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Theoretical problems in nonsymmetric gravitational theory

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It has recently been noted that the nonsymmetric metric model of gravity faces severe observational constraints. We show here that it is also subject to physically unacceptable formal difficulties even as an effective field theory: When expanded about a Riemannian background, the model exhibits curvature-coupled negative-energy (ghost) modes and unacceptable asymptotic behavior.

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There has been considerable work on a reinterpretation of the old Einstein–Straus unified field theory [1], based on nonsymmetric metrics $g_{\mu\nu}$ and affinities $\Gamma^{\alpha}_{\mu\nu}$, in which the antisymmetric part of the metric now represents a new field [2] rather than electromagnetism. There is actually a wide class of such "geometric" models [3], but most of them could be rejected on the ground that they contain ghost, negative-energy, excitations [4]. However, one model, nonsymmetric gravitational theory (NGT) [2], has been hitherto believed to be consistent. Indeed, it has been claimed that NGT survives the stronger requirement of ghost freedom in an expansion about a Riemannian background [5]. Recently NGT has been used to suggest several null tests of general relativity, which placed strong constraints on it [6]. We show here that in fact the above claim of ghost freedom is incorrect: not only do curvature-coupled ghost modes develop in NGT, but they preclude acceptable asymptotic behavior. Our results were obtained in the course of analyzing the whole complex of such "geometrical" models [7], motivated by their superficial resemblance to string σ-model actions in that the latter can also be expressed in terms of a nonsymmetric metric. In the process, we found that the only possible consistent candidates appear to be a new class, involving nonvanishing cosmological constant terms such as $\lambda\sqrt{-g}$, which give a finite range to the antisymmetric field, breaking the latter's gauge invariance and turning it into a massive vector. As discussed in [7] these theories may in fact provide a useful foil to general relativity in terms of possible observational effects. The present short note only addresses the inconsistency of standard NGT.

Because there has been some confusion in the literature, we emphasize that our analysis is based on the existence of two totally independent coupling constants in NGT, namely the usual Einstein κ^2 and that associated with the antisymmetric field. We assume (as in standard field theory) that expansion in these coupling constants is allowed, so that in particular the theory must reduce to Einstein gravity to zeroth order in the antisymmetric field and remain consistent order by order. [For example,the generalization of the Schwarzschild solution in NGT is separately analytic in these two parameters and obeys Eqs. (3) below to the expected order in B.] Any putative nonperturbative miracles are at present a

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matter of faith.

The source-free NGT Lagrangian in first-order (Palatini) form is

$$\mathcal{L}_{\rm NGT} = \sqrt{-g}\, g^{\mu\nu}\, R_{\mu\nu}(\Gamma)\ . \qquad (1)$$

We omit matter coupling since the consistency problem already arises here. Without loss of generality, we have chosen the above model as representative of the generic two-derivative class discussed in [2]; our conclusions [7] apply to all of these theories. The field equations obtained by independent variation of (1) with respect to $g_{\mu\nu}$ and $\Gamma^\lambda_{\alpha\beta}$ are

$$R_{\mu\nu}(\widetilde{\Gamma}) = \tfrac{2}{3}\,(\partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu)\ , \qquad (2a)$$

$$\partial_\lambda g_{\mu\nu} - g_{\alpha\nu}\widetilde{\Gamma}^\alpha_{\mu\lambda} - g_{\mu\alpha}\widetilde{\Gamma}^\alpha_{\lambda\nu} = 0\ , \qquad (2b)$$

where

$$\widetilde{\Gamma}^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} + \tfrac{2}{3}\delta^\lambda_\mu \Gamma_\nu\ , \quad 2\Gamma_\nu \equiv \Gamma^\lambda_{\nu\lambda} - \Gamma^\lambda_{\lambda\nu}\ , \quad g^{\mu\alpha}g_{\nu\alpha} = \delta^\mu_\nu\ . \qquad (2c)$$

Equation (2b) can be solved for $\widetilde{\Gamma}^\lambda_{\mu\nu}$ in terms of the metric. The algebraic condition $\widetilde{\Gamma}^\lambda_{\mu\lambda} - \widetilde{\Gamma}^\lambda_{\lambda\mu} = 0$ which follows from (2c) then implies the constraint $\partial_\mu[\sqrt{-g}(g^{\mu\nu} - g^{\nu\mu})] = 0$, while the field Γ_ν enters as the associated Lagrange multiplier. In the following we shall deal with the second-order form of these field equations, i.e., just (2a) with $\widetilde{\Gamma}^\lambda_{\mu\nu} = \widetilde{\Gamma}^\lambda_{\mu\nu}(g)$. [As for all theories involving torsion, there is of course a difference between the above model and the naive "second order" one defined by the Lagrangian (1) but with $\Gamma^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu}(g)$ determined by $\partial_\lambda g_{\mu\nu} - g_{\alpha\nu}\Gamma^\alpha_{\mu\lambda} - g_{\mu\alpha}\Gamma^\alpha_{\lambda\nu} = 0$ [3]. The latter theory was eliminated in [5, 7]].

We begin our analysis with an expansion of the NGT field equations in powers of the antisymmetric part $B_{\mu\nu}$ of the metric but treating its symmetric part $G_{\mu\nu}$ nonperturbatively. The relevant equations are

$$\bar{R}_{\mu\nu}(G) = 0\ , \qquad (3a)$$

$$\bar{D}^\alpha H_{\mu\nu\alpha} - 4\bar{R}^\alpha{}_\mu{}^\beta{}_\nu(G)B_{\alpha\beta} = \tfrac{4}{3}(\partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu)\ , \qquad (3b)$$

$$\bar{D}^\mu B_{\mu\nu} = 0\ , \qquad (3c)$$

where $H_{\mu\nu\alpha}$, the cyclic curl of $B_{\mu\nu}$, is the field strength; all operations are in the background G space, as emphasized by the bars. Here (3a) is the order B^0 part, while (3a) and (3b) are the order B^1 parts.

Were it not for the curvature term $\bar{R}B$ in (3b), the B–field equations would describe the anti-symmetric gauge field (invariant under $\delta B_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$) expressed in the Lorentz gauge (3c) imposed by Γ_ν. [The presence of $(\partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu)$ as a source of (3b) is the usual consequence of fixing the gauge through the term $\Gamma_\nu\partial_\mu(\sqrt{-G}\,B^{\mu\nu})$ in the action.] This Lorentz condition would leave a residual invariance (with the gauge function A_μ restricted to obey the source-free Maxwell equations), which would remove the longitudinal modes of $B_{\mu\nu}$. The latter, in contrast with the transverse mode, do not carry positive-definite energy. Indeed, the standard energy definition associated with the $O(B^2)$ action responsible for (3) exhibits, beyond the usual positive contribution associated with the H^2 term in the action (transverse mode of B), nonpositive contributions from the gauge-fixing term $\sqrt{-G}\,B^{\mu\nu}(\partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu)$ (and of course from the $\bar{R}BB$ terms which appear there). All this is even apart from the asymptotic behavior problem discussed below. Still neglecting the $\bar{R}B$ term in (3b), we see that Γ_ν would also obey the source-free Maxwell equations as a consequence of the identity $\bar{D}^\nu\bar{D}^\alpha H_{\mu\nu\alpha} \equiv 0$; hence it could consistently be eliminated for all time by setting it to zero at any initial time.

However, the presence of the curvature coupling term $\bar{R}B$ completely destroys the above arguments and renders NGT inconsistent: This term is manifestly noninvariant under the residual A transformation, which implies that these ghostlike longitudinal modes remain coupled [8] ("ghostlike" means negative energy in the present classical context); correspondingly, Γ_ν also fails to decouple, because the Maxwell equation it obeys has a curvature-dependent source

$$\bar{D}^\mu\bar{D}_\mu\Gamma_\nu - \bar{D}^\mu\bar{D}_\nu\,\Gamma_\mu = -3\,\bar{D}^\mu(\bar{R}^\alpha{}_\mu{}^\beta{}_\nu B_{\alpha\beta})\ , \qquad (4)$$

so it is impossible to remove it by choosing initial conditions. Moreover,the propagation of Γ_ν has the dire physical consequence that it generates unacceptable asymptotic behavior of $B_{\mu\nu}$ in the wave zone, even if the curvature decays rapidly at infinity: the constraint (4) is an inhomogeneous wave equation for Γ_μ (in the Lorentz gauge $\bar{D}^\mu\Gamma_\mu = 0$), and so Γ_μ has the usual $1/r$ falloff at future null infinity. But then, inserting this information in the right-hand side of (3b), one finds that $B_{\mu\nu}$ (and in particular its longitudinal part) fails to vanish at future null infinity. As a result of this unphysical asymptotic behavior, the scattering of B waves by gravity can formally radiate infinite negative energy [9], a (classical) instability at variance with observation. [For completeness, let us mention that the alternative nonsymmetric model [3] obtained from the "naive second order" Lagrangian (1) with $\Gamma(g)$ determined by (2b) suffers from a different type of inconsistency when expanded about a curved background. The corresponding equations are (3a) and (3b) without the Γ terms in the right-hand side. Then, the B field equations imply that the right-hand side of (4) must vanish, which is an unacceptably strong local constraint on the possible initial values of B.] Note that these inconsistencies occur at the first nontrivial level beyond flat space, as is typical of higher spin matter-gravity coupling problems.

In conclusion, not only are ghost modes present in NGT but they do not decay at infinity, which suggests violent instability against their radiation in gravitational processes. A recent claim to the contrary is manifestly incorrect [10]. Therefore NGT is not viable even as a phenomenological model. The more general question, whether there are consistent "geometrical" models (homogeneous of second-derivative order) other than "Einstein plus $H^2_{\lambda\mu\nu}$," is investigated in [7]. There, it is argued that no useful deformation of the corresponding linear B invariance exists, so that the only way out is to construct an infinite series of "geometrical" terms contrived to reproduce "$R + H^2_{\lambda\mu\nu}$" and cancel all higher terms order by order in the B expansion [11]; this is of course not a very unified theory.

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[8] The incorrect claims to the contrary of [5] can be traced to a sign error in his version of our (3b), as now recognized in P.F. Kelly, erratum to [5], Class. Quantum Grav. (to be published).
[9] When comparing our results with those of T.P. Krisher, Phys. Rev. D **32**, 329 (1985), we found that he had made crucial sign mistakes in going from his Eq. (2.21c) to his final "NGT luminosity formula" (3.16). The signs of the NGT terms in (3.16) should be reversed. With corrected signs his results (4.39) for the "radiation fields to lowest post-Newtonian order" imply that NGT waves radiate a negative flux of dipole radiation. This paper also did not consider the asymptotic problems in NGT, which invalidates in particular the formula (4.39c) and the apparent finite dipolar flux it implies. See [7] for a fuller discussion of these issues.
[10] The error in J.W. Moffat and D.C. Tatarski, University of Toronto Report No. UTPT-92-01 (unpublished), is readily recognized by noticing that their antisymmetric field equation (4.4f) is an ordinary differential equation for the function of one variable $\alpha(u)$ with coefficients also depending on the other variables r and θ. Therefore in the general case of an arbitrary general relativistic "news function" $c(u,\theta)$ it admits only the trivial solution $\alpha(u) = 0$ [this conclusion already follows from considering the leading terms in the Bondi expansion of the antisymmetric field equation, as we have checked explicitly].
[11] An example of this construction to order B^2 is given in R.B. Mann, Class. Quantum Grav. **6**, 41 (1989).