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Contents

Abstract

Recent theoretical investigations of high energy nucleon-nucleon collisions[1)] support the two-center model of multiple meson production as discussed by Ciok, et al.[2)]. To facilitate a prediction of the most probable distribution of shower particles as a function of laboratory energy, Farley's kinematical treatment of the "two-fireball" model is employed[3)]. A statistical determination of the multiplicity of pion (including neutrals) production per collision is assumed to be valid with possible corrections suggested through restrictions imposed by experimental values of the inelasticity, K_π.

Acknowledgments

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I Introduction

An analysis of multiple pion production induced by nucleon-nucleon collisions is developed on the basis of a two fireball statistical model. At 300 Gev primary energy, the statistical model based purely upon phase space arguments indicates that about six pions per fireball are created per collision. Experimental evidence however, shows that the mean pion energy is essentially almost independent of excitation energy (i.e., 0.5 Gev) in the excited nucleon system. This coupled with an average number of six pions (multiplicity of 12), predicts an inelasticity about twice as large as that observed experimentally. This suggests, therefore, that the number of pions produced per collision (on a statistical basis) should be modified by matrix element considerations.

Experimental cosmic ray data appears to suggest that the value for the inelasticity is in the neighborhood of 0.3 to 0.4 at 300 Gev lab energy[4)], at the same time supporting the two fireball concept. In order for this value to agree with this model, it is necessary to reduce either n_s (multiplicity) or the average pion energy in the excited nucleon system $\bar{E}$. Some experiments favor values of $\bar{E}$ as low as 0.4, instead of the usually accepted value ~ 0.5. [4)] However, this reduction in $\bar{E}$ is still not sufficient to lower K_π to the "correct" value. It is apparent therefore that n_s must be lowered in order to obtain better agreement. Pure phase space and statistical equilibrium arguments (a la Fermi) yields a value ~ 12 pions/collision (i.e., 6 pions per fireball), hence matrix element, or non-equilibrium considerations must be employed in order to lower this value.

More recent work based upon "The Eightfold Way"[5] has shown that the elastic scattering cross section approaches a constant value of approximately 40 millibarns at infinite energy[1]. In addition, experimental cosmic ray data also suggests that the inelastic cross section approaches a constant value in the vicinity of 20 millibarns[4]. This indicates, therefore, that the probability per collision of creating "vector mesons" is about one-third. If we make the further assumption that single vector meson production is more probable than multiple vector meson production*, we should expect approximately one pion to be produced per collision. (This arises from decays such as $\rho \longrightarrow 2\pi$, $\omega \longrightarrow 3\pi$.) By making the further plausible assumption that the average pion energy in the excited nucleon system is almost constant ~ 0.5 Gev, we may arrive at an asymptotic value of the inelasticity 0.2, which is in good agreement with the high energy experimental results[6].

A direct consequence of lowering the value of n_s is to lower the excitation energy of the fireballs. This automatically increases the value of $\bar{\gamma}$, and hence pushes the narrow cone pion distribution to higher energies. This is apparently undesirable since some experiments indicate that the pion energy spectrum cuts off at energies lower than that predicted by the Farley model with $n_s \sim 12$. At the present time, there appears to be no way of resolving this paradox**.

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* This is supported by recent experiments at 4 Gev; however, this energy is probably not large enough to guarantee this statement.

** One possibility is currently being investigated by H. Ticho.

In the development, Farley's model will be exploited in an attempt to obtain approximate energy and angular distributions of pions in the laboratory.

II Development

We shall consider an inelastic collision of an incoming primary nucleon (described by energy E_p and Lorentz factor γ_p) with a nucleon stationary in the laboratory system (LS). Let β_c denote the velocity of the center of mass system (CS) where

$$(1 - \beta_c^2)^{-1/2} = \gamma_c = \left[\frac{1}{2}(\gamma_p + 1)\right]^{1/2} \qquad (1)$$

The two nucleons of rest mass m_o and total energy $\gamma_c m_o = E_c$ approach each other with equal velocities β_c in the CS. After collision, the mass of each nucleon is increased to $\bar{\gamma}^{-1} E_c = m_o^*$ the velocities of the two excited nucleons being reduced to $\bar{\beta}$. The energy relation thus becomes:

$$\gamma_c m_o = E_c = \bar{\gamma}(\bar{\gamma}^{-1} E_c) = \bar{\gamma} m_o^* \qquad (2)$$

Assuming $n_s/2$ pions per nucleon are emitted isotropically with average energy $\bar{E}$ [3] in the excited nucleon rest system (NS), the excitation energy may be written in the form

$$E_x = \frac{n_s \bar{E}}{2} = m_o^* - m_o = \bar{\gamma}^{-1} E_c - m_o \qquad (3)$$

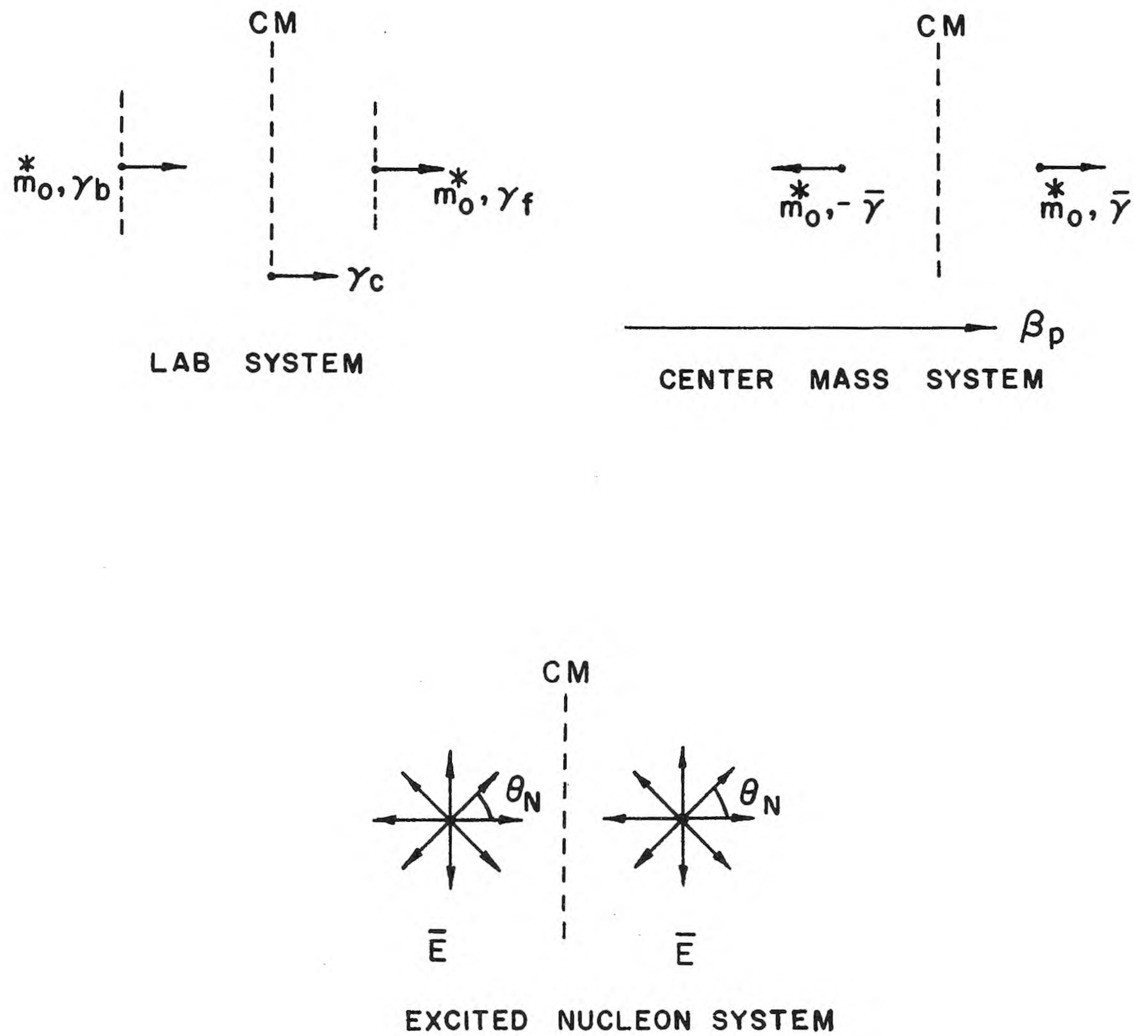


FIG. 1 KINEMATICS AFTER COLLISION

If we let γ_f and γ_b describe the motion in the LS of the excited nucleons which are emitted in the same and the opposite direction to β_p in the CS (Fig. 1), it can be easily shown that

$$\gamma_f = \bar{\gamma}\,\gamma_c\,(1 + \bar{\beta}\,\beta_c) \tag{4}$$

$$\gamma_b = \bar{\gamma}\,\gamma_c\,(1 - \bar{\beta}\,\beta_c) \tag{5}$$

III <u>Laboratory System Shower Distribution:</u> (E_p = 300 Gev)

For E_p = 300 Gev, a Fermi statistical analysis of the meson cloud yields $n_s/2 = 6$. Thus the $E_x = 3$ curve of Farley's[3)] energy spectrum of mesons in the NS may be chosen as that which will yield, under transformation, the most probable energy distribution of shower particles in the LS. (See Fig. 2.)

The following relations and values will be important. From (1)

$$\gamma_c = \left[\frac{1}{2}\,(300 + 1)\right]^{1/2} = 12.3 \quad ; \tag{6}$$

from (3), (2),

$$\bar{\gamma} = \gamma_c\, m_o/(\tfrac{1}{2}\, n_s\, \bar{E} + m_o) = 2.7 \quad ; \tag{7}$$

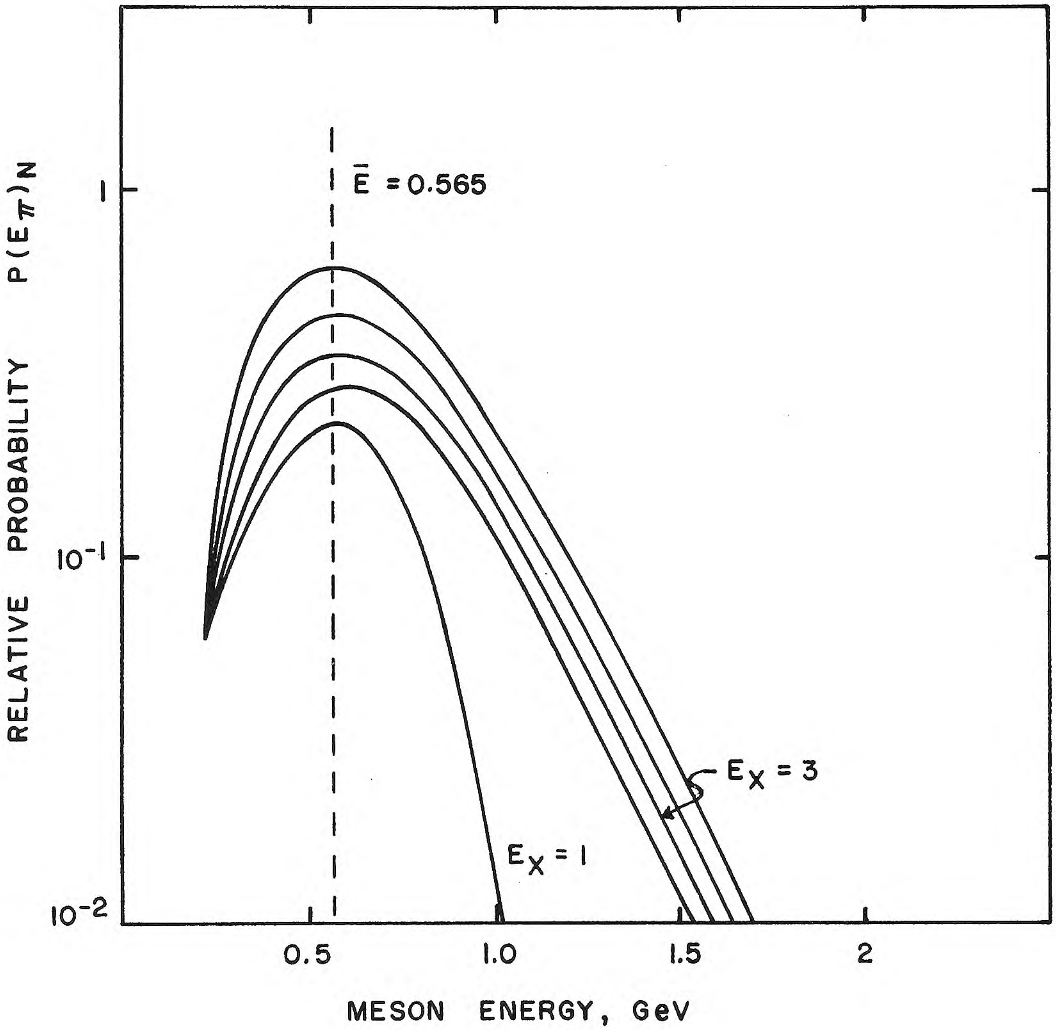
and from (4), (5),

$$\gamma_f = 62.9 \tag{8}$$

$$\gamma_b = 2.4 \tag{9}$$

Choosing points from Fig. 2 and applying the relation

$$(P_\pi)_N = \sqrt{E_\pi^{\,2} - \mu_\pi^{\,2}}\,, \quad \text{where} \quad \mu_\pi \cong m_o/7 \tag{10}$$



RELATIVE PROBABILITY, $P(E_{\pi})_N$ OF MESON EMISSION AS A FUNCTION OF THEIR ENERGIES.

FIG. 2.

the following table is obtained:

TABLE 1

$P(E_\pi)_N$	0.2	0.38	0.42	0.36	0.12	0.02
$(E_\pi)_N$	0.15 Gev	0.25	0.50	0.565 (mean)	1.0	1.5
$(P_\pi)_N$	0.07 Gev	0.21	0.49	0.55	0.99	1.5

Note that the mean pion energy is in good agreement (in the relativistic limit), with the experimental value of the mean transverse momentum $\bar{P}_T$ = 0.5 Gev/c obtained by Edwards, et al.[7], and others.

At 300 Gev, the majority of pions, when transformed by the relations

$$(E_L)_f = \gamma_f \left\{ (E_\pi)_N + \beta_f \, (P_\pi)_N \cos \Theta_N \right\} \tag{11}$$

and

$$(E_L)_b = \gamma_b \left\{ (E_\pi)_N + \beta_b \, (P_\pi)_N \cos \Theta_N \right\} \tag{12}$$

will be concentrated in a 1-3 degree cone in the forward direction (see Appendix). The energy distribution associated with the forward cone is shown in Fig. 3. Since the backwards cone occupies only a small part (0-10 Gev) of the (LS) transformed energy spectrum, Fig. 3 contains essentially all the information of interest. The spectrum was transformed in the following straightforward manner. Each of the five points in Table 1 was transformed according to (11) for 15° intervals ranging from 0 to 180 degrees. Since NS emission is isotropic, $P(E_\pi)_N$ is proportional to the solid angle of emission, and therefore weighted as $\sin \Theta_N$. The

FORWARD CONE

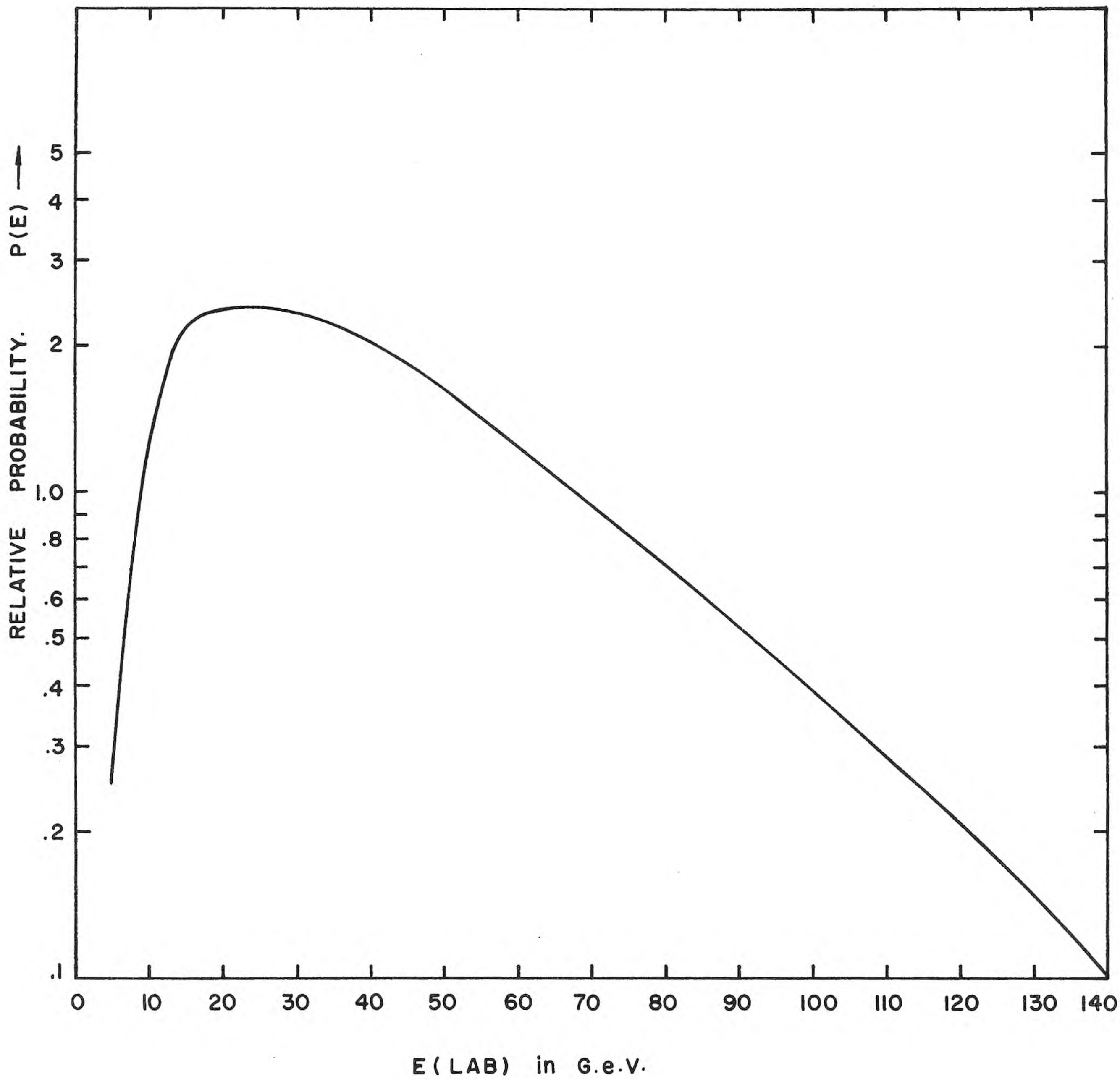


FIG. 3

constant Θ_N curves were added and the resultant curve was renormalized to $\frac{1}{2}\, n_s = 6$.

IV Effect of Inelasticity on n_s

The inelasticity, K_π, in a nucleon-nucleon collision, is defined as that fraction of the total energy available in the CS which is subsequently evaporated in the form of pions. From Ciok, et al.[2], we have

$$K_\pi = \frac{1}{2}\, n_s\, \bar{E}\, \bar{\gamma}/\gamma_c \qquad ; \qquad (13)$$

from (3)

$$\frac{1}{2}\, n_s\, \bar{E} = \bar{\gamma}^{-1}\, E_c - m_o = m_o(\bar{\gamma}^{-1}\, \gamma_c - 1) \qquad ;$$

hence

$$\gamma_c / \bar{\gamma} = \frac{1}{2}\, n_s\, \bar{E}/m_o + 1 \qquad ,$$

and finally

$$K_\pi = \frac{1}{2}\, n_s\, \bar{E}/(\frac{1}{2}\, n_s\, \bar{E} + m_o) \qquad . \qquad (14)$$

For our particular case $n_s = 12$, $\bar{E} \approx 0.565$, thus $K_\pi = 0.78$.

Gierula, Haskin, and Lohrmann[8], and others find values of the inelasticity which tend towards 0.3 in this energy range. GHL have used a low multiplicity, $n_s = 4$, two-center model which closely predicts LS angular distributions. This work, coupled with later studies of the Bristol experiments by Perkins, seems to demand a re-evaluation of n_s.

A statistical model in which only small-angle scattering is important will tend to lower the inelasticity somewhat, since the "core" of the nucleon will not be excited. Recent work by Gell-Mann[5] has indicated that vector mesons may play a strong role in nucleon-nucleon interactions. On the basis of our extension of this work, it has been shown[4] that $(\frac{d\sigma}{d\Omega})_{elastic}$ is strongly peaked in the forward direction at high energies, the same behavior is also indicated for the contribution to $(\frac{d\sigma}{d\Omega})_{inelastic}$. This, therefore, suggests that the momentum transfer at high energies is fairly small, thus reducing the effective excitation energy, and hence the value of n_s and K_π derived solely on the basis of the statistical model. (This again causes an increase in $\bar{\gamma}$.)

In conclusion, it should be explained that the two fireball model of N-N collisions is probably correct for all energies. Sakurai[9] has suggested that the repulsive nature of the nucleon core should give rise to two excited nucleon fireball systems. However, the approximation employed in the Fermi statistical model to these separate fireball systems is probably invalid since the excitation energy will tend to remain small, even at high energies[1]. This would probably not be the case for N-$\bar{N}$ systems where a strong attractive potential exists (possibly even large enough to give bound states (e.g., π meson)[10]), this process would clearly enhance the tendency towards a single fireball description, and it is hoped that this point can ultimately be checked experimentally.

APPENDIX

Number of pions as a function of LS angle, Θ_L. For

$$\beta_f \cong \beta_p \cong 1, \quad \gamma_f \tan \Theta_L \cong \tan \Theta_N/2 \tag{1A}$$

Let $\frac{1}{2} n_s$ be the number of pions per fireball emitted isotropically in the NS. By conservation of particles

$$\frac{n_s}{4\pi} \sin \Theta_N \, d\, \Theta_N = N(\Theta_L) \sin \Theta_L \, d\, \Theta_L \quad , \tag{2A}$$

differentiating (1A) and combining with (2A) gives

$$N(\Theta_L)_f = \frac{n_s}{4\pi} \frac{\sin \Theta_N}{\sin \Theta_L} \cdot \frac{2\gamma_f \sec^2 \Theta_L}{\sec^2 \Theta_N/2}$$

or

$$N(\Theta_L)_f = \frac{n_s}{\pi} \gamma_f^{\,2} \sec^3 \Theta_L \left\{ 1 + \gamma_f^{\,2} \tan^2 \Theta_L \right\}^{-2} \quad ; \tag{3A}$$

similarly

$$N(\Theta_L)_b = \frac{n_s}{\pi} \gamma_b^{\,2} \sec^3 \Theta_L \left\{ 1 + \gamma_b^{\,2} \tan^2 \Theta_L \right\}^{-2} \quad . \tag{4A}$$

Integrating (3A) and (4A) and adding

$$0 \leq \Theta_L \leq 1^\circ \quad : \quad N(\Theta_L) \cong 0.25 \; n_s$$

$$0 \leq \Theta_L \leq 3^\circ \quad : \quad N(\Theta_L) \cong 0.5 \; n_s$$

Hence, one may conclude that there are about three pions per collision in the first degree cone and six in the three degree forward cone, on the assumption of six pions per fireball as derived on the basis of the statistical model.

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