Universal Linked Multiple Access Source Codes

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Abstract — We consider the multiple access source coding (MASC) problem (also known as the Slepian-Wolf problem) for situations where the joint source statistics are unknown *a priori*. Since neither encoder receives information about the joint source statistics, we allow an asymptotically negligible amount of communication between the encoders. We prove the existence of *universal 2-encoder Linked* LMASCs (LMASCs) with rates approaching the Slepian-Wolf bound, demonstrate the tightness of this bound, and calculate the rate of convergence of the proposed universal LMASC. The result generalizes to $M > 2$ encoders. We also consider scenarios where the number of bits passed between the system encoders is allowed to grow linearly in the code dimension; in these scenarios one encoder can act as a conduit for the flow of another encoder's information.

I. INTRODUCTION

Given finite-alphabet sources $(X, Y) \in \mathcal{X} \times \mathcal{Y}$ drawn i.i.d. from p.m.f. $Q = \{q(x, y)\}$, a blocklength- n , m -round 2-encoder LMASC $\check{C}_n = (l_{m,n} = (l_{m,n}^{(X)}, l_{m,n}^{(Y)}), f_n = (f_n^{(X)}, f_n^{(Y)}), g_n)$ for (X, Y) comprises two link encoders, two message encoders and a decoder. The link passes m rounds of messages between encoders, and the link encoders at a source observe the source sequence and the previous link messages. That is, for each $k \in \{1, \dots, m\}$, the link encoders are $l_{k,n}^{(X)} : \mathcal{X}^n \times \mathcal{I}^{k-1} \rightarrow \mathcal{I}_k$ and $l_{k,n}^{(Y)} : \mathcal{Y}^n \times \mathcal{I}^{k-1} \rightarrow \mathcal{J}_k$. The message encoders are Xavier ($f_n^{(X)} : \mathcal{X}^n \times \mathcal{I}^m \rightarrow S_X$) and Yvonne ($f_n^{(Y)} : \mathcal{Y}^n \times \mathcal{I}^m \rightarrow S_Y$), and the decoder is Zorba ($g_n : S_X \times S_Y \rightarrow \mathcal{X}^n \times \mathcal{Y}^n$). Here S_X , S_Y , and $\mathcal{I}_k$, $\mathcal{J}_k$ ($k \in \{1, \dots, m\}$) are binary prefix codes. The error probability, inter-encoder rate, and decoder input rate are defined as, respectively,

$$\begin{aligned} P_e(\check{C}_n, Q) &= Q^n(g_n(f_n(X^n, Y^n)) \neq (X^n, Y^n)) \\ r(\check{C}_n, Q) &= \frac{1}{n} E_Q(\sum_{k=1}^m |I_k|, \sum_{k=1}^m |J_k|) \\ R(\check{C}_n, Q) &= (R_X(\check{C}_n, Q), R_Y(\check{C}_n, Q)) \\ &= \frac{1}{n} E_Q(|f_n^{(X)}(X^n, J^m)|, |f_n^{(Y)}(Y^n, I^m)|). \end{aligned}$$

where $(I_k, J_k) = l_{k,n}(X^n, Y^n)$. Rate pair (R_X, R_Y) is said to be $r_{X \leftrightarrow Y}$ -LMASC-achievable on Q if there exists a sequence $\{\check{C}_n\}$ of LMASCs (possibly dependent on Q) such that $R(\check{C}_n, Q) \rightarrow (R_X, R_Y)$, $r(\check{C}_n, Q) \rightarrow r_{X \leftrightarrow Y} = (r_{X \rightarrow Y}, r_{Y \rightarrow X})$ and $P_e(\check{C}_n, Q) \rightarrow 0$. The set of $r_{X \leftrightarrow Y}$ -LMASC-achievable rates on Q is denoted by $\mathcal{R}_L(Q, r_{X \leftrightarrow Y})$. A sequence $\{\check{C}_n\}_{n=1}^\infty$ of $r_{X \leftrightarrow Y}$ -LMASCs is *universal* with respect to the class Ω

of memoryless distributions on $\mathcal{X} \times \mathcal{Y}$ if, for every $Q \in \Omega$, some point $(R_X^*(Q), R_Y^*(Q))$ on the lower boundary of $\mathcal{R}_L(Q, r_{X \leftrightarrow Y})$ is $r_{X \leftrightarrow Y}$ -LMASC-achievable on Q by $\{\check{C}_n\}$.

II. MAIN RESULT

We here outline bounds on the performance of universal LMASCs. A brief sketch of the corresponding proofs follows. Details appear in [2].

Theorem 1 For any point (R_X, R_Y) in $\mathcal{R}_L(Q, r_{X \leftrightarrow Y})$ there exists a universal sequence $\{\check{C}_n\}$ of $r_{X \leftrightarrow Y}$ -LMASCs for source class Ω . The $r_{X \leftrightarrow Y}$ -LMASC-achievable region is given by

$$\begin{aligned} \mathcal{R}_L(Q, r_{X \leftrightarrow Y}) &= \{(R_X, R_Y) : R_X \geq H_Q(X|Y) - r_{X \rightarrow Y}, \\ &\quad R_Y \geq H_Q(Y|X) - r_{Y \rightarrow X}, \\ &\quad R_X + R_Y \geq H_Q(X, Y)\}. \end{aligned}$$

Remarks: (1) Like the Slepian-Wolf Theorem [1], Theorem 1 can be generalized to M encoders.

(2) The Slepian-Wolf region is the same as $\mathcal{R}_L(Q, (0, 0))$.

The main argument constructs a universal sequence of 2-round $(0, 0)$ -LMASCs with $(R_X^*(Q), R_Y^*(Q)) = (H_Q(X), H_Q(Y|X))$. The symmetry of the problem makes $(H_Q(X|Y), H_Q(Y))$ similarly attainable. Time-sharing between the codes achieving these two rate pairs or padding the codes with extraneous symbols gives every other rate pair in $\mathcal{R}_L(Q, (0, 0))$. These $(0, 0)$ -LMASCs are then used to construct universal $r_{X \leftrightarrow Y}$ -LMASCs.

Given the above scenario, we choose $l_{m,n}$ and $m(n)$ so that $(I^m, J^m) = (x^{m(n)}, y^{m(n)})$ for some $m(n)$ satisfying $\lim_{n \rightarrow \infty} (m(n)/n) = 0$. Then Xavier and Yvonne can independently make an estimate of Q as $\hat{P}_n = \hat{p}_n(x, y) = (1/m(n)) \sum_{i=1}^{m(n)} 1((X_i, Y_i) = (x, y))$. We show in [2] that $Q^{m(n)}((x, y) \in \mathcal{X} \times \mathcal{Y} : |\hat{p}_n(x, y) - q(x, y)| \geq \delta(n)) \leq 2^{-m(n)\delta^2(n)c}$ (where c is a constant independent of Q). Thus choosing $\delta(n)$ so that $\delta(n) \rightarrow 0$ and $m(n)\delta^2(n) \rightarrow \infty$ ensures that $\hat{P}_n \rightarrow Q$ with a probability that approaches 1. We then use $\hat{P}_n$ to select a $\check{P} \in \Omega$ and a corresponding code $\check{C}_n(\check{P})$ such that for $\Delta(n) = c'\delta(n)\log(\delta(n))$ and any $Q \in \Omega$ (c' is a constant independent of Q)

$$\begin{aligned} P_e(\check{C}_n(\check{P}), Q) &\leq 2^{-m(n)\delta^2(n)c+1} \\ r(\check{C}_n(\check{P}), Q) &= \frac{1}{n} (\lceil \log |\mathcal{X}| m(n) \rceil, \lceil \log |\mathcal{Y}| m(n) \rceil) \\ R_X(\check{C}_n(\check{P}), Q) &\leq R_X^*(Q) + \Delta(n) \\ R_Y(\check{C}_n(\check{P}), Q) &\leq R_Y^*(Q) + \Delta(n). \end{aligned}$$

REFERENCES

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