Network Source Coding Using Entropy Constrained Dithered Quantization *

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Assuming the squared error distortion measure, we bound the performance achieved by using *scalar* entropy constrained dithered quantization (SECDQ) [1] to build multi-resolution (MR), multiple access (MA), and broadcast system (BS) source codes.

In MR coding, let Q_j be a uniform scalar quantizer with basic cell $(-\Delta_j/2, \Delta_j/2]$, where for each $j \in \{1, \ldots, L\}$, $\Delta_j = \Delta_{j-1}/M_j$ for some integer $M_j \geq 1$. Assume dither Z is a random variable uniformly distributed on $(-\Delta_1/2, \Delta_1/2]$ and known to the encoder and decoder. In resolution j, the encoder uses a universal conditional entropy coder to describe $Y_j = Q_j(X + Z)$ to the decoder at rate $H(Y_j|Y_{j-1}, Z)$. The decoder reconstructs X as $\hat{X} = Y_j - Z$. The rate redundancy $L_j = \sum_{i=1}^{j} H(Y_i|Y_{i-1}, Z) - R(D_j)$ is the difference between the total rate at resolution j with expected distortion D_j and the source's rate-distortion bound $R(D_j)$ at the same distortion.

The following results apply for arbitrary source distributions.

Theorem 1 *If there exist integers $\{M_j\}$ such that the target distortions satisfy $D_j = D_{j-1}/M_j^2$ for all j, then MR-SECDQ gives $L_j < 0.7547$.*

Theorem 2 *MR-SECDQ guarantees $L_j < 1.7547$ (for any target distortions $\{D_j\}$).*

Theorem 3 *Time-sharing between MR-SECDQ resolutions gives $L_j < 1.096$.*

An MA-SECDQ encoder describes X_1 and X_2 using two independent SECDQs. The decoder builds reconstructions $\hat{X}_j = \alpha_j(Q_1(X_1+Z_1)-Z_1)+\beta_j(Q_2(X_2+Z_2)-Z_2)$, $j = 1, 2$. The resulting performance is close to the optimum for certain sources, e.g., joint Gaussian sources with small covariance coefficients. We use similar strategies in BS systems and obtain constant upper bounds on the differences between the rates of BS-SECDQ and the inner bound of the achievable rate region of BS source codes [2].

Finally if a K-dimensional lattice ECDQ (KECDQ) and an optimal linear estimator are used in MR coding, we have the following result.

Theorem 4 *MR-KECDQ gives $L_j \leq (1/2)(\sum_{i=1}^{j} \log(2\pi e G_{Ki}) + 1)$, where G_{Ki} is the normalized second moment of the lattice at resolution i, giving* $\lim_{K\to\infty} L_j \leq 1/2, \forall j$.

References

[1] J. Ziv. On universal quantization. *IEEE Trans. on Information Theory.* May 1985.

[2] R. M. Gray and A. D. Wyner. Source Coding for a simple network. *Bell Systems Technical Journal.* November 1974.

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