Graviton-form invariants in $D = 11$ supergravity

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We complete an earlier derivation of the 4-point bosonic scattering amplitudes, and of the corresponding linearized local supersymmetric invariants in $D = 11$ supergravity, by displaying the form-curvature, F^2R^2, terms.

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Some time ago [1], we presented an efficient method for constructing explicit bosonic invariants at quartic order in $D = 11$ supergravity. We were stimulated in part by contemporaneous [2] calculations of the lowest, 2-loop order, divergences of the theory. Our approach to finding counterterms was first to perform direct tree level calculations of all 4-point bosonic scattering amplitudes. We then localized these nonlocal invariants by removing their denominators, through multiplication by the Mandelstam variables stu. These were the desired-guaranteed (linearly) supersymmetric counterterms. Indeed, the localization enabled us to express them in terms of curvatures and form field strengths rather than through the original polarization tensors. The only obstacle we encountered was in the form-graviton sector, whose explicit "covariantization" we could not provide–hence this belated note on a subject that is still of current interest [3].

We will not rerecord here the remaining, $R^4 + F^4 + RF^3$, invariants as they are already given and expounded on in [1], where notation and details are found. We emphasize that both the curvatures R and four-form field strengths F are on their respective linearized mass shells: Space is Ricci flat and F is divergenceless. The (relatively normalized) promised local R^2F^2 terms are given by the ten combinations

$$
\begin{aligned}
L_{gF} = &+\frac{1}{24}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{R}_{\mu_5\mu_2\mu_3\mu_4}\mathcal{D}_{\mu_1}F_{\mu_6\mu_7\mu_8\mu_9}\mathcal{D}_{\mu_5}F_{\mu_6\mu_7\mu_8\mu_9} - \frac{1}{3}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{R}_{\mu_5\mu_2\mu_6\mu_4}\mathcal{D}_{\mu_3}F_{\mu_1\mu_7\mu_8\mu_9}\mathcal{D}_{\mu_6}F_{\mu_5\mu_7\mu_8\mu_9}\\
&-\frac{1}{2}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{R}_{\mu_5\mu_2\mu_6\mu_4}\mathcal{D}_{\mu_7}F_{\mu_1\mu_5\mu_8\mu_9}\mathcal{D}^{\mu_7}F_{\mu_3\mu_6\mu_8\mu_9} - \frac{2}{3}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{R}_{\mu_5\mu_2\mu_6\mu_4}F_{\mu_1\mu_7\mu_8\mu_9}\mathcal{D}_{\mu_3}\mathcal{D}_{\mu_6}F_{\mu_5\mu_7\mu_8\mu_9}\\
&+\frac{1}{2}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{R}_{\mu_5\mu_4\mu_6\mu_7}\mathcal{D}_{\mu_6}F_{\mu_1\mu_2\mu_8\mu_9}\mathcal{D}_{\mu_7}F_{\mu_3\mu_5\mu_8\mu_9} - \frac{1}{2}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{R}_{\mu_5\mu_4\mu_6\mu_7}\mathcal{D}_{\mu_6}F_{\mu_3\mu_5\mu_8\mu_9}\mathcal{D}_{\mu_7}F_{\mu_1\mu_2\mu_8\mu_9}\\
&+\frac{1}{6}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{R}_{\mu_5\mu_6\mu_3\mu_4}\mathcal{D}_{\mu_5}F_{\mu_1\mu_7\mu_8\mu_9}\mathcal{D}_{\mu_6}F_{\mu_2\mu_7\mu_8\mu_9} + \frac{1}{8}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{R}_{\mu_5\mu_6\mu_3\mu_4}\mathcal{D}_{\mu_7}F_{\mu_1\mu_2\mu_8\mu_9}\mathcal{D}^{\mu_7}F_{\mu_5\mu_6\mu_8\mu_9}\\
&-\frac{1}{2}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{R}_{\mu_5\mu_6\mu_7\mu_4}\mathcal{D}_{\mu_8}F_{\mu_1\mu_2\mu_7\mu_9}\mathcal{D}^{\mu_8}F_{\mu_5\mu_6\mu_3\mu_9} - \frac{1}{4}\mathcal{R}_{\mu_1\mu_2\mu_3\mu_4}\mathcal{D}_{\mu_5}F_{\mu_1\mu_2\mu_6\mu_7}\mathcal{D}^{\mu_5}\mathcal{R}_{\mu_8\mu_9\mu_3\mu_4}F_{\mu_8\mu_9\mu_6\mu_7}.
\end{aligned}
$$

We have not seriously attempted to simplify this result using say cyclic identities, nor to obtain "current-current" factorizations; it seems to us unlikely that major condensation can occur.

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Note added.—Some time ago, corresponding quartic (in curvature and 3-form H) invariants were calculated in the heterotic string slope expansion [4]. A mutual check would be to compare them with the $D = 10$ Kaluza-Klein reduction of our various invariants, upon identifying $F_{\dots 11}$ with $H_{\dots}$, dropping all $\mathcal{D}_{11}$, and $\mathcal{R}_{\mu\nu\lambda 11}$. The famous "$t_8 t_8$" hallmark of the string expansion seems likely to emerge here as well. We are grateful to P. Vanhove for this interesting suggestion.

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