Table III. r of the plasma sheath surface (in units of r_0) on the night side of the earth and $|r \sin\theta|$ as functions of polar angle in the plane of the earth's dipole and earth-sun line.

θ	15.5°	19°	23°	37.5°	55°	69°	90°
r	~1.10	1.26	1.50	2.00	3.00	5.00	∞
$\|r \sin\theta\|$	~1.06	1.19	1.38	1.58	1.72	1.77	1.785

It is a pleasure for the author to have this opportunity to thank Dr. Alex Dessler and especially Dr. Francis S. Johnson, both of Lockheed, for bringing the problem to the author's attention, and for much illuminating discussion.

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EVIDENCE FOR A CONFIGURATIONAL EMF IN A CONDUCTING MEDIUM*

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There is a pressure drop associated with an increase in the flow rate of a moving fluid. The venturi meter and the aspirator are familiar devices which derive from this phenomenon. This paper concerns itself with the detection of an analogous effect in the electron gas in a metal.

The usual analysis for the pressure drop in classical fluids requires the conservation of energy. Associated with a higher flow rate is an increase in kinetic energy. The pressure is a measure of potential energy. It must be lower in the region of higher flow velocity where the kinetic energy is higher. If the higher flow velocity is brought about by a narrowing of the flow channel then the phenomenon is called the Bernoulli effect.

In the electron gas of a metal the dissipative effects dominate over the inertial ones. Unlike the case for a viscous fluid, the dissipation comes from the transfer of momentum to the lattice vibrations and imperfections rather than to the walls of the container. Regardless of the nature of the collisions, however, the two fluids have much in common. They both have need of the presence of a force merely to maintain the flow. A force maintains the current in the face of the scattering of forward momentum into random motion by collisions. In the electron fluid this force is the applied electric field. By Ohm's law the force is proportional to the flow velocity. The electric field in the electron gas is the analog of the pressure gradient in viscous fluid flow.

The electron gas in a metal sees a background of positive charge from the ion cores. These keep the electron density uniform on penalty of building up high internal fields in the metal. The electron gas is, therefore, relatively incompressible.

Suppose that an electric current flow is constricted in cross section as in Fig. 1. Continuity and incompressibility demand that the flow velocity or drift of electrons be greater in the constricted region. Inspired by analogy with the hydrodynamic problem, the following question is posed: What accelerates the electrons in the region of changing flow rate so that they acquire the increased speed necessary for drift through the constriction? Is there an additional field present from which the electrons get the extra accelerating kick which they must have in order to move faster in the constriction? The experi-

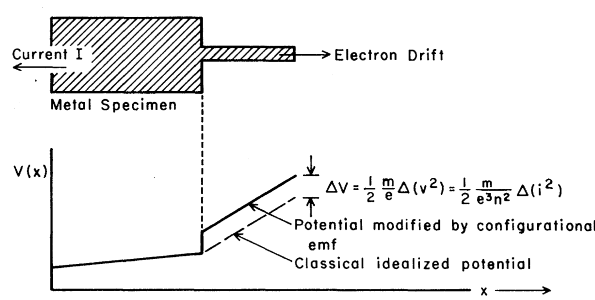

FIG. 1. Expected potential vs distance due to constricted current flow in the conducting specimen shown.

mental objective was the detection of the potential difference associated with such an accelerating field.

The existence of such a potential reflects the inexactness of Ohm's law. The effect of this non-Ohmic potential is shown diagramatically in Fig. 1. The potential step corresponding to the spatial step is the same regardless of the direction of current flow. This is in contrast to a constriction resistive potential which reverses with current.

There is a parallel structure in the analysis of the Bernoulli effect and of the configurational emf. It seems reasonable, therefore, to make a first estimate of the configurational voltage from the following equation:

$$-e\Delta V_C + \tfrac{1}{2}m\Delta v^2 = 0 \qquad (1)$$

Here V_C is the configurational potential, e is the electronic charge, m is the electronic mass, and v is the drift velocity. When the constricted area is sufficiently smaller than the area of the wider flow region, Eq. (1) may be written:

$$\Delta V_C = \tfrac{1}{2}(m/e^3n^2)i^2. \qquad (2)$$

Here i is the current density in the constricted region and n is the number of current carriers per unit volume.

The essential quality which makes the configurational emf detectible is its dependence on the square of the current. It has the same polarity regardless of the direction of current flow. This polarity depends only upon the sign of the charge of the current carriers. Equations (1) and (2) apply to negative carriers. In principle, the experiment requires only that an ac signal be applied to a stepped specimen like that of Fig. 1. A dc voltmeter across the step then reads the configurational emf directly.

The experiment was carried out in thin-film samples of bismuth. These were shaped basically as shown in Fig. 2. Note that all foreign wire leads from the external circuitry meet the sample material only at regions thermally removed from the central strip carrying the high current density. This procedure avoids introducing thermocouple emf's into the circuit. A special technique employing a bridge circuit with the specimen of Fig. 2 insured that the signal was not due to rectifying contacts at the specimen terminals.

A definite unidirectional configurational voltage was detectible. This voltage was proportional to the square of the current out to current densities estimated as high as 10^6 amperes/cm^2. Some of the specimens appeared to show conduction by positive carriers. This was deduced from the sign of the configurational voltage. In each case the carrier sign was confirmed by a Hall effect measurement.

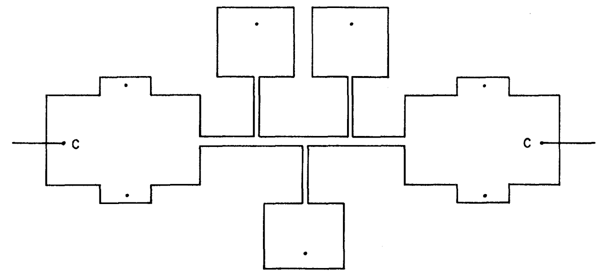

FIG. 2. Evaporated thin film specimen geometry. Points C are current terminals; all other points are potential contacts.

Various supplementary experiments shows that the detected voltage was clearly related to the configuration of the specimen. For example, a null effect results if the two potential contact points do not span a change in cross section. Figure 2 shows the extra potential probes provided for consistency tests of this kind.

The positive carrier specimens gave large signals but noisy and erratic ones. They were poor specimens on which to make measurements. The negative carrier specimens gave consistent and reproducible results.

The quantitative correlation between the experimental data and Eq. (2) was quite poor. The calculations were made using the value of the carrier density, n, obtained from direct Hall measurements on the samples.

That there is quantitative disagreement with Eq. (2) is not surprising. The crudeness of its derivation suggests that at best it might be applicable to an ideal metal like sodium or potassium.

Recently more elaborate and detailed calculations have been made. The analysis makes use of the Boltzmann transport equation as applied to the Fermi gas of electrons. The results indicate that in certain cases a better statement than that of Eq. (2) is given by

$$\Delta V_C = \gamma(m/m^*)(m/e^3n^2)i^2. \qquad (3)$$

The factor γ is of order unity. It involves integrals over the angular distribution of the phonon scattering cross section. The quantity m^* represents an average effective mass of electrons at the top of the Fermi sea.

Equation (3) suggests that the discrepancy between the experimental data and Eq. (2) arises from the deviation of m^*/m from unity in bismuth. If so interpreted, the data indicate that the value of this ratio is of the order $m^*/m = 10^{-4}$.

In summary then, there is little doubt empirically that there is an emf associated with a current flow constriction. Those gross properties of this emf which have been investigated seem to correlate well with those derived from the hydrodynamic principles which motivated this research. Although this explanation of the configurational emf appears to be the most reasonable one at present, it has not been totally verified. Further investigation on a variety of materials has been initiated to resolve the matter. It is hoped that new experimental results will be available for presentation soon to supplement the extended theoretical analysis which is in preparation.

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SHALLOW IMPURITY TRAPS AND ELECTRON TRANSFER DYNAMICS IN n-TYPE SILICON AT LIQUID HELIUM TEMPERATURES*

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From combined electron spin resonance and photoconductivity experiments on samples of phosphorus-doped silicon with boron compensation, we have been able to determine some interesting properties of shallow impurities in silicon. These include (a) rate of electron transfer[1] from a neutral phosphorus impurity to a neutral boron impurity at various impurity concentrations and temperatures, and (b) ratio of positive phosphorus ion trapping cross section to neutral boron trapping cross section for conduction electrons. In addition, the technique we introduce allows an accurate determination of the ratio of donor concentration to compensating acceptor concentration without recourse to transport phenomena. The technique also permits controlling the number of ionized impurities in a given sample while at low temperatures (liquid helium); this makes it possible to investigate in a simple manner physical properties which may depend on ionized impurity concentration, such as low-temperature mobility. Information on the lifetimes of hot carriers as a function of their energy can also be obtained.

We now describe the experimental procedure and theory relevant to the above studies. Consider a sample of n-type silicon containing P phosphorus impurities/cm^3 and B boron impurities/cm^3. In equilibrium at liquid helium temperatures, the neutral phosphorus concentration P^0 equals $(P - B)$, the positive phosphorus ion concentration P^+ is equal to B, and the negative boron ion concentration B^- also equals B. The P^0 contain unpaired electrons, and thus the electron spin resonance signal[2] is proportional to P^0. We now produce electron-hole pairs uniformly throughout the sample by illuminating with intrinsic radiation (1.03 microns)[3] near the indirect transition threshold. The electrons and holes are rapidly captured; the electrons predominantly by the P^+ ions, and the holes predominantly by the B^- ions. If the rate of generation of electron-hole pairs is much greater than the rate of electron transfer from the P^0 to the B^0, then the P^0 concentration builds up very close to P. The new resonance signal is thus $(1 - B/P)^{-1}$ times the original resonance signal, and the compensation B/P is directly determined. If the intrinsic radiation is now turned off and the resonance signal monitored, the rate of electron transfer from P^0 to B^0 can be determined. The electron transfer rate can be increased greatly by delocalizing the electrons with extrinsic radiation. A wavelength of about 2 microns insures a strong predominance of free electrons over holes because of the free-carrier absorption peak[4] in n-type silicon. If we consider a simple model in which P^+ and B^0 are the