymmetric Yang-Mills theory in three and four dimensions have recently been computed using the conjectured duality between superstring theory and large N gauge theory. The Kaluza-Klein states of supergravity do not correspond to any states in the Yang-Mills theory and therefore should decouple in the continuum limit. On the other hand, in the supergravity limit $g_{YM}^2 N \to \infty$, we find that the masses of the Kaluza-Klein states are comparable to those of the glueballs. We also show that the leading $(g_{YM}^2 N)^{-1}$ corrections do not make these states heavier than the glueballs. Therefore, the decoupling of the Kaluza-Klein states is not evident to this order.

1 Introduction

Spectra of glueball masses in non-supersymmetric Yang-Mills theory in three and four dimensions have recently been calculated [1] using the conjectured duality between string theory and large N gauge theory [2–5]. The results are apparently in good numerical agreement with available lattice gauge theory data, although a direct comparison may be somewhat subtle, since the supergravity computation is expected to be valid for large ultraviolet coupling $\lambda = g_{YM}^2 N$, whereas we expect that QCD in the continuum limit is realized for $\lambda \to 0$ [5, 6]. As explained in [6, 1], the supergravity computation at $\lambda \gg 1$ gives the glueball masses in units of the fixed ultraviolet cutoff Λ_{UV}. For finite λ, the glueball mass M is expected to be a function of the form

$$M^2 = F(\lambda)\Lambda_{UV}^2. \tag{1.1}$$

In the continuum limit $\Lambda_{UV} \to \infty$, M should remain finite and of order Λ_{QCD}. This would require $F(\lambda) \to 0$ as $\lambda \to 0$. In [1], the leading string theory corrections to the masses were computed and shown to be negative and of order $\lambda^{-3/2}$, in accordance with expectation.

Witten has proposed [5] that three-dimensional pure QCD is dual to type IIB string theory on the product of an AdS_5 black hole and $\mathbf{S}^5$. This proposal requires that certain states in string theory decouple in the continuum limit $\lambda \to 0$. One class of such states are Kaluza-Klein excitations on $\mathbf{S}^5$. The supergravity fields on the AdS_5 black hole $\times$ $\mathbf{S}^5$ can be classified by decomposing them into spherical harmonics (the Kaluza-Klein modes) on $\mathbf{S}^5$ [7, 8]. They fall into irreducible representations of the isometry group $SO(6)$ of $\mathbf{S}^5$, which is the R-symmetry of the four-dimensional $\mathcal{N} = 4$ supersymmetric gauge theory from which QCD_3 is obtained by compactification on a circle. Consequently, only $SO(6)$ singlet states should correspond to physical states in QCD_3 in the continuum limit. These are the glueball states studied in [1]. However, we find that, in the supergravity limit, masses of the $SO(6)$ non-singlet states are of the same order as the $SO(6)$ singlet states. Since these states should decouple in the limit $\lambda \to 0$, it was speculated in [1] that the string theory corrections should make the non-singlet states heavier than the singlet states.

The purpose of this paper is to test this idea. We compute the masses of the $SO(6)$ non-singlet states coming from the Kaluza-Klein excitations of the dilaton in ten dimensions. We find the masses in the supergravity limit to be of the same order as those of the $SO(6)$ singlet states. We then calculate the leading string theory corrections to the masses. We find that the leading corrections do not make the Kaluza-Klein states heavier than the glueballs. Therefore, the decoupling of the Kaluza-Klein states is not evident to this order. This suggests that the quantitative agreement between the glueball masses from supergravity and the lattice gauge theory data should be taken with a grain of salt.

2 The Supergravity Limit

We calculate the masses of the Kaluza-Klein states following the analysis of [1]. According to [5], QCD_3 is dual to type IIB superstring theory on the AdS_5 black hole $\times$ $\mathbf{S}^5$ geometry given by

$$\frac{dx^2}{l_s^2\sqrt{4\pi g_{YM}^2 N}} = \frac{d\rho^2}{\left(\rho^2 - \frac{b^4}{\rho^2}\right)} + \left(\rho^2 - \frac{b^4}{\rho^2}\right) d\tau^2 + \rho^2 \sum_{i=1}^{3} dx_i^3 + d\Omega_5^2, \tag{2.1}$$

where $d\Omega_5$ is the line element on the unit $\mathbf{S}^5$ and l_s is the string length. The horizon of the black hole is located at $\rho = b$. In order for the geometry to be regular at the horizon, the coordinate τ must be periodic with period $2\pi R$, where $R = (2b)^{-1}$. The inverse radius R^{-1} serves as the ultraviolet cutoff of QCD_3; namely, $\Lambda_{UV} = (2R)^{-1} = b$.

To compute the mass of an $SO(6)$ non-singlet state, we express the dilaton field Φ as

$$\Phi = f_0(\rho) e^{ikx} Y_l(\Omega_5), \tag{2.2}$$

where $Y_l(\Omega_5)$ is the l-th spherical harmonic on $\mathbf{S}^5$, and solve the dilaton equation in the geometry (2.1). This equation reduces to a second-order ordinary differential equation for $f_0(\rho)$; in units in which $b = 1$,

$$\rho^{-1}\frac{d}{d\rho}\left((\rho^4 - 1)\rho\frac{df_0}{d\rho}\right) - (k_0^2 + l(l+4)\rho^2) f_0 = 0. \tag{2.3}$$

The mass in three dimensions is equal to $-k_0^2$ [5]. Since the geometry (2.1) is smooth everywhere, we require that $f_0(\rho)$ be regular everywhere, and in particular at $\rho = \infty$ and at the horizon $\rho = 1$. The equation admits a regular solution $f_0(\rho)$ for discrete values of k_0^2. This determines the mass spectrum.

As in [1], we determine $M^2 = -k_0^2$ numerically by the shooting method. We first solve the differential equation (2.3) as an asymptotic expansion in ρ^{-2} and compute the first few terms in the expansion. We then numerically integrate the equation, with boundary conditions derived from the asymptotic expansion imposed at a sufficiently large value of ρ ($\rho \gg k_0^2$). The solution must be regular at the horizon $\rho = 1$. This requirement determines the spectrum of k_0^2. In the numerical evaluation, we find it convenient to set the boundary condition to be $f_0' = 0$ at the horizon. As we will show in the Appendix, this shooting method can be used to compute k_0^2 and the wavefunction $f_0(\rho)$ to arbitrarily high precision. The results of the numerical work are listed in Table 1. As expected, the masses are all of the order of the ultraviolet cutoff $\Lambda_{UV} = b$.

l	0	1	2	3	4	5	6	7
M_l^2	11.59	19.43	29.26	41.10	54.93	70.76	88.60	108.4
M_l^{*2}	34.53	48.07	63.60	81.11	100.6	122.1	145.6	171.1
M_l^{**2}	68.98	88.24	109.5	132.7	157.9	185.1	214.3	245.5

Table 1: 3d $(\text{Mass})^2$ of the l-th Kaluza-Klein modes on $\mathbf{S}^5$ and their excited states, in units of b^2

3 Leading String Theory Corrections

Witten's proposal requires that the Kaluza-Klein states decouple in the continuum limit $\lambda \to 0$. Here we examine whether this efffect is evident from the leading string theory corrections.

According to [9], the leading $\alpha' = (4\pi g_{YM}^2 N)^{-1/2}$ correction to the AdS$_5$ black hole metric is

$$\frac{ds^2}{l_s^2\sqrt{4\pi g_{YM}^2 N}} = (1+\delta_1)\frac{d\rho^2}{\left(\rho^2 - \frac{b^4}{\rho^2}\right)} + (1+\delta_2)\left(\rho^2 - \frac{b^4}{\rho^2}\right)d\tau^2 + \rho^2\sum_{i=1}^{3} dx_i^2 + d\Omega_5^2, \quad (3.1)$$

where

$$\begin{aligned} \delta_1 &= +15\gamma\left(5\frac{b^4}{\rho^4} + 5\frac{b^8}{\rho^8} - 19\frac{b^{12}}{\rho^{12}}\right) \\ \delta_2 &= -15\gamma\left(5\frac{b^4}{\rho^4} + 5\frac{b^8}{\rho^8} - 3\frac{b^{12}}{\rho^{12}}\right), \end{aligned} \quad (3.2)$$

and $\gamma = \frac{1}{8}\zeta(3)\alpha'^3$. In this geometry, the dilaton is no longer constant, but is given by

$$\Phi_0 = -\frac{45}{8}\gamma\left(\frac{b^4}{\rho^4} + \frac{b^8}{2\rho^8} + \frac{b^{12}}{3\rho^{12}}\right). \quad (3.3)$$

There is also a correction to the ten-dimensional dilaton action [10, 11],

$$I_{dilaton} = -\frac{1}{16\pi G_{10}}\int d^{10}x\sqrt{g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \gamma e^{-\frac{3}{2}\Phi}W\right], \quad (3.4)$$

where W is given in terms of the Weyl tensor. In our background and in units where $b = 1$, $W = 180/\rho^{16}$. The relation between the location of the horizon $\rho = b$ and the periodicity $2\pi R$ of τ is also modified to

$$R = \left(1 - \frac{15}{8}\zeta(3)\alpha'^3 + \cdots\right)\frac{1}{2b}. \tag{3.5}$$

It is the inverse radius R^{-1} that serves as the ultraviolet cutoff of QCD_3.

To solve the dilaton wave equation in the α'-corrected geometry (3.1), we write

$$\Phi = \Phi_0 + f(\rho)e^{ikx}Y_l(\Omega_5), \tag{3.6}$$

where Φ_0 is the dilaton background given by (3.3), and expand $f(\rho)$ and k^2 in γ as

$$f(\rho) = f_0(\rho) + \gamma h(\rho), \quad k^2 = k_0^2 + \gamma\delta k^2. \tag{3.7}$$

Here $f_0(\rho)$ obeys the lowest order equation (2.3) and is a numerically given function, and k_0^2 is likewise determined from (2.3). The second-order differential equation obtained from the action (3.4) in the background metric (3.1) and dilaton field (3.3) is, in units in which $b = 1$,

$$\begin{aligned}
&\rho^{-1}\frac{d}{d\rho}\left((\rho^4-1)\rho\frac{dh}{d\rho}\right) - (k_0^2 + l(l+4)\rho^2)h = \\
&= (75 - 240\rho^{-8} + 165\rho^{-12})\frac{d^2 f_0}{d\rho^2} \\
&+(75 + 1680\rho^{-8} - 1815\rho^{-12})\rho^{-1}\frac{df_0}{d\rho} \\
&+(\delta k^2 - 120(k_0^2 + l(l+4)\rho^2)\rho^{-12} - 405\rho^{-14})f_0(\rho).
\end{aligned} \tag{3.8}$$

With $f_0(\rho)$ and k_0^2 given, one may regard this as an inhomogeneous version of the equation (2.3). We solve this equation for $h(\rho)$ and δk^2.

We are now ready to present our results. Let us denote the lowest mass of the l-th Kaluza-Klein state by M_l. In units of the ultraviolet cutoff $\Lambda_{UV} = (2R)^{-1}$, with R given by (3.5), we find

$$\begin{aligned}
M_0^2 &= 11.59 \times (1 - 2.78\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_1^2 &= 19.43 \times (1 - 2.66\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_2^2 &= 29.26 \times (1 - 2.62\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_3^2 &= 41.10 \times (1 - 2.61\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_4^2 &= 54.93 \times (1 - 2.63\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_5^2 &= 70.76 \times (1 - 2.66\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_6^2 &= 88.60 \times (1 - 2.69\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_7^2 &= 108.4 \times (1 - 2.72\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2.
\end{aligned} \tag{3.9}$$

Similar behavior is observed for the excited levels of each Kaluza-Klein state.

Thus the corrections do not make the Kaluza-Klein states heavier than the glueballs, and the decoupling of the Kaluza-Klein states is not evident to this order. According to Maldacena's duality, the $\lambda^{-1/2}$ expansion of the gauge theory corresponds to the α'-expansion of the two-dimensional sigma model with the AdS_5 black hole $\times$ $\mathbf{S}^5$ as its target space. It is possible that the decoupling of the Kaluza-Klein states takes place only non-perturbatively in the sigma model.

Acknowledgments

We thank Csaba Csáki, Aki Hashimoto, Yaron Oz, John Terning, and especially David Gross for useful discussions. We thank the Institute for Theoretical Physics at Santa Barbara for its hospitality.

This work was supported in part by the NSF grant PHY-95-14797 and the DOE grant DE-AC03-76SF00098, and in part by the NSF grant PHY-94-07194 through ITP. H.R. and J.T. gratefully acknowledge the support of the A. Carl Helmholz Fellowship in the Department of Physics at the University of California, Berkeley.

Appendix: The Boundary Condition at the Horizon

In this appendix, we show that the boundary condition at the horizon $\rho = b$ used in the shooting method [1] is consistent, and that the eigenvalue k^2 and the wavefunction $f(\rho)$ can be evaluated to an arbitrarily high precision using this method.

In the neighborhood of $\rho = b$, the dilaton wave equation takes the form

$$\partial_\rho(\rho - b)\partial_\rho f(\rho) + \cdots = 0. \tag{3.10}$$

Its general solution is of the form

$$f(\rho) = c_1\left[1 + \alpha(\rho - b) + \cdots\right] + c_2\left[\log(\rho - b) + \cdots\right] \tag{3.11}$$

with arbitrary coefficients $c_{1,2}$ (the constant α is determined by the wave equation and is in general non-zero). The regularity of the dilaton field requires $c_2 = 0$. In the shooting method, we numerically integrate the differential equation starting from a sufficiently large value of ρ down to the horizon. For generic k^2, the function thus obtained, when expanded as in (3.11), would have $c_2 \neq 0$. The task is to adjust k^2 so that $c_2 = 0$.

Since $f(\rho)$ is divergent at $\rho = b$ for generic k^2, it is numerically difficult to impose the boundary condition directly at $\rho = b$. Instead, in [1] and in this paper, we required $f' = 0$ at $\rho = b + \epsilon$ for a given small ϵ (for example, $\epsilon = 0.0000001b$ in this paper). By (3.11), this condition implies

$$c_2 = -c_1 \alpha \epsilon + \cdots . \tag{3.12}$$

Therefore, c_2 can be made arbitrarily small by adjusting ϵ. This justifies the numerical method used in [1] and in this paper.

We thank Aki Hashimoto for discussions on the numerical method.

References

[1] C. Csáki, H. Ooguri, Y. Oz and J. Terning, "Glueball Mass Spectrum from Supergravity," hep-th/9806021.

[2] J. M. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity," hep-th/9711200.

[3] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, "Gauge Theory Correlators from Non-Critical String Theory," hep-th/9802109.

[4] E. Witten, "Anti-de Sitter Space and Holography," hep-th/9802150.

[5] E. Witten, "Anti-de Sitter Space, Thermal Phase Transition, And Confinement in Gauge Theories," hep-th/9803131.

[6] D. J. Gross and H. Ooguri, "Aspects of Large N Gauge Theory Dynamics as Seen by String Theory," hep-th/9805129.

[7] H. J. Kim, L. J. Romans and P. Van Nieuwenhuizen, "Mass Spectrum of Chiral Ten-Dimensional $\mathcal{N}=2$ Supergravity on S^5," Phys. Rev. **D32** (1985) 389.

[8] M. Günaydin and N. Marcus, "The Spectrum of The S^5 Compactification of Chiral $\mathcal{N}=2, D=10$ Supergravity and The Unitary Supermultiplets of $U(2,2/4)$,"Class.Quant.Grav. **2** (1985) L11.

[9] S. S. Gubser, I. R. Klebanov and A. A. Tseytlin, "Coupling Constant Dependence in the Thermodynamics of $\mathcal{N}=4$ Supersymmetric Yang-Mills Theory," hep-th/9805156.

[10] M. T. Grisaru, A. E. M. van de Ven and D. Zanon, "Four-Loop Beta Function for the $\mathcal{N}=1$ and $\mathcal{N}=2$ Supersymmetric Nonlinear Sigma-Model in Two Dimensions," Phys. Lett. **B173** (1986) 423; M. T. Grisaru and D. Zanon, "Sigma-Model Superstring Corrections to the Einstein-Hilbert Action," Phys. Lett. **B177** (1986) 347.

[11] D. J. Gross and E. Witten, " Superstring Modifications of Einstein Equation," Nucl. Phys. **B277** (1986) 1.