Axion detection via Topological Casimir Effect

ChunJun Cao,[a] Ariel Zhitnitsky[b]

[a] *Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California, 91125, USA*

[b] *Department of Physics and Astronomy, University of British Columbia, Vancouver, B.C. V6T 1Z1, Canada*

E-mail: cjcao@caltech.edu, arz@phas.ubc.ca

ABSTRACT: We propose a new table-top experimental configuration for the direct detection of dark matter axions with mass in the $(10^{-6}\text{eV} - 10^{-2}\text{eV})$ range using non-perturbative effects in a system with non-trivial spatial topology. Different from most experimental setups found in literature on direct dark matter axion detection, which relies on $\dot{\theta}$ or $\vec{\nabla}\theta$, we found that our system is in principle sensitive to a static $\theta \geq 10^{-14}$ and can also be used to set limit on the fundamental constant θ_{QED} which becomes the fundamental observable parameter of the Maxwell system if some conditions are met. Connection with Witten effect when the induced electric charge e' is proportional to θ and the magnetic monopole becomes the dyon with non-vanishing $e' = -e\frac{\theta}{2\pi}$ is also discussed.

Contents

1 Introduction and Motivation

The leitmotiv of the present work is related to the fundamental parameter θ in the Maxwell Electrodynamics, as well as the axion field related to this parameter. The θ parameter was originally introduced in Quantum Chromodynamics (QCD) in the 70s. Although the term can be represented as a total derivative and does not change the equation of motion, it is known that this parameter is a fundamental physical parameter of the system on the non-perturabative level. In particular, $\theta \neq 0$ introduces $\mathcal{P}$ and $\mathcal{CP}$ violation in QCD, which is most well captured by the renowned strong $\mathcal{CP}$ problem. A formal and deep reason of why the θ becomes a real physical parameter is that the topological mapping $\pi_3[SU(N)] = \mathbb{Z}$ in a 3+1 dimensional world is nontrivial. This rich topological structure leads to the generation of physically identical (but topologically distinct) sectors which play the key role in non-perturbative formulation of the theory.

The strong $\mathcal{CP}$ problem in QCD problem was resolved by promoting the fundamental parameter θ to a dynamical axion $\theta(x)$ field, see original papers [1–7] and review articles

[8–13]. However, the axion has not yet been discovered 40 years after its initial formulation. Still, it remains one of the most interesting resolutions of the strong $\mathcal{CP}$ problem to date, which has also led to numerous proposals for direct dark matter searches. Here we list but a fraction of the new (and old) ideas [14–26] related to the axion search experiments.

On the other hand, one may also discuss a similar theta term in QED. It is normally assumed that a similar θ_{QED} parameter in the abelian Maxwell Electrodynamics is unphysical (if magnetic monopoles are absent), and can be always removed from the system. The arguments are precisely the same as given above and based on the observation that the θ_{QED} term does not change the equation of motion. However, in contrast with QCD, the topological mapping for the abelian gauge group $\pi_3[U(1)] = 0$ is trivial. This justifies the widely accepted understanding that θ_{QED} does not affect any physical observables and can be safely removed from the theory.

While these arguments are indeed correct when the theory is defined in infinity large 3+1 dimensional Minkowski space-time we want to point out that the theta term does yield a physical effect when the Maxwell system is defined on a non-simply connected, compact manifold with non-trivial $\pi_1[U(1)] = \mathbb{Z}$ such as a ring. In this case, the physics related to pure gauge configurations describing the topological sectors does not reduce to triviality when one removes all unphysical degrees of freedom as a result of gauge fixing when the Maxwell theory is quantized. The phenomena, in all respects, are very similar to the Aharonov-Bohm effect when the system is highly sensitive to pure gauge (but topologically nontrivial) configurations.

Indeed, we claim that θ_{QED} parameter is also a physical parameter in the Maxwell electrodynamics when the system is formulated on a nontrivial manifold. This is precisely a deep reason why a constant (naively unphysical) parameter θ may enter the physical observables in the axion Maxwell electrodynamics[1]. Precisely these contributions lead to the explicit θ-dependent effects, which cannot be formulated in terms of conventional propagating degrees of freedom. In fact, most of the effects[2] which are subject of the present work are non-perturbative in nature as they enter the observables as $\exp(-1/e^2)$ and cannot be seen in perturbation theory. Our explicit computations in next sections support this claim. One should emphasize here that while parametrically $\exp(-1/e^2)$ is exponentially small, numerically this factor could be of order of one due to a special design of a geometrical configuration.

Our result also implies that some physical observables, when considering a setup with nontrivial topology, can be proportional to θ, as opposed to $\dot{\theta}$ as commonly assumed or discussed for perturbative computations. It may also find important applications in the axion search experiments where some observables are proportional to the static time-independent θ, and, in general, do not vanish even when $\dot{\theta} = 0$.

Another important implication of our findings is that some physical observables may

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[1]Here and in what follows we use θ rather than θ_{QED} to simplify notations. It should not confuse the readers as the only θ parameter we have in the present work is the θ which couples exclusively to the gauge $E\&M$ fields, because we do not include effects related to gluons and fermions in our discussion.

[2]The exception is section 6 where a nontrivial topology is enforced by the external magnetic field rather than by a nontrivial non-simply connected manifold.

not be expressible in terms of propagating degrees of freedom, such as transverse photons. In other words, there are so-called "non-dispersive" contributions to some correlation functions which are physical and observable, but cannot be expressed in terms of any "absorptive" contributions[3] which carry (through the dispersion relations) only the information about the "dispersive" portion of the correlation functions.

The practical implication of this claim is that there are some θ-dependent contribution to the vacuum energy, which cannot be expressed in terms of any propagating degrees of freedom. Precisely this type of non-perturbative contribution is related to the topological sectors of the axion Maxwell electrodynamics and the tunnelling transitions between them. The very same physical effects lead to the extra term in the vacuum pressure which was dubbed in [27] as the Topological Casimir Effect (TCE), representing an additional non-perturbative contribution to the conventional Casimir Effect [28] and which cannot be expressed in terms of the physical propagating transverse photons.

The main goal of the present work is to elaborate on possible new θ-dependent phenomena (mostly related to the axion search experiments) which originated from the topological sectors in the Maxwell electrodynamics.

The organization of this paper is as follows. In section 2 we introduce some notations related to the axion physics. We also review a variety of topological phenomena related to the θ term. In section 3 we generalize the construction of [27, 29] of the topological portion of the partition function $\mathcal{Z}_{\text{top}}(\tau, \theta)$ to include the θ term in the Euclidean path integral formulation. We explicitly show that a number of observables in the Maxwell electrodynamics are proportional to θ rather than $\dot{\theta}$ as the partition function $\mathcal{Z}_{\text{top}}(\tau, \theta)$ itself depends on θ. As the corresponding formulae play the key role in our studies, we reproduce in section 4 the relevant expressions using the Hamiltonian approach formulated in the Minkowski space time, in contrast with Euclidean formulation of the path integral. Finally, in sections 5, 6 we propose a number of experimental setups for possible axion search experiments. The crucial element in the design of the suggested apparatus is the presence of a topologically nontrivial configuration (such as a ring) when the topologically distinct sectors of the Maxwell system may manifest themselves and play the key role. We conclude in Section 7 where we summarize the main results of our findings.

2   Axion θ field and variety of topological phenomena

We introduce, in the conventional way, the axion field $a(t) = f_a \theta(t)$ from which Maxwell equations receive corrections. The relevant term enters the Maxwell equation as follows [30]

$$\vec{\nabla} \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t} - g_{a\gamma\gamma} \vec{B} \frac{\partial a}{\partial t}, \quad g_{a\gamma\gamma} = \frac{K_{a\gamma\gamma}\alpha}{2\pi f_a}, \tag{2.1}$$

where the spatial variation for the axion field $\sim \vec{\nabla} a$ is consistently neglected because we assume it to be small and thus irrelevant for the present work. The model dependent

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[3]It would contradict a "folk theorem" that the S matrix contains all information about all physical observables. We thank Mark Wise for providing and explaining this observation.

numerical coefficients are: $K_{a\gamma\gamma} \simeq 0.75$ for DFSZ model and $K_{a\gamma\gamma} \simeq -1.92$ for KSVZ model. We treat the axion field as a classical coherent field [11] with normalization

$$a(t) = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \cos(\omega_a t + \phi), \quad \rho_{\text{DM}} \simeq \frac{0.3 \text{ GeV}}{\text{cm}^3}. \qquad (2.2)$$

The coherence of the field is determined by the de Broglie wave length λ_D of the axions with a typical mass $m_a \sim 10^{-4}$eV,

$$\lambda_D = \frac{\hbar}{m_a v_a} \simeq 10 \cdot \left(\frac{10^{-4}\text{eV}}{m_a}\right) m. \qquad (2.3)$$

Our normalization corresponds to the assumption that the axions represent the dark matter (or at least some portion of it) of the Universe.

The lower limit on the axion mass, as it is well known, is determined by the requirement that the axion contribution to the dark matter density does not exceed the observed value $\Omega_{\text{dark}} \approx 0.23$. There is a number of uncertainties in the corresponding estimates. We shall not comment on these subtleties by referring to the review papers [8–13]. The corresponding uncertainties are mostly due to the remaining discrepancies between different groups on the computations of the axion production rates due to the different mechanisms such as misalignment mechanism versus domain wall/string decays. If one take for granted that the misalignment mechanism is dominant, which is normally assumed to be always the case if inflation occurs after the Peccei-Quinn (PQ) phase transition, then the estimate is:

$$\Omega_{(\text{DM axion})} \simeq \left(\frac{6 \cdot 10^{-6}\text{eV}}{m_a}\right)^{\frac{7}{6}} \text{ for misalignment mechanism.} \qquad (2.4)$$

This formula essentially states that the axion of mass $m_a \simeq 2 \cdot 10^{-5}$ eV saturates the dark matter density observed today, while the axion mass in the range of $m_a \geq 10^{-4}$ eV contributes very little to the dark matter density. This claim, of course, is entirely based on estimate (2.4) which accounts only for the axions directly produced by the misalignment mechanism.

There is another mechanism of the axion production when the PQ symmetry is broken after inflation. In this case the string-domain wall network produces a large number of axions such that the dark matter density might be saturated by heavier axion mass $m_a \simeq 10^{-4}$ eV. We briefly comment on the so-called $N_{\text{DW}} = 1$ domain walls, which might be formed[4] even if the inflation occurs after the PQ phase transition as argued in [31]. If this is indeed the case, then the axion production due to the domain wall decay will be the dominating mechanism even when the PQ symmetry is broken before inflation, which is normally considered to be the case when only the misalignment mechanism (2.4) could be operational. This is one of the reasons to justify the value $m_a \simeq 10^{-4}$eV shown in (2.3).

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[4]It is normally assumed that the domain walls are the topological configurations which interpolate between physically distinct vacuum states. Such domain walls obviously cannot be produced if inflation occurs after the PQ phase transition, when there is a unique physical vacuum state within a horizon. The key point advocated in [31] is that the $N_{\text{DW}} = 1$ domain walls correspond to the configurations which interpolate between *topologically distinct* but physically *identical* states. The inflation cannot remove these states outside of the horizon. Somehow this option had been overlooked in previous studies on the subject.

We now return to analysis of the Maxwell equations in the presence of the axion field given by eq.(2.1). The physical meaning of extra term $\sim \dot{a}$ in (2.1) is quite obvious: the axion generates the extra current $\vec{j}_a$ in the presence of the axion field

$$\vec{j}_a = -\dot{\theta} \, \frac{K_{a\gamma\gamma}\alpha}{2\pi} \, \vec{B}, \quad \alpha \equiv \frac{e^2}{4\pi}. \tag{2.5}$$

This anomalous current points along magnetic field in contrast with ordinary $E\&M$, where the current is always orthogonal to $\vec{B}$. Most of the recent proposals [14–26] to detect the dark matter axions are precisely based on this extra current (2.5).

We would like to make a few comments on the unusual features of this current. First of all, the generation of the very same non-dissipating current (2.5) in the presence of θ has been very active area of research in recent years. However, it is with drastically different scale of order $\Lambda_{\rm QCD}$ instead of m_a. The main driving force for this activity stems from the ongoing experimental results at RHIC (relativistic heavy ion collider) and the LHC (Large Hadron Collider), which can be interpreted as the observation of such anomalous current (2.5).

The basic idea for such an interpretation can explained as follows. It has been suggested by [32, 33] that the so-called $\theta_{\rm ind}$-domain can be formed in heavy ion collisions as a result of some non-equilibrium dynamics. This induced $\theta_{\rm ind}$ plays the same role as fundamental θ in (2.5), and leads to a number of $\mathcal{P}$ and $\mathcal{CP}$ odd effects, such as chiral magnetic effect, chiral vortical effect, and charge separation effect, to name just a few. This field of research initiated in [34] became a hot topic in recent years as a result of many interesting theoretical and experimental advances, see recent review papers [35, 36] on the subject.

For our present studies it is important to realize that the anomalous non-dissipating current (2.5) can be interpreted as a manifestation of the Witten's effect [37] when the magnetic monopole becomes an electrically charged object, the dyon. Indeed, as argued in [34] the electric field will always be induced in the presence of $\theta \neq 0$ if an external magnetic field is also present in the system, see (3.7). This induced electric field $\langle \vec{E} \rangle_{\rm ind} \sim \theta \vec{B}$ will be generating the non-dissipating current (2.5) in plasma and may lead to a separation of charges[5].

A similar argument also suggests that a magnetic dipole moment will always generate the electric dipole moment in the presence of the θ background. We elaborate on this argument in section 3.3, see eqs. (3.11), (3.12), (3.13). Independent explicit computations [22, 23] also support this argument.

Our final comment (on this large number of topological phenomena emerging as a result of the θ term) is as follows. It is commonly assumed that the physics related to the θ parameter in electrodynamics must decouple in the limit $\dot{\theta} \to 0$, so that all physical phenomena must be θ independent if $\dot{\theta} = 0$. The conventional argument is often based

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[5]This separation of charges observed at RHIC and the LHC can be interpreted as the manifestation of the charge separation effect [34–36]. This effect can be also interpreted as the chiral magnetic effect (CME) represented by eq. (2.5). The effect was dubbed as CME because $\dot{\theta} = \mu_5$ can be interpreted as the chiral chemical potential μ_5. Relation $\dot{\theta} = \mu_5$ can be easily derived using $U(1)_A$ chiral time-dependent transformation in the path integral.

on observation that the θ term in this limit can be represented as the total derivative in the action, and, therefore, cannot change the equation of motion or influence physical observations, see [35] for review. However, as we mentioned in the Introduction, these arguments do not hold for relevant non-perturbative effects when the system is formulated on a nontrivial manifold[6]. Explicit computations in section 3 support this claim.

3   Topological partition function in presence of the axion.

We want to construct the partition function $\mathcal{Z}_{\text{top}}(\theta)$ in the presence of the axion field $\theta(t)$. Mathematically, we consider the usual 3-dimensional space with an infinitely long cylindrical portion removed, the space of consideration is thus homotopic to S^1. For simplicity, we align the axis of the (rectangular) cylinder with the z-direction. The single periodic direction is provided by the S^1 factor of the cylinder. Certainly, the real spatial topology for Minkowski space is trivial, and one may argue that such constructions can never be physically realized unless the non-trivial topology is provided by the actual Universe. Such will be the case if the Universe is a torus or has handles from wormholes. We here make a few arguments that although change in real spatial topology cannot be controlled in an experimental setting, it can be effectively realized in real space at low energies. For instance, while a conducting ring does not change the actual spatial topology, it does enforce the S^1 topology for some low energy electrons below some binding energy.

For our present work, one possible realization is by inserting an actual cylinder with cross-sectional size a lot smaller than its length along z. We claim that the periodic boundary condition is indeed enforced up to some energy cut off provided by the material, as in the case of Casimir effects [38]. As expected, the imposed material boundary becomes transparent for modes above such cut offs. Nevertheless, the boundary condition is enforced in the regime in which vacuum Casimir effect is dominant. The validity of such arguments are indeed supported by the actual measurement of Casimir pressure in vacuum.

As expected, the correction to Casimir pressure from such non-perturbative effects is a few orders of magnitude smaller than the current detection limit [27]. Nevertheless, by designing an intrument with many identical units, some related effects such as induced dipole moments can be magnified. We also discuss the effects when a strong external field is threading through a region of space. In such cases, non-trivial topology can also enter even in the absence of material boundaries. By threading a sufficiently strong external field in some region, it is sufficient to consider the zero field region for low energy effects. Such region will be homotopic to S^1 as desired. Processes that correspond to unwinding have to pass through the non-zero flux region which has a higher energy proportional to the sqaure of the strengths of the applied fields. However, we will not give explicit computations in this paper to demonstrate the explicity equivalence in the low energy limit, which involves non-perturbative corrections and is highly non-trivial as a topic by itself. In practice, of

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[6]This phenomena is in fact related to the so-called Gribov ambiguities which are well known to emerge in non-abelian gauge theories. These ambiguities also appear in the Maxwell electrodynamics when it is quantized on a non-simply connected compact manifold, see some comments and relevant references on the subject in [27].

course, the cylinder is never infinitely long and external fluxes will not extend to infinity. However, topology is enforced for sufficiently long cylinders or extended fluxes, when we consider thin spatial slices cutting through the cylinder which are effectively 2-dimensional and are far from the ends. A well-known example will be the familiar Aharonov-Bohm setup where interference pattern can be detected despite the finite size of the solenoid. This sets the range of validity for the computations throughout this paper.

Working under the assumptions that such non-trivial topology can indeed be effectively realized, we now follow [27, 29] in constructing the topological partition function, which is sensitive to the topological sectors of the system. As a crucial observation from previous work, we note that the gauge field A^μ is only periodic up to a large gauge transformation in the periodic direction, and results in formation of the topological sectors $|k\rangle$. In this section we use conventional Euclidean path integral formulation and then reproduce the key results using the Hamiltonian analysis in section 4.

For the sake of clarity, we briefly review the fluctuations due to the magnetic fluxes [27] in section 3.1 and neglect the possible electric fluxes discussed in ref. [29] in the periodic direction. In subsection 3.2 we generalize this construction to include the axion θ field. Finally, in subsection 3.3 we argue that the obtained results can be interpreted as the manifestation of the Wittens' effect [37] when the magnetic monopole becomes electrically charged object, the dyon.

This construction explicitly shows that the θ is a physical parameter of the system even though it enters the Lagrangian (3.6) with operator which can be represented as a total derivative. One can explicitly see that all effects which are subject of the present work are non-perturbative in nature as they enter the partition function (3.4) as $\exp(-1/e^2)$ and cannot be seen in perturbation theory.

3.1 Maxwell system on a compact manifold

In what follows we simplify our analysis by considering a simplest case with contributions from winding topological sectors $|k\rangle$ in the z-direction only. The classical instanton configuration in Euclidean space which describes the corresponding tunnelling transitions can be represented as follows:

$$A^\mu_{\text{top}} = \left(0, \ -\frac{\pi k}{eL_1 L_2}x_2, \ \frac{\pi k}{eL_1 L_2}x_1, \ 0\right), \tag{3.1}$$

where k is the winding number that labels the topological sector, and L_1, L_2 are the dimensions of the rectangular cylinder in the x and y-directions respectively. The height of the cylinder is L_3. This terminology ("instanton") is adapted from similar studies in 2d QED [27] where corresponding configuration in $A_0 = 0$ gauge describe the interpolation between pure gauge vacuum winding states $|k\rangle$.

The physical meaning of the configurations (3.1) is that they describe the tunnelling processes which occur in the system between different winding sectors $|k\rangle$. For relatively small systems and finite temperature β the probability for the tunnelling processes is not small, and must be taken into account to describe the physical vacuum states.

A key observation from [27] is that the topological portion $\mathcal{Z}_{\text{top}}$ decouples from quantum fluctuations, $\mathcal{Z} = \mathcal{Z}_{\text{quant}} \times \mathcal{Z}_{\text{top}}$ such that the quantum fluctuations from propagating photons do not depend on topological sector k and can be computed in topologically trivial sector $k = 0$. Indeed, the cross term

$$\int \mathrm{d}^4 x \; \vec{B} \cdot \vec{B}_{\text{top}} = \frac{2\pi k}{eL_1 L_2} \int \mathrm{d}^4 x \; B_z = 0 \tag{3.2}$$

vanishes because the magnetic portion of quantum fluctuations in the z-direction, represented by $B_z = \partial_x A_y - \partial_y A_x$, is a periodic function as $\vec{A}$ is periodic over the domain of integration. This technical remark in fact greatly simplifies our analysis.

The classical action for configuration in the presence of the uniform static external magnetic field B_z^{ext} therefore takes the form

$$\frac{1}{2} \int \mathrm{d}^4 x \left(\vec{B}_{\text{ext}} + \vec{B}_{\text{top}} \right)^2 = \pi^2 \tau \left(k + \frac{\theta_{\text{eff}}}{2\pi} \right)^2 \tag{3.3}$$

where parameter τ is defined as $\tau = 2\beta L_3 / e^2 L_1 L_2$, while the effective theta parameter $\theta_{\text{eff}} \equiv eL_1 L_2 B_{\text{ext}}^z$ is expressed in terms of the external magnetic field B_{ext}^z. Therefore, the partition function in the presence of the uniform magnetic field is given by [27, 39]

$$\mathcal{Z}_{\text{top}}(\tau, \theta_{\text{eff}}) = \sqrt{\pi \tau} \sum_{k \in \mathbb{Z}} \exp \left[-\pi^2 \tau \left(k + \frac{\theta_{\text{eff}}}{2\pi} \right)^2 \right]. \tag{3.4}$$

This system in what follows will be referred as the topological vacuum ($\mathcal{TV}$) because the propagating degrees of freedom, the photons with two transverse polarizations, completely decouple from $\mathcal{Z}_{\text{top}}(\tau, \theta_{\text{eff}})$.

The dual representation for the partition function is obtained by applying the Poisson summation formula such that (3.4) becomes

$$\mathcal{Z}_{\text{top}}(\tau, \theta_{\text{eff}}) = \sum_{n \in \mathbb{Z}} \exp \left[-\frac{n^2}{\tau} + in \cdot \theta_{\text{eff}} \right]. \tag{3.5}$$

Formula (3.5) justifies our notation for the effective theta parameter θ_{eff} as it enters the partition function in combination with integer number n. One should emphasize that integer number n in the dual representation (3.5) is not the integer magnetic flux k. Furthermore, the θ_{eff} parameter which enters (3.4, 3.5) is not a fundamental θ parameter which is normally introduced into the Lagrangian in front of $\vec{E} \cdot \vec{B}$ operator. This fundamental θ term describing the physical axion field will be introduced in next section. Rather, this parameter θ_{eff} should be understood as an effective parameter representing the construction of the $|\theta_{\text{eff}}\rangle$ state for each 2-dimensional slice with non-trivial $\pi_1[U(1)]$ in the four dimensional system.

3.2 Euclidean partition function in the presence of θ

Our goal now is to generalize formula (3.4) to include the fundamental $\theta(t)$ into the partition function $\mathcal{Z}_{\text{top}}(\tau, \theta, \theta_{\text{eff}})$. As such, we insert an extra θ term into the Euclidean action (3.3),

$$S_E = \int d^4x \left[\frac{1}{2}\vec{B}^2 + \frac{1}{2}\vec{E}^2 + i\frac{K_{a\gamma\gamma}\alpha}{\pi}\theta\vec{E}\cdot\vec{B} \right]. \tag{3.6}$$

Note that our normalization for the topological term is different from conventional definition of θ in the so-called "axion electrodynamics" in condensed matter physics which corresponds to $K_{a\gamma\gamma} = 1$ in our equation (3.6). Another comments is related to complex factor i in the definition (3.6). This is the result of the Euclidean signature when the time and electric field are in fact imaginary variables.

Variation of the action S_E with respect to electric field $\delta/\delta\vec{E}$ returns

$$\langle\vec{E}\rangle_{\text{ind}} = -i\frac{K_{a\gamma\gamma}\alpha}{\pi}\theta\left(\vec{B}_{\text{ext}} + \langle\vec{B}\rangle_{\text{ind}}\right), \tag{3.7}$$

which implies that the electric field will be always induced in the presence of θ term. In what follows we want to simplify all formulae and consider the limit of $\theta \to 0$, in which case the computations of the rhs $\langle\vec{B}\rangle_{\text{ind}}$ entering (3.7) can be carried out with the partition function (3.4) computed at $\theta = 0$. In this approximation the induced electric field assumes the form

$$\langle E^z\rangle_{\text{ind}} = -i\theta \cdot \frac{K_{a\gamma\gamma}\alpha}{\pi} \cdot \frac{\sqrt{\tau\pi}}{\mathcal{Z}_{\text{top}}(\tau, \theta_{\text{eff}})} \tag{3.8}$$
$$\times \sum_{k\in\mathbb{Z}}(B^z_{\text{ext}} + \frac{2\pi k}{eL_1L_2})\exp[-\tau\pi^2(k + \frac{\theta_{\text{eff}}}{2\pi})^2].$$

This formula (written in Euclidean metric) represents the main result of this section. In next section we interpret the obtained result in from different perspective.

On the other hand, we can also consider the dual picture whereby an external electric field E_{ext} is applied and a magnetic flux is induced from nonzero θ. Although not the physical configuration we consider in this paper, we note the duality becomes manifest, when the system is endowed with an additional z-periodicity. As such, one similarly obtains, in Euclidean metric

$$\langle B^z\rangle_{\text{ind}} = -i\theta\frac{K_{a\gamma\gamma}\alpha}{\pi}\left(E^z_{\text{ext}} + \langle E^z\rangle_{\text{ind}}\right), \tag{3.9}$$

whereby a magnetic flux is induced by external electric field, as expected. The induced electric field $\langle E^z\rangle_{\text{ind}}$ in this expression is related to the electric fluxes, which can be in principle computed [29], is dropped here in the current configuration where z-periodicity is lacking.

A few comments regarding (3.7) are in order. Naively, (3.7) seems to suggest that the electric field $\langle\vec{E}\rangle_{\text{ind}}$ will be always induced regardless of the topological features of the space, which we claim plays the crucial role. Nevertheless, this naive objection is incorrect as we argue below. Just as the electric charge will be always induced in the background of the magnetic monopole (see eq. (3.13) and comments after this formula below), similar non-triviality is induced by a magnetic flux (string) which is always accompanied at large

distances by a pure gauge (but topologically nontrivial) vector potential, see section 6 and eq. (6.1) with details analysis. Precisely this pure gauge but topologically nontrivial vector potential is responsible for a nontrivial mapping $\pi_1[U(1)] = \mathbb{Z}$ in configurational space, which plays the key role in the entire construction. In other words, the external magnetic flux itself, like the case of Aharonov-Bohm effect, is providing (and enforcing) the required non-trivial topology. Therefore, for external field with finite extent, the θ-term does not reduce to zero on the boundary as usual, but rather generates a number of nontrivial effects. This θ dependence in (3.7) emerges, of course, because the object of our studies is not the QED vacuum (where θ parameter indeed can be safely removed), but rather a heavy sector with non-vanishing magnetic flux, see section 6 with additional explanations and comments on this matter.

3.3 Interpretations

First of all, the expression for the induced electric field (3.8) is written in Euclidean metric where all path integral computations related to the tunnelling transitions are normally performed. As usual, we assume that analytical continuation is valid and, therefore, the same formula holds for the physical electric field in Minkowski space-time, which is obtained by removing complex "i" in front of eq. (3.8).

Our next comment is about the interpretation of the obtained formula (3.8). First, we would like to interpret the induced magnetic field $\langle B \rangle_{\text{ind}}$ in terms of the induced magnetic moment in each given topological sector k

$$\langle m_{\text{ind}} \rangle = - \left(B_{\text{ext}}^z + \langle B \rangle_{\text{ind}} \right) L_1 L_2 L_3 = -\frac{\sqrt{\tau\pi}}{\mathcal{Z}_{\text{top}}} \tag{3.10}$$
$$\times \frac{2\pi L_3}{e} \sum_{k \in \mathbb{Z}} \left(\frac{\theta_{\text{eff}}}{2\pi} + k \right) \exp[-\tau\pi^2 (k + \frac{\theta_{\text{eff}}}{2\pi})^2].$$

Furthermore, the corresponding magnetic moment in k sector can be also understood in terms of the induced non-dissipating persistent currents which flow along infinitely thin boundary of the system as discussed in [39].

Novel element emerges as a result of θ parameter. In this case the induced electric field (3.8) can be interpreted in terms of the induced dipole moment

$$\langle d \rangle_{\text{ind}} = -\langle E \rangle_{\text{ind}} L_1 L_2 L_3 = \theta \cdot \frac{K_{a\gamma\gamma}\alpha}{\pi} \frac{\sqrt{\tau\pi}}{\mathcal{Z}_{\text{top}}(\tau, \theta_{\text{eff}})} \tag{3.11}$$
$$\times \frac{2\pi L_3}{e} \sum_{k \in \mathbb{Z}} \left(\frac{\theta_{\text{eff}}}{2\pi} + k \right) \exp[-\tau\pi^2 (k + \frac{\theta_{\text{eff}}}{2\pi})^2],$$

where we adopted the Minkowski signature for the physical electric field in contrast with expression (3.8) derived in Euclidean space-time. The same expression for the electric dipole moment can be thought as accumulation of the charges on the metallic plates of the area $L_1 L_2$ and separated by distance L_3 along z.

Comparison between (3.10) and (3.11) suggests that the induced electric field in the presence of θ can be thought as the Witten's effect as the electric and magnetic dipole

moments are related:

$$\langle d \rangle_{\text{ind}} = -\theta \cdot \frac{K_{a\gamma\gamma}\alpha}{\pi} \langle m_{\text{ind}} \rangle \tag{3.12}$$

which obviously resembles the Witten's relation if one represents the magnetic moment as $\langle m_{\text{ind}} \rangle = gL_3$ where g is the magnetic charge. As the magnetic charge g is quantized, $g = \frac{2\pi}{e}$, formula (3.12) can be rewritten as

$$\langle d \rangle_{\text{ind}} = -\theta \cdot \frac{K_{a\gamma\gamma}e^2}{4\pi^2}\frac{2\pi L_3}{e} = -\left(\frac{e\theta}{2\pi}\right)L_3 \cdot K_{a\gamma\gamma}. \tag{3.13}$$

This formula can be obviously interpreted as the Witten's effect when the magnetic dipole $\langle m_{\text{ind}} \rangle$ becomes also an electric dipole $\langle d \rangle_{\text{ind}}$ with the moment determined by the electric charges $e' = -(e\theta/2\pi)K_{a\gamma\gamma}$ which precisely coincides with the Witten's expression for $e' = -(e\theta/2\pi)$ if one uses the conventional normalization for the θ term with $K_{a\gamma\gamma} = 1$ according to [37].

The discussions above strongly suggest that the the effects in the bulk of the system can be represented in terms of the boundary effects: boundary induced currents, or boundary induced charges. It is not really an unexpected property of the system as we previously argued that this topological vacuum $(\mathcal{TV})$ is, in many respects, similar to a topologically ordered system [40, 41]. Therefore, the representation of the effect (3.8) in terms of the boundary sources is another manifestation of the property which is normally attributed to a system which belongs to a topologically ordered phase.

Formula (3.11) plays the crucial role in our analysis in section 5 where we describe a possible experimental set-up for the axion search experiment. It is explicitly proportional to the axion field $\theta(t)$, instead of $\dot{\theta}(t)$. This formula also explicitly demonstrates that $\theta_{\text{QED}} = $ constant is a fundamental and physically observable parameter of the theory when the system is formulated on a nontrivial manifold.

4 Hamiltonian Approach

In this section, we reproduce (3.11) using the Hamiltonian approach and confirm the results from the path-integral calculation.

Following [42], we can write down the zero-mode contribution to partition function on a 3-torus. As we have previously demonstrated, it decouples with the rest of the dynamical photon partition function. There are multiple fluxes in the other directions. For our purpose, we write only the contribution to the z-flux for zero external field,

$$Z_{\text{top}}[\tilde{\theta}] \sim \sum_{m,n} \int \frac{dl}{2\pi}\delta\left(\frac{m\tilde{\theta}}{2\pi} + l + n\right) \tag{4.1}$$
$$\times \exp\left(-\frac{\beta V}{2}\left[(\frac{el}{L_1 L_2})^2 + (\frac{2\pi m}{e L_1 L_2})^2\right]\right)$$

up to constant normalization. Here $\tilde{\theta} = -\theta\alpha K_{a\gamma\gamma}/\pi$ is from the axion coupling term. As expected, the partition function is invariant under $\tilde{\theta} \to \tilde{\theta} + 2\pi$ and nonzero theta introduces electromagnetic mixing.

Unsurprisingly, the θ-dependent induced field vanishes for all values of θ_{QED} due to cancellation in the summation. As before, we need to apply an external field to break the invariance of the exponential weight factor under $m \to -m$ and the same thing for n.

For instance, we can shift magnetic flux by adding an external B field in the z-direction. This shifts the magnetic flux, such that $2\pi m \to 2\pi m + e L_1 L_2 B_{\text{ext}}$. Setting $\theta_{\text{eff}} = e L_1 L_2 B_{\text{ext}}$, we have for total flux density

$$\langle E \rangle \sim \sum_{m,n} \left[(\frac{\theta_{\text{eff}}}{2\pi} + m)\frac{\tilde{\theta}}{L_1 L_2 e} + \frac{2\pi n}{e\beta L_3} \right] \tag{4.2}$$
$$\times \exp\left[-\frac{1}{\eta}\left(n + (m + \frac{\theta_{\text{eff}}}{2\pi})\frac{\tilde{\theta}}{2\pi} \right)^2 - \tau\pi^2 \left(m + \frac{\theta_{\text{eff}}}{2\pi} \right)^2 \right],$$

where $\eta = 2L_1 L_2 / e^2 \beta L_3$ and $\tau = 2\beta L_3 / e^2 L_1 L_2$ and we omit constant normalization factors proportional to $1/Z_{\text{top}}$. Magnetic moment can also be computed in a similar fashion. Therefore, for small θ perturbations, as is the case for coherent dark-matter axions, we induce time-dependent electric flux

$$\langle E(t) \rangle = -\frac{\alpha K_{a\gamma\gamma}}{\pi}\gamma(\theta_{\text{eff}}, \tau)\theta(t) + \mathcal{O}(\theta^2) \tag{4.3}$$

where it is important to note $\theta_{\text{eff}} \sim B_{\text{ext}}$ is external magnetic flux we apply *parallel* to the induced E-flux.

Here

$$\gamma(\theta_{\text{eff}}, \tau) \equiv \frac{\partial\langle E(\tilde{\theta})\rangle}{\partial\tilde{\theta}}\bigg|_{\tilde{\theta}=0} \tag{4.4}$$
$$= \frac{2\pi}{eL_1 L_2}\langle \frac{\theta_{\text{eff}}}{2\pi} + m \rangle\bigg|_{\tilde{\theta}=\pi} - \frac{2}{eL_3\beta\eta}\langle n^2(m + \frac{\theta_{\text{eff}}}{2\pi}) \rangle\bigg|_{\tilde{\theta}=0}.$$

Expectation values $\langle \dots \rangle$ are understood to be with respect to the topological partition function.

Note that we recover the case where the spatial topology has only one periodic direction, $S^1 \times I \times I$, by only considering the $n = 0$ sector, because there is no z-periodicity in our configuration. It precisely agrees with equations (3.8) and (3.11) derived in section 3 using drastically different technique. Formula (4.2) is in fact a more general expression than eq. (3.8) derived previously because in formula (4.2) the axion field $\tilde{\theta}$ is not assumed to be small, and furthermore, electric fluxes studied in [29] which describe $n \neq 0$ sectors, are included into the formula (4.2). Reproducing of eq. (3.8) using a different approach also adds to our confidence that the derivation is indeed correct.

5   Experimental Setup and Numerical Estimates for $|\theta_{\text{eff}}\rangle$ states

We give a few simple numerical estimates related to the induced field (3.8) and induced dipole moment (3.11) as a result of the external $\theta(t)$ field which is identified with the dark

matter axion (2.2). We also want to present proposals on possible experimental setups which would allow to measure the induced field $\langle \vec{E} \rangle_{\text{ind}}$ resulting from the DM axion entering a proposed axion detector.

For coherent DM axions which are believed to have the wavelength in the order of a few meters, we can still operate in the adiabatic regime for a device with size on the scale of mm. The typical parameters in this section are: the axion mass $m_a \simeq 10^{-4}$ eV which corresponds to the typical frequencies $\nu \sim 20$ GHz. A typical external magnetic field to be considered in this section should be the same order of magnitude as magnetic field in the quantum fluxes describing the tunnelling events. Numerically, the magnetic field is quite small, $B_{\text{ext}} \sim \theta_{\text{eff}}/(eL_1L_2) \sim 4 \cdot 10^{-5}G$ for a typical mm size samples. We also require the temperature of the system is not too low, $T \geq 10K$ which corresponds to $\beta \leq 0.2$mm. At the same time, the temperature cannot be too high as the Aharonov-Bohm coherence must be maintained in the entire system[7]. These parameters guarantee that the key dimensionless parameter $\tau \lesssim 1$ and the tunnelling transitions will occur without exponential suppression. These tunnelling transitions select a specific $|\theta_{\text{eff}}\rangle$- state. Our classification in the present section is based on this classification scheme.

In next section 6 we consider another set of parameters when the requirement on small τ can be dropped. In this case the tunnelling transitions do not occur as they are strongly suppressed, and our classification scheme will be based on the winding number topological sectors $|k\rangle$ rather than on $|\theta_{\text{eff}}\rangle$- classification which represents the superposition of the winding $|k\rangle$ states. Nevertheless, the equations (3.7) and (3.9) still hold even when tunnelling transitions are suppressed. We further elaborate on this classification scheme in section 6.

For subsections 5.2 and 5.1, we consider the simple configuration of applying the external flux using a single cylindrical solenoid with mm size. An external magnetic field is applied parallel to the principle axis by the solenoid and an electric field (parallel to the external magnetic field) will be induced in the presence of nonzero θ. For subsequent sections 5.3 and 5.4, we put additional plates near the ends of the cylinder and connect them by a (super)conducting wire.

Note that while each device, which we study in the following subsections, only yields a minute amount of observable effect, the small scale of the devices allows in principle an amplification by considering a large number, N, of such devices. For instance, for a cubic meter size detector, which is still well below the axion wavelength such that coherence is maintained, one can in principle pack N^3 mm scale construction of each unit and attains a factor of $\sim 10^9$ amplification.

5.1   The Dipole Moment

We start our numerical estimates with magnetic dipole moment given by (3.10). If somehow we manage to adjust parameters of the system such that $\tau \leq 1$ than the magnitude of $\langle m_{\text{ind}}^z \rangle$

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[7]One should remark here that there are related effects when the entire system can maintain the Aharonov-Bohm phase coherence at very high temperature $T \simeq 79$ K [43]. It is obviously more than sufficient for our purposes.

from eq. (3.10) is determined by parameter $2\pi L_3/e$ such that

$$\langle m_{\text{ind}}^z \rangle \sim \frac{2\pi L_3}{e} \sim 1.5 \cdot 10^{11} \left(\frac{e \cdot \text{cm}^2}{\text{s}} \right) \cdot \left(\frac{L_3}{1\text{mm}} \right). \tag{5.1}$$

By expressing the magnetic moment $\langle m_{\text{ind}}^z \rangle$ in units (5.1) we want to emphasize that this magnetic moment can be interpreted as the generation of the persistent current along the ring as described in [39]. In other words we interpret $\langle m_{\text{ind}}^z \rangle$ as a macroscopically large magnetic moment which is generated by coherent non-dissipating surface current $\langle J \rangle$ flowing along the ring and measured in units e/s. In this case one can represent $\langle m_{\text{ind}}^z \rangle = L_1 L_2 \langle J \rangle$ which explains our representation in form (5.1).

The value (5.1) should be compared numerically with Bohr magneton for a single electron represented in the same units,

$$\mu_B = \frac{e\hbar}{2m_e} \simeq 0.6 \left(\frac{e \cdot \text{cm}^2}{\text{s}} \right), \quad \frac{\langle m_{\text{ind}}^z \rangle}{\mu_B} \sim 10^{11}. \tag{5.2}$$

The comparison between the two numbers can be interpreted that our system effectively describes $\sim 10^{11}$ degrees of freedom which coherently produce a macroscopically large magnetic moment (5.1) and coherent persistent current $\langle J \rangle$. This enhancement is accompanied by another large factor $N^3 \sim 10^9$ mentioned above. These two large factors represent the maximum enhancement which can be achieved for a coherent axion field with $\lambda_D \sim 1\text{m}$. The enhancement factor 10^{11+9} is the same order of magnitude which is normally discussed in other axion search experiments [14–26] based on coupling of the axion with the matter fields.

The crucial element in our estimate (5.1) is that the key parameter τ should be sufficiently small, $\tau \leq 1$. This would guarantee that the vacuum transitions would not be strongly suppressed. The main assumption here is that Aharonov-Bohm (AB) phase coherence can be maintained at sufficient high temperature, which can drastically decrease parameter τ, see footnote 7.

From (5.1), one can also set to measure the magnitude of the induced electric dipole moment $\langle d \rangle_{\text{ind}}$ which will be generated exclusively as a result of the interaction with the axion field θ. Again, for a system of mm size, the dipole moment according to (3.11), (3.12), (3.13) is given by

$$\langle d \rangle_{\text{ind}} \sim 10^{-2} \theta \cdot K_{a\gamma\gamma} \left(\frac{L_3}{1\text{mm}} \right) e \cdot \text{cm} \tag{5.3}$$

For expected range of $\theta \sim 10^{-18}$, this is about 9 orders of magnitude greater than the current experimental limit of the electron electric dipole moment (EDM), $|d_e| \lesssim 10^{-29} e \cdot \text{cm}$ [44, 45]. This large enhancement factor can be attributed to the coherence of the large number of effective microscopic degrees of freedom participating in generation of the electric dipole moment (5.3) similar to generation of the magnetic dipole moment (5.1) as discussed above. The estimate (5.3) can be also compared with the current limit on neutron EDM $|d_n| < 10^{-26} e \cdot \text{cm}$ [46], $\langle d \rangle_{\text{ind}}/|d_n| \sim 10^{24} \theta \sim 10^6$.

One should emphasize that the system is placed in a static uniform magnetic field. In conventional perturbative QED (without topological sectors) even the magnetic moment (5.1) cannot be induced when a ring is placed inside of a uniform static magnetic filed. The generation of a static electric dipole moment (5.3) even a more puzzling effect within conventional QED formulated on a trivial topology. The electric dipole moment $\langle d \rangle_{\mathrm{ind}}$ could be only generated due to a background θ field, which itself could be only resulted from the passing nearby axion, or due to fundamental $\theta_{\mathrm{QED}} \neq 0$, see more comments on this last possibility in subsection 6.3.

5.2 Emission in the microwave bands

If the dipole moment (5.3) is induced due to the passing nearby axion with mass m_a, then the system will emit photons with frequency $\omega = m_a$. Assuming that the system is macroscopically large one can estimate the time-averaged intensity of the dipole radiation for each such unit using the classical expression

$$\langle I \rangle \simeq \frac{\omega^4}{12\pi c^3} \langle d \rangle_{\mathrm{ind}}^2 \sim \alpha \omega^4 \frac{(L_3 K_{a\gamma\gamma}\theta_m)^2}{12\pi^2 c^3}, \tag{5.4}$$

where θ_m is the amplitude of the coherent axion oscillation. For a device on the mm scale, again we estimate the average power from a single unit to be on the order of $10^{-13}(K_{a\gamma\gamma}\theta_m)^2 \mathrm{eV}^2$, which can be represented in conventional units as $\langle I \rangle \sim 10^{-16}(K_{a\gamma\gamma}\theta_m)^2$ watt. Such small intensity is unlikely to be observed even when a large factor N^6 for coherent emission from N^3 dipoles is inserted.

The effect could potentially be enhanced if a device is specifically designed in a such a way that the splitting of the corresponding quantum levels exactly coincides with the axion mass. In this case the quantum resonance simulated transition is possible which may greatly enhance very a low rate of the emission (5.4). Such computations are well beyond the scope of the present work as they require some special technique [47] which is not yet fully developed. We leave the corresponding analysis for the future studies.

5.3 Potential Difference and Induced Charge

If we add 2 plates near the ends of the cylinder, the induced field $\langle E \rangle_{\mathrm{ind}}$ will induce the electric charges on these plates, which in principle, can be measured. The magnitude of these charges can be estimated as

$$\langle Q \rangle \sim \frac{e\theta(t)}{2\pi} K_{a\gamma\gamma}. \tag{5.5}$$

It is clear that for θ of interest, $\langle Q \rangle \ll e$ and thus the small fractional charges are only to be understood probabilistically. The corresponding potential difference can be estimated as

$$\langle \Delta V \rangle \simeq \frac{e\theta K_{a\gamma\gamma} L_3}{2\pi L_1 L_2} \sim 10^{-4} K_{a\gamma\gamma} \frac{\theta(t)}{2\pi} (\mathrm{volt}). \tag{5.6}$$

To get a bearing on the magnitude of this potential difference, we compare it to the Hall voltage usually on the order of $(10-100)\mathrm{mV}$ in quantum Hall measurements [48, 49], which is many orders of magnitude greater that (5.6). The amplification $N^3 \sim 10^9$ mentioned above, in principle, may drastically increase the sensitivity.

5.4 Oscillating and Transient Current

As the axion passes through the detector, the induced dipole and charges on the plate also oscillate, which gives rise to a current in the wire if we connect the plates. For a superconducting wire, the maximal current one can attain is approximately

$$\langle J \rangle \approx \frac{Qc}{L_3} \cdot \left(\frac{v}{c} \right) \sim \frac{\theta_m K_{a\gamma\gamma}}{2\pi} \cdot \left(\frac{v}{c} \right) \cdot 10(\text{nA}), \tag{5.7}$$

where v is a typical discharge velocity, which obviously depends on physical properties of material of a wire. For the superconductor wire we expect that $v \sim c$. The average current can then be measured by a SQUID. Assuming SQUID sensitivity $\sim 10^{-18}$T, in principle each unit can reach sensitivity at $\langle J \rangle \sim 10^{-3}$nA. Therefore, for a clever design of the detector which consists of $N^3 \sim 10^9$ such units to amplify the overall current, we are in principle sensitive to $\theta \gtrsim 10^{-12}$.

Based on this estimate, the topological device is in principle competitive with the upper limit of existing and proposed experiments for dark matter axion searches. More importantly, distinct from the direct detection experiments currently known in literature, this configuration is also sensitive to θ as opposed to $\dot{\theta}$ as a result of the topological features of the system as discussed above.

Furthermore, one can also use the setup for the measurement of a static θ. For such measurements, we first apply B_{ext} to induce the charges on the plates, and then turn off the applied field and measure the transient discharge current. In next section we elaborate on this option to measure the static θ using slightly different setups when the topology selects a specific winding $|k\rangle$ states in contrast with specific $|\theta_{\text{eff}}\rangle$ states which have been considered so far. The effects will be still proportional to small parameter θ but some enhancement factors may emerge as we argue below.

6 Numerical estimates for $|\kappa\rangle$ states

In this section we make some numerical estimates assuming the effective S^1 topology is still enforced but in the limit when tunneling and non-trivial winding is suppressed. In this case, $|\kappa\rangle$ states, which are states that correspond to configurations with non-zero flux κ threading through space, are good quantum states. Therefore, different from the previous sections, we perform our analysis in this section for the $|\kappa\rangle$ states rather than for superposition of the winding states classified by parameter θ_{eff}. Physically it implies that we choose uniform magnetic field B_z along z direction which selects specific boundary conditions for pure gauge (but topologically nontrivial) vector potential at large distances such that $\oint_\Gamma A_\mu dx_\mu = 2\pi\kappa$, where Γ is the path at large distances in xy plane[8]. As we already mentioned in this

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[8]The parameter κ which classifies our states in the present section is arbitrary real number. It measures the magnetic physical flux, which not necessary assumes the integer values. It should not be confused with integer numbers n, l, m, k which enter the expressions describing the path integral in previous sections, where we sum over all topological sectors to select $|\theta_{\text{eff}}\rangle$ state. In present section we select $|\kappa\rangle$ state as the physical state which explains the title of this section 6.

case the θ parameter (not to be confused with θ_{eff}) still remains a physical parameter of the system. In such circumstances the electric field (3.7) will be induced along the magnetic field in the region of space where the magnetic field is present. The topological arguments suggest that the corresponding configurations cannot "unwind" as the uniform static magnetic field B_z enforces the system to become effectively two-dimensional, when the θ parameter is obviously a physical parameter, similar to analogous analysis in the well-known 2d Schwinger model.

Indeed the θ term in the action (3.6) with fixed k can be rewritten as follows

$$S_\theta \sim \theta e^2 \int \mathrm{d}^4 x \ \vec{E} \cdot \vec{B} = \theta \left[e \int d^2 x_\perp B_z \right] \cdot \left[e \int dz dt E_z \right] = 2\pi\kappa \ \theta \cdot \left[e \int dz dt E_z \right]. \ (6.1)$$

The expression on the right hand side is still a total divergence, and does not change the equation of motion. In fact, the expression in the brackets is identically the same as the θ term in 2d Schwinger model, where it is known to be a physical parameter of the system as a result of nontrivial mapping $\pi_1[U(1)] = \mathbb{Z}$, see e.g. [27] for a short review on θ term in 2d Schwinger model in the given context. The expression (6.1) for the θ term written in the external background field shows once again that θ parameter in 4d Maxwell theory becomes the physical parameter of the system when some conditions are met. In many respects this phenomenon (when the 4d θ becomes the physically observable parameter of the system) is very similar to the Witten's effect [37] when the presence of a monopole enforces the boundary conditions which cannot be "unwinded" due to the monopole's magnetic topological charge. In such circumstances the θ term becomes a physical parameter of the system (in monopole's sector), and, in particular, a monopole becomes the dyon with electric charge $\sim \theta$ as we already disused in section 3.3. The role of the magnetic charge (in the Witten's effect) plays the magnetic flux κ in our case. This flux enforces the boundary conditions (6.1) and makes the θ_{QED} to become an observable parameter in the sector with non-vanishing magnetic flux.

The discussions of the present section are devoted to the physically observable effects due to θ parameter (6.1) in the given $|\kappa\rangle$ sector. In this case we do not have any other requirements (such as small τ) except that AB coherence phase must be maintained. This opens up a new perspective for the axion search experiments due to two reasons. First, the effect is sensitive to the static axion field even without tunnelling suppression factor $\sim \exp(-1/e^2)$ which always accompanied all our formulae in the previous section. This is because we study the system in the k sector with non vanishing κ flux, similar to the Witten's effect with non vanishing monopole's charge. In both cases the effect is proportional to θ without $\exp(-1/e^2)$ suppression because the object of studies is not the vacuum, but the heavy k sector.

Secondly, the effect for the induced electric field (3.7) and related formulae for the potential difference $\langle \Delta V \rangle$, the induced charges $\langle Q \rangle$ and induced currents $\langle J \rangle$ can be drastically enhanced due to the additional parameter B_{ext}^z which could be in the range of Tesla rather than in a fraction of Gauss, see next subsection 6.1 with detail estimates. This is because the overall expression is approximated by a linear dependence on the external field. Such a linear dependence on the external field for the $|\kappa\rangle$ states can be easily understood from

expression (6.1) where θ parameter always enters in combination with (non-fluctuating) external field B_z, which eventually drastically enhances all the effects related to θ.

For completeness of the presentation we also briefly discuss in section 6.2 the dual picture when the magnetic field is induced in the background of the electric field. We emphasize in section 6.3 that our studies on static θ due to the DM axion passing the detector can be equally apply to constraint the fundamental parameter of QED, the θ_{QED} which becomes a physically observable parameter of the system when the theory is formulated on a nontrivial manifold, or it is placed into the background field which itself enforces a nontrivial topology as discussed above.

6.1 Potential Difference, Induced Charge and Induced Current with B_{ext}

From (3.7), we arrive at the following expression for the induced electric field in the presence of $\theta \neq 0$.

$$\langle E \rangle_{\mathrm{ind}} = -\frac{\theta K_{a\gamma\gamma}\alpha}{\pi} B_{\mathrm{ext}}. \qquad (6.2)$$

If we place the plates at the ends of the cylinder, the induced field $\langle E \rangle_{\mathrm{ind}}$ will induce the charge on the plates similar to our discussions in the previous section. The magnitude of the charges can be estimated as follows

$$\langle Q \rangle \sim \frac{e\theta(t)}{2\pi} K_{a\gamma\gamma} \cdot \left[\frac{eB_{\mathrm{ext}}L_1 L_2}{2\pi} \right]. \qquad (6.3)$$

The difference, in comparison with the previous discussions, is that the external magnetic field B_{ext} could be quite large which makes the effect much stronger. This charge separation effect due to $\theta \neq 0$ generates the potential difference, which can be estimated as follows

$$\langle \Delta V \rangle \simeq \frac{e\theta K_{a\gamma\gamma}L_3}{2\pi L_1 L_2} \cdot \left[\frac{eB_{\mathrm{ext}}L_1 L_2}{2\pi} \right] \sim 0.2 K_{a\gamma\gamma}\theta \cdot \left(\frac{L_3}{\mathrm{mm}} \right) \cdot \left(\frac{B_{\mathrm{ext}}}{\mathrm{Gauss}} \right) (\mathrm{volt}). \qquad (6.4)$$

If we place the system into the background of the strong external field $B_{\mathrm{ext}} \sim 1\mathrm{T}$ and assume the $N^3 \sim 10^9$ amplification mentioned above, along with high sensitivity to measure the potential difference on the level 10mV (which is typical for the quantum Hall measurements [48, 49]), then one can push the sensitivity for the static θ to the level $\theta \sim 10^{-14}$. We emphasize that such a measurements are sensitive to the static θ regardless of its nature: whether it is the fundamental constant of QED, or it is generated by the coherent DM axion passing through detector.

If we connect two plates with a wire, then the induced current can be estimated as follows

$$\langle J \rangle \sim \frac{\langle Q \rangle c}{L_3} \left(\frac{v}{c} \right) \approx 10^{-6} \cdot \left(\frac{\theta K_{a\gamma\gamma}}{10^{-14}} \right) \cdot \left(\frac{L_1 L_2 / L_3}{\mathrm{mm}} \right) \cdot \left(\frac{B_{\mathrm{ext}}}{1\mathrm{T}} \right) \cdot \left(\frac{v}{c} \right) \mathrm{nA} \qquad (6.5)$$

where v is a typical discharge velocity, which obviously depends on physical properties of material of a wire. For the superconducting wire we expect that the discharge velocity is close to maximum possible $v \sim c$. A single device obviously cannot produce a measurable

effect for small $\theta \simeq 10^{-14}$. However, as we already mentioned, the potential difference (6.4) for such small $\theta \simeq 10^{-14}$ can be in principle measured with the $N^3 \sim 10^9$ amplification. A similar amplification produces the effect for the current $\langle J \rangle$ on the level 10^3 nA which is the same order of magnitude as in the usual experiments that measure persistent currents [50–52]. The challenge, of course, will be in maintaining Aharonov-Bohm-like coherence in the presence of a strong magnetic field using superconducting wire for the given system size.

To reiterate the basic result of this subsection: the primary phenomenon of this system is the generation of the induced electric field (6.2) in the background of the external magnetic field and in the presence of non-vanishing θ. One can, in principle, observe this small electric field for $\theta \simeq 10^{-14}$ by measuring the induced charges (6.3), the potential difference (6.4), or the discharge current (6.5), all of which obviously represent the secondary effects which follow from (6.2).

6.2 Dual Picture with electric external field E_{ext}

In this subsection we want to elaborate on the dual picture of the same phenomenon when the external magnetic field B_{ext} is replaced by external electric field E_{ext}. In this case, formula (3.9) suggests the magnetic field is induced. In physical notations using the Minkowski signature this formula reads

$$\langle B \rangle_{\text{ind}} = \frac{\theta K_{a\gamma\gamma} \alpha}{\pi} E_{\text{ext}}, \tag{6.6}$$

which represents, in all respects, the dual expression for (6.2) when electric and magnetic fields exchange their roles: the external field becomes the induced field, and vice versa. The significance of this formula is that the induced magnetic field can be measured with very high accuracy by making use of the existing infrastructure in the usual experiments that measure persistent currents from Aharonov-Bohm coherence [50–52].

Instead of applying a magnetic field through the ring, however, one can instead apply an external electric field in the same direction. As eq. (6.6) states, the magnetic flux will be induced along the principal axis of the small cylinder/ring. The induced magnetic flux can be understood [39] as coming from the surface persistent current $\langle J \rangle$ which can be estimated as follows

$$\langle J \rangle \simeq \frac{\theta K_{a\gamma\gamma} \alpha}{\pi} L_3 E_{\text{ext}} \sim 10^{-6} \cdot \left(\frac{\theta K_{a\gamma\gamma}}{10^{-14}} \right) \cdot \left(\frac{E_{\text{ext}}}{10^5 \frac{\text{V}}{\text{cm}}} \right) \cdot \left(\frac{L_3}{\text{mm}} \right) \text{nA}. \tag{6.7}$$

This estimate shows that the magnitude of the current is the same order of magnitude as (6.5) for the discharge current for small $\theta \simeq 10^{-14}$. However, as before, the current (6.7) can be in principle measured with the $N^3 \sim 10^9$ amplification. A huge advantage of this specific design is that the physics of the persistent currents is well understood. For zero external magnetic field the persistent currents are obviously vanish. The currents may only be generated if the θ parameter does not vanish. It could only happen if the DM axions $\theta(x)$ are passing through detector, or due to the fundamental $\theta_{\text{QED}} \neq 0$, see next subsection with some comments on this possibility. We emphasize again that the effect (6.7) is proportional

to the static θ, rather than $\partial_\mu \theta$ as explained in the details at the very beginning of this section 6.

6.3 Measurement of fundamental θ_{QED}

In section 5 we studied a number of effects which are sensitive to constant θ. All these effects are due to the tunnelling transitions between states that are topologically distinct but related by (large) gauge transformations. Therefore, all the effects are formally suppressed by a factor $\exp(-1/e^2)$, though numerically the suppression could be quite mild as long as parameter $\tau \sim 1$. On other hand, in subsections (6.1) and (6.2) we studied a number of effects which are also sensitive to constant θ. However, in that case the effects considered are in the suppressed tunnelling limit. As explained at the very beginning of section 6 the optimized effects are proportional to θ without $\exp(-1/e^2)$ suppression because the system belongs to the sector with non-vanishing magnetic flux, similar to the Witten's effect [37] with non-vanishing monopole's charge. In both cases the effect is proportional to θ (and not to $\partial_\mu \theta$) because the systems belong to the heavy κ sectors, rather than to unique vacuum sector where factor $\exp(-1/e^2)$ unavoidably emerges in the Maxwell system for non-simply connected manifolds with nontrivial $\pi_1[U(1)]$.

The numerical estimates in subsections (6.1) and (6.2) are quite promising as they show a number of potentially enhanced factors which in principle can drastically increase the sensitivity to constraint (or observe) very tiny $\theta \gtrsim 10^{-14}$. The same analysis can also be used to put upper limit on the fundamental constant of θ_{QED}, a parameter that has not been measured to the best of our knowledge. While of θ_{QED} does not produce any physically measurable effects for QED with trivial topology, or in vacuum, we expect the proposed Aharonov-Bohm-type configuration discussed in sections 5 and 6 to be sensitive to such a parameter which is normally "undetectable" in a typical scattering experiment based on perturbative analysis of QED.

The limits imposed by dipole moment, charge, potential and transient current will be the same as the limit set for the coherent DM axion θ. For a detector consists of $N^3 \sim 10^9$ such units, θ_{QED} one should in principle be able to exclude $\theta_{\mathrm{QED}} \gtrsim 10^{-14}$, depending on the specific realizable experimental setup.

We conclude this section with the following remark. The possible physical effects from θ_{QED} has also been previously discussed [53, 54] in the same spirit. Here we have proposed a more detailed experimental setup with estimations on sensitivity that is in principle experimentally accessible. More importantly, we explicitly computed both type of the effects, with and without $\exp(-1/e^2)$ suppression to emphasize the role of topology, irrespectively whether it is enforced by external field in κ sectors as in section 6, or by a design of a non simply connected spatial manifolds as in section 5.

7 Conclusion

The main goals of the present paper can be summarized on three distinct but related items.

1. First of all, we introduce fundamental, time independent θ_{QED} term in QED which we claim is a physical observable when some conditions are met. This should be contrasted

with conventional viewpoint that θ_{QED} is not a physically relevant parameter in abelian gauge theories.

A deep reason why θ_{QED} in vacuum becomes a physical parameter when the Maxwell system is defined on non-simply connected manifold is due to the emergence of the so-called Gribov's ambiguities (when the gauge cannot be completely fixed), which is well known phenomenon in non-abelian gauge theories, see footnote 6 for comments and references. The θ_{QED} also becomes a physically observable parameter in the sectors with non vanishing magnetic flux κ as explained in section 6, and more specifically, in subsection 6.3. In this case the effect is very much the same as the Witten's effect when the θ becomes a physically observable parameter in the monopole's sectors.

We suggest a specific design for a tabletop experiment which, in principle, can constrain the fundamental θ_{QED} on the level $\theta_{\text{QED}} \sim 10^{-14}$. This constraint should be treated as an independent from known constraint on a physically distinct parameter $\theta_{\text{QCD}} \leq 10^{-10}$.

2. A related, but distinct, goal is the application of our findings to the axion search experiments, where $\theta(x)$ describes the DM axion with typical frequency $\omega = m_a$ passing through a detector. This part of the paper is motivated by a number of ongoing axion search experiments which may finally unlock the nature of the dark matter. The novel element which was not previously fully explored is that the observable effects may depend directly on θ rather than on $\partial_\mu \theta$ which normally enters the conventional formulae devoted to the DM axion search experiments. The topological arguments play the crucial difference, similar to item 1 above.

3. The final goal of this work is to advocate an idea that there is a novel type of vacuum energy, the Topological Casimir Effect (TCE) which cannot be formulated in terms of propagating degrees of freedom (the photons in the Maxwell theory). This new type of energy is highly sensitive to the $\theta(t)$ parameter. Therefore, there is a real chance to ascertain the existence of such effects using specifically designed instruments with high precision, as discussed in this paper. The work in this direction might shed some light on the nature of the dark energy. In fact, the original papers [27, 29] on TCE were mostly motivated by the idea to imitate this new type of vacuum energy in a tabletop experiment.

In the context of QCD, this type of the vacuum energy cannot be associated with any physical propagating degrees of freedom, analogous to TCE in the present work in Maxwell system. This motivates the proposal in [55, 56] that the observed dark energy in the Universe may have, in fact, precisely such a non-dispersive nature[9]. The proposal where an extra energy[10] cannot be associated with any propagating particles is aimed to

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[9]This novel type of vacuum energy which can not be expressed in terms of propagating degrees of freedom has been extensively studied in QCD lattice simulations, see [55] and references therein on the original lattice results.

[10]There are two instances in the evolution of the Universe when the vacuum energy plays a crucial role. The first instance is identified with the inflationary epoch when the Hubble constant H was almost constant, which corresponds to the de Sitter type behaviour $a(t) \sim \exp(Ht)$ with exponential growth of the size $a(t)$ of the Universe. The second instance where the vacuum energy plays a dominant role corresponds to the present epoch when the vacuum energy is identified with the so-called dark energy ρ_{DE} which constitutes almost 70% of the critical density. In the proposal [55, 56] the vacuum energy density can be estimated as $\rho_{DE} \sim H\Lambda_{QCD}^3 \sim (10^{-4}\text{eV})^4$, which is amazingly close to the observed value.

provide an approach that is different from a commonly accepted paradigm that the extra vacuum energy in the Universe is always associated with the potential of some propagating degree of freedom, such as inflaton, see original papers [57, 58], and reviews [59, 60].

Acknowledgements

References

[1] R. D. Peccei and H. R. Quinn, *Constraints Imposed by CP Conservation in the Presence of Instantons*, *Phys. Rev.* **D16** (1977) 1791–1797.

[2] S. Weinberg, *A New Light Boson?*, *Phys. Rev. Lett.* **40** (1978) 223–226.

[3] F. Wilczek, *Problem of Strong p and t Invariance in the Presence of Instantons*, *Phys. Rev. Lett.* **40** (1978) 279–282.

[4] J. E. Kim, *Weak-interaction singlet and strong* CP *invariance*, *Phys. Rev. Lett.* **43** (Jul, 1979) 103–107.

[5] M. Shifman, A. Vainshtein, and V. Zakharov, *Can confinement ensure natural cp invariance of strong interactions?*, *Nuclear Physics B* **166** (1980), no. 3 493 – 506.

[6] M. Dine, W. Fischler, and M. Srednicki, *A simple solution to the strong cp problem with a harmless axion*, *Physics Letters B* **104** (1981), no. 3 199 – 202.

[7] A. R. Zhitnitsky, *On Possible Suppression of the Axion Hadron Interactions. (In Russian)*, *Sov. J. Nucl. Phys.* **31** (1980) 260. [Yad. Fiz.31,497(1980)].

[8] K. van Bibber, *Ultrasensitive Searches for the Axion*, in *APS April Meeting Abstracts*, Apr., 2007.

[9] S. J. Asztalos, L. J. Rosenberg, K. van Bibber, P. Sikivie, and K. Zioutas, *Searches for astrophysical and cosmological axions*, *Ann. Rev. Nucl. Part. Sci.* **56** (2006) 293–326.

[10] G. G. Raffelt, *Astrophysical axion bounds*, *Lect. Notes Phys.* **741** (2008) 51–71, [hep-ph/0611350]. [,51(2006)].

[11] P. Sikivie, *Dark matter axions*, *Int. J. Mod. Phys.* **A25** (2010) 554–563, [arXiv:0909.0949].

[12] L. J. Rosenberg, *Dark-matter QCD-axion searches*, in *Sackler Colloquium: Dark Matter Universe: On the Threshhold of Discovery Irvine, USA, October 18-20, 2012*, 2015.

[13] P. W. Graham, I. G. Irastorza, S. K. Lamoreaux, A. Lindner, and K. A. van Bibber, *Experimental Searches for the Axion and Axion-Like Particles*, *Ann. Rev. Nucl. Part. Sci.* **65** (2015) 485–514, [arXiv:1602.00039].

[14] D. Budker, P. W. Graham, M. Ledbetter, S. Rajendran, and A. Sushkov, *Proposal for a Cosmic Axion Spin Precession Experiment (CASPEr)*, Phys. Rev. **X4** (2014), no. 2 021030, [arXiv:1306.6089].

[15] P. W. Graham and S. Rajendran, *New Observables for Direct Detection of Axion Dark Matter*, Phys. Rev. **D88** (2013) 035023, [arXiv:1306.6088].

[16] G. Rybka, A. Wagner, A. Brill, K. Ramos, R. Percival, and K. Patel, *Search for dark matter axions with the Orpheus experiment*, Phys. Rev. **D91** (2015), no. 1 011701, [arXiv:1403.3121].

[17] P. Sikivie, N. Sullivan, and D. B. Tanner, *Proposal for axion dark matter detection using an lc circuit*, Phys. Rev. Lett. **112** (Mar, 2014) 131301.

[18] C. Beck, *Possible resonance effect of axionic dark matter in Josephson junctions*, Phys. Rev. Lett. **111** (2013) 231801, [arXiv:1309.3790].

[19] Y. V. Stadnik and V. V. Flambaum, *Axion-induced effects in atoms, molecules, and nuclei: Parity nonconservation, anapole moments, electric dipole moments, and spin-gravity and spin-axion momentum couplings*, Phys. Rev. **D89** (2014), no. 4 043522, [arXiv:1312.6667].

[20] P. Sikivie, *Axion Dark Matter Detection using Atomic Transitions*, Phys. Rev. Lett. **113** (2014), no. 20 201301, [arXiv:1409.2806].

[21] B. T. McAllister, S. R. Parker, and M. E. Tobar, *Axion Dark Matter Coupling to Resonant Photons via Magnetic Field*, Phys. Rev. Lett. **116** (2016), no. 16 161804, [arXiv:1607.01928]. [Erratum: Phys. Rev. Lett.117,no.15,159901(2016)].

[22] C. T. Hill, *Axion Induced Oscillating Electric Dipole Moments*, Phys. Rev. **D91** (2015), no. 11 111702, [arXiv:1504.01295].

[23] C. T. Hill, *Axion Induced Oscillating Electric Dipole Moment of the Electron*, Phys. Rev. **D93** (2016), no. 2 025007, [arXiv:1508.04083].

[24] Y. Kahn, B. R. Safdi, and J. Thaler, *Broadband and Resonant Approaches to Axion Dark Matter Detection*, Phys. Rev. Lett. **117** (2016), no. 14 141801, [arXiv:1602.01086].

[25] R. Barbieri, C. Braggio, G. Carugno, C. S. Gallo, A. Lombardi, A. Ortolan, R. Pengo, G. Ruoso, and C. C. Speake, *Searching for galactic axions through magnetized media: the QUAX proposal*, arXiv:1606.02201.

[26] A. Arvanitaki and A. A. Geraci, *Resonantly Detecting Axion-Mediated Forces with Nuclear Magnetic Resonance*, Phys. Rev. Lett. **113** (2014), no. 16 161801, [arXiv:1403.1290].

[27] C. Cao, M. van Caspel, and A. R. Zhitnitsky, *Topological Casimir effect in Maxwell Electrodynamics on a Compact Manifold*, Phys. Rev. **D87** (2013), no. 10 105012, [arXiv:1301.1706].

[28] H. B. G. Casimir, *On the Attraction Between Two Perfectly Conducting Plates*, Indag. Math. **10** (1948) 261–263. [Kon. Ned. Akad. Wetensch. Proc.100N3-4,61(1997)].

[29] C. Cao, Y. Yao, and A. R. Zhitnitsky, *Aharonov-Bohm phases in a quantum LC circuit*, Phys. Rev. **D93** (2016), no. 6 065049, [arXiv:1512.00470].

[30] P. Sikivie, *Experimental Tests of the Invisible Axion*, Phys. Rev. Lett. **51** (1983) 1415–1417. [Erratum: Phys. Rev. Lett.52,695(1984)].

[31] X. Liang and A. Zhitnitsky, *Axion field and the quark nugget's formation at the QCD phase transition*, Phys. Rev. **D94** (2016), no. 8 083502, [arXiv:1606.00435].

[32] D. Kharzeev, R. D. Pisarski, and M. H. G. Tytgat, *Possibility of spontaneous parity violation in hot QCD*, *Phys. Rev. Lett.* **81** (1998) 512–515, [`hep-ph/9804221`].

[33] K. Buckley, T. Fugleberg, and A. Zhitnitsky, *Can theta vacua be created in heavy ion collisions?*, *Phys. Rev. Lett.* **84** (2000) 4814–4817, [`hep-ph/9910229`].

[34] D. Kharzeev and A. Zhitnitsky, *Charge separation induced by P-odd bubbles in QCD matter*, *Nucl. Phys.* **A797** (2007) 67–79, [`arXiv:0706.1026`].

[35] D. E. Kharzeev, *Topologically induced local P and CP violation in QCD x QED*, *Annals Phys.* **325** (2010) 205–218, [`arXiv:0911.3715`].

[36] D. E. Kharzeev, J. Liao, S. A. Voloshin, and G. Wang, *Chiral magnetic and vortical effects in high-energy nuclear collisionsâĂŤA status report*, *Prog. Part. Nucl. Phys.* **88** (2016) 1–28, [`arXiv:1511.04050`].

[37] E. Witten, *Dyons of Charge e theta/2 pi*, *Phys. Lett.* **B86** (1979) 283–287.

[38] M. Bordag, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, *Advances in the Casimir effect*, *Int. Ser. Monogr. Phys.* **145** (2009) 1–768.

[39] A. R. Zhitnitsky, *Dynamical Casimir Effect in a small compact manifold for the Maxwell vacuum*, *Phys. Rev.* **D91** (2015), no. 10 105027, [`arXiv:1501.07603`].

[40] A. R. Zhitnitsky, *Maxwell Theory on a Compact Manifold as a Topologically Ordered System*, *Phys. Rev.* **D88** (2013), no. 10 105029, [`arXiv:1308.1960`].

[41] A. R. Zhitnitsky, *Topological order and Berry connection for the Maxwell Vacuum on a four-torus*, *Phys. Rev.* **D90** (2014), no. 10 105007, [`arXiv:1407.3804`].

[42] K.-T. Chen and P. A. Lee, *Topological insulator and the θ vacuum in a system without boundaries*, *Phys. Rev. B* **83** (Mar, 2011) 125119.

[43] M. Tsubota, K. Inagaki, T. Matsuura, and S. Tanda, *Aharonov-Bohm effect in charge-density wave loops with inherent temporal current switching*, *EPL (Europhysics Letters)* **97** (Mar., 2012) 57011, [`arXiv:0906.5206`].

[44] J. J. Hudson, D. M. Kara, I. J. Smallman, B. E. Sauer, M. R. Tarbutt, and E. A. Hinds, *Improved measurement of the shape of the electron*, *Nature* **473** (2011) 493–496.

[45] **ACME** Collaboration, J. Baron et al., *Order of Magnitude Smaller Limit on the Electric Dipole Moment of the Electron*, *Science* **343** (2014) 269–272, [`arXiv:1310.7534`].

[46] J. M. Pendlebury et al., *Revised experimental upper limit on the electric dipole moment of the neutron*, *Phys. Rev.* **D92** (2015), no. 9 092003, [`arXiv:1509.04411`].

[47] Y. Yao and A. R. Zhitnitsky, *Aharonov-Bohm phases and Dynamical Casimir Effect in a quantum LC circuit*, `arXiv:1605.01411`.

[48] K. von Klitzing, *The quantized hall effect*, *Rev. Mod. Phys.* **58** (Jul, 1986) 519–531.

[49] H. L. Stormer, D. C. Tsui, and A. C. Gossard, *The fractional quantum hall effect*, *Rev. Mod. Phys.* **71** (Mar, 1999) S298–S305.

[50] M. Büttiker, Y. Imry, and R. Landauer, *Josephson behavior in small normal one-dimensional rings*, *Physics Letters A* **96** (July, 1983) 365–367.

[51] V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Gallagher, and A. Kleinsasser, *Magnetic response of a single, isolated gold loop*, *Phys. Rev. Lett.* **67** (Dec, 1991) 3578–3581.

[52] W. E. Shanks, *Persistent Currents in Normal Metal Rings*. PhD thesis, Yale University, 2011.

[53] S. D. H. Hsu, *Physical consequences of the QED theta angle*, `arXiv:1012.2906`.

[54] S. D. H. Hsu, *Theta terms and asymptotic behavior of gauge potentials in (3+1) dimensions*, `arXiv:1107.0756`.

[55] A. R. Zhitnitsky, *Inflaton as an auxiliary topological field in a QCD-like system*, *Phys. Rev.* **D89** (2014), no. 6 063529, [`arXiv:1310.2258`].

[56] A. R. Zhitnitsky, *Dynamical de Sitter phase and nontrivial holonomy in strongly coupled gauge theories in an expanding universe*, *Phys. Rev.* **D92** (2015), no. 4 043512, [`arXiv:1505.05151`].

[57] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, *Phys. Rev.* **D23** (1981) 347–356.

[58] A. D. Linde, *A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems*, *Physics Letters B* **108** (Feb., 1982) 389–393.

[59] A. D. Linde, *Inflationary Cosmology*, *Lect. Notes Phys.* **738** (2008) 1–54, [`arXiv:0705.0164`].

[60] V. Mukhanov, *Physical Foundations of Cosmology*. Cambridge Univ. Press, Cambridge, 2005.