Evidence for Skyrmions and Single Spin Flips in the Integer Quantized Hall Effect

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(Received 29 June 1995)

We have employed tilted-field magnetotransport measurements of the energy gap for the odd-integer quantized Hall states at Landau level filling factors \( \nu = 1, 3, \) and 5 to determine the spin of thermally excited quasielectron-quasihole pairs. At \( \nu = 1 \) our data show that as many as 7 electron spin flips accompany such excitations, while at \( \nu = 3 \) and 5 apparently only a single spin flips. These results lend support to the recent suggestions that “Skyrmionic” quasiparticles are the lowest-lying charged excitations of the fully polarized \( \nu = 1 \) quantum Hall fluid but are not at the higher odd-integer fillings.

PACS numbers: 73.40.Hm

The integer and fractional quantized Hall effects (QHE) that occur in two-dimensional electron systems (2DES) at high magnetic field are customarily distinguished by the origin of the underlying energy gaps in the two cases [1]. On the one hand, the magnetic field-induced resolution of the single-particle energy spectrum into a series of discrete, but highly degenerate, spin-split Landau levels provides all the energy gaps necessary to explain the integer QHE, with each such Hall plateau corresponding to the complete filling of an integer number of these single-particle levels. In contrast, for the fractional effect, which occurs only at certain partial fillings of the Landau levels, the requisite energy gap derives entirely from many-body effects. This distinction, while often useful, can be misleading, especially in the case of the odd-integer QHE. For these states, in which the Fermi level resides in the spin-flip gap within the uppermost Landau level, electron-electron interaction effects have long been known [2,3] to greatly enhance the energy gap above the single-particle Zeeman energy. In fact, it is now believed that the odd-integer QHE states would survive even if the Zeeman energy were removed entirely [4]. In this case, the 2DES would develop spontaneous ferromagnetic order (at zero temperature) solely because of interaction effects. But perhaps even more remarkable than this are the recent predictions concerning the nature of the elementary excitations of these ferromagnetic states [4,5]. Provided that the Zeeman energy is sufficiently small, the lowest-lying charged excitation is not simply a single flipped spin but is instead a large, smooth distortion of the spin field in which many spins are flipped. While such excitations, whose charge is \( \pm e \), obviously cost more Zeeman energy than a single spin flip, the near parallelism of neighboring spins saves on exchange energy. The total spin, and hence the spatial extent, of these objects is determined by the competition between these two energies. Evidence for these unusual excitations (known as “Skyrmions” in the limit of zero Zeeman energy) has recently been uncovered in NMR Knight-shift studies of the 2DES ground state spin polarization [6]. In this Letter we report on transport studies that directly probe the charged excitations of odd-integer quantized Hall states. Our findings strongly suggest that while large-spin Skyrmionic quasiparticles dominate the \( \nu = 1 \) integer QHE (where only the lower spin branch of the lowest Landau level is occupied), they are not relevant to the higher odd-integer states at \( \nu = 3 \) and 5.

In this experiment we determine the energy gap \( \Delta \) for creating a widely separated quasielectron-quasihole pair in a given quantized Hall state by measuring the temperature dependence of the longitudinal resistance in the thermally activated regime where \( R_{xx} = R_0 \exp(-\Delta/2T) \). We then assume that the spin \( s \) of the quasiparticle pair can be extracted from the change in \( \Delta \) produced by tilting the total magnetic field \( B_{\text{tot}} \) away from normal to the 2D plane (keeping, however, the perpendicular field \( B_{\perp} \), and thus the Landau level filling factor \( \nu \), fixed). The basis for this assumption is that for an ideal, infinitely thin 2DES, an in-plane magnetic field \( B_{\parallel} \) couples to the system only through the Zeeman energy, while the perpendicular field controls the orbital dynamics [7]. This implies that the Zeeman contribution to the energy gap \( \Delta \) for creating a quasiparticle pair with spin \( s \) (in units of \( \hbar \)) out of a polarized ground state is merely additive:

\[
\Delta = \Delta_{0,\parallel}(B_{\perp}) + s|g|\mu_B B_{\text{tot}}. \tag{1}
\]

The first term, \( \Delta_{0,\parallel}(B_{\perp}) \), is the contribution to the gap arising from all non-Zeeman sources (e.g., many-body effects) and, in this model, depends only upon the perpendicular magnetic field \( B_{\perp} \). This formula is a generalization, to the case \( s > 1 \), of the result given by Ando and Uemura [8] for the energy required to flip a single spin in the 2DES in Si MOSFETs. From this equation it follows that the derivative \( \partial\Delta/\partial B_{\text{tot}} \) (evaluated at constant \( B_{\perp} \)) is just \( s|g|\mu_B \). Since the \( g \) factor and Bohr magneton \( \mu_B \) are known \( |g| = 0.44 \) in GaAs [9] and \( \mu_B = 0.672 \) K/T, measuring \( \partial\Delta/\partial B_{\text{tot}} \) determines the spin \( s \). For example, in the traditional (i.e., pre-Skyrmion) view of the fully spin-polarized \( \nu = 1 \) QHE state, the lowest-lying charged excitation is assumed to be a single flipped spin, i.e., \( s = 1 \). In this case \( \Delta_{0,1} \) is the Coulomb exchange energy.
[10] $E_m = \sqrt{\frac{e^2}{\epsilon}} / \epsilon \ell_0$ (where $\epsilon \approx 13$ is the dielectric constant of GaAs and $\ell_0 = \sqrt{\hbar/eB_\perp}$ is the magnetic length). Although the exchange term dominates the net gap in typical GaAs/AlGaAs systems, the Zeeman contribution should nevertheless be detectable via the derivative $\partial \Delta / \partial B_{tot} = + |g| \mu_B (= 0.3 \text{ K/T} \text{ in GaAs})$.

The four samples used in this study are modulation-doped GaAs/AlGaAs heterostructures grown by molecular beam epitaxy. Two are conventional single heterointerfaces, and two are GaAs single quantum wells, with widths of 200 and 140 Å, respectively. As grown, these samples, labeled S11, S12, QW1, and QW2, have 2DES densities of $N = 0.6, 1.3, 2.1,$ and $1.4 \times 10^{11}$ cm$^{-2}$, and low temperature mobilities of $\mu = 3.4, 2.8, 2.0,$ and $0.38 \times 10^6$ cm$^2$/V s. In addition, for samples QW1 and QW2, these parameters could be altered significantly by applying voltages to metal gate electrodes placed on the sample’s top and/or bottom surface. Each sample was a roughly 5 × 5 mm square with eight diffused In contacts around the outer edge. Conventional magnetotransport measurements were performed down to 0.5 K using 100 nA, 5 Hz excitation. Tilting of the samples with respect to the applied magnetic field was performed in situ at low temperature.

Figure 1 shows typical temperature dependences of the resistivity minimum at $B_\perp = 2.3$ T of the $\nu = 1$ QHE state in sample S11. Data obtained with the magnetic field perpendicular to the 2D plane ($\theta = 0$) and tilted out to $\theta = 56^\circ$ are shown. The dashed lines are least-squares fits to the linear portion of the data; from the slopes of these lines we find energy gaps of $\Delta = 19$ and $23$ K for $\theta = 0^\circ$ and $56^\circ$, respectively. These gap values are much larger than the Zeeman energy in GaAs ($g|\mu_B B_{tot} = 0.7 \text{ K at B_{tot} = 2.3 T}$) and clearly demonstrate the well-known [2, 3] dominance of many-body effects at $\nu = 1$. Figure 2 shows the overall tilted-field dependence of the $\nu = 1$ energy gap observed in samples S11, QW1, and QW2. (For sample QW1 the two data sets shown were obtained using different gating configurations [11]). The leftmost point in each data set corresponds to $\theta = 0$ and $B_{tot} = B_\perp$. The energy gap $\Delta$ is given in units of $e^2/\epsilon \ell_0$, and the total magnetic field is represented by the dimensionless Zeeman energy $g = |g| \mu_B B_{tot} / (e^2/\epsilon \ell_0)$. (The Coulomb energy $e^2/\epsilon \ell_0$ depends only upon $B_\perp$ and is thus constant in a tilt experiment.)

As Fig. 2 shows, the $\nu = 1$ energy gaps initially rise quickly as the magnetic field is tilted. The initial slope $\partial \Delta / \partial B_{tot}$ is roughly 2 K/T and is the same in all the samples. This slope is about 7 times larger than what Eq. (1) predicts for excitation of quasiparticle pairs involving a single spin flip. Assuming that Eq. (1) gives an adequate basis for interpreting these data, the large slope suggests that unusual large-spin ($s \approx 7$) charged objects are being thermally excited. On the other hand, Nicholas et al. [2], who first noticed the large $\partial \Delta / \partial B_{tot}$ at $\nu = 1$, attributed it to a breakdown within the assumptions underlying Eq. (1). But before arguing that our results do, in fact, imply the existence of large-spin charged excitations at $\nu = 1$, we turn

FIG. 1. Arrhenius plots of the longitudinal resistance $R_{xx}$ at filling factor $\nu = 1$ ($B_\perp = 2.3$ T) for sample S11. The data sets are recorded for tilt angles $\theta = 0^\circ$ and $56^\circ$. The experimental geometry is shown in the lower left inset. The upper right inset displays traces of $R_{xx}$ at $\theta = 0$ vs magnetic field around $\nu = 1$.

FIG. 2. Results of tilted-field experiments on the $\nu = 1$ QHE. The energy gaps $\Delta$ at fixed $B_\perp$ are plotted vs the Zeeman energy $g \mu_B B_{tot}$, both in units of $e^2/\epsilon \ell_0$. Each data set starts with $\theta = 0$ and $B_{tot} = B_\perp$ at the lower left. On the quantum well samples we use gate electrodes to tune the electron densities [11]. From top to bottom the samples had electron densities $0.6, 1.0, 0.6,$ and $1.0 \times 10^{11}$ cm$^{-2}$ and mobilities $3.4, 0.52, 0.18,$ and $0.16 \times 10^6$ cm$^2$/V s, respectively. For comparison we include lines with $\partial \Delta / \partial (g \mu_B B_{tot}) = s = 7$ (dashed) and 1 (dotted). The inset shows a Hartree-Fock result of Skyrmeon theory (full line) [514].
to our experimental results for the higher-order spin-flip QHE states at \( \nu = 3 \) and 5.

Figure 3 displays the tilted-field results for \( \nu = 3 \) and 5 obtained using samples SI2, QW1, and QW2. For these samples the 2DES density (adjusted by gating, if necessary) produced these higher filling factors at about the same perpendicular magnetic field as employed earlier at \( \nu = 1 \). This assures us that the various energy scales (Zeeman, Coulomb, and cyclotron) are all of the same magnitude as they were for \( \nu = 1 \). Again, the figure plots the normalized energy gap \( \Delta / (e^2 / \epsilon_0) \) versus dimensionless Zeeman energy \( g |\mu_B B_{\text{tot}}| / (e^2 / \epsilon_0) \). While the observed gaps at these filling factors are somewhat smaller than that found at \( \nu = 1 \), they still exceed the Zeeman energy \( g |\mu_B B_{\text{tot}}| / (e^2 / \epsilon_0) \) by about an order of magnitude. Thus, interaction effects dominate these integer QHE states as well. On the other hand, instead of a large initial slope (the dashed line corresponds to the slope seen at \( \nu = 1 \)), the data in Fig. 3 exhibit rather little variation with tilt. The dotted lines have slopes appropriate to single spin flips (\( s = 1 \)) and, while \( \nu = 3 \) in sample QW2 and \( \nu = 5 \) in sample QW1 are consistent with this, the two other data sets show even weaker dependences. Thus, we observe a qualitative difference between the charged excitations of the \( \nu = 3 \) and 5 QHE states and those at \( \nu = 1 \).

We believe that the data shown in Figs. 2 and 3 strongly support the recent theoretical predictions about Skyrmionic excitations in the quantized Hall effect. Nevertheless, before comparing our data to these predictions, we first discuss two effects not present in an ideal, infinitely thin 2DES, and show that they cannot be held responsible for our results. We consider first the non-Zeeman effects of large in-plane magnetic fields. Owing to the finite thickness of real 2D electron systems, an in-plane magnetic field couples not only to the Zeeman energy, but also to the perpendicular dynamics. This coupling involves mixing between the subbands of the heterostructure confinement potential. Although the effect of these mixings on QHE gaps is not well understood [12], the controlling parameter is the thickness of the 2D sheet or, equivalently, the energy splitting between the lowest and first-excited confinement subbands. For sample SI1, a conventional single heterointerface, self-consistent solution of the Schrödinger and Poisson equations [13] yields an estimated subband splitting of \( E_{01} \approx 7 \) meV and an rms thickness for the ground subband wave function of \( \sigma_z \approx 76 \) Å. In order to have a significantly thinner 2DES, with its concomitantly larger subband splittings, we chose to study quantum well samples. For the 200 Å quantum well sample QW1 \( E_{01} \approx 31 \) meV and \( \sigma_z \approx 42 \) Å while in the 140 Å sample QW2 these numbers are 57 meV and 31 Å, respectively. But, as Fig. 2 clearly demonstrates, for these much thinner 2DES samples the slopes \( \partial \Delta / \partial B_{\text{tot}} \) observed at \( \nu = 1 \) are nearly the same as that observed in sample SI1. The same conclusion applies to the \( \nu = 3 \) and 5 data in Fig. 3, although the anomalously small slope for \( \nu = 3 \) in sample SI2 may, in fact, be a residual finite-thickness effect. In our opinion, the thickness independence of our results at \( \nu = 1, 3, \) and 5 strongly discounts subband mixing effects of the in-plane magnetic field.

Another important question concerns the role of disorder in the 2DES. Indeed, the samples used here differ significantly in their zero field mobility \( \mu \): for sample SI1 \( \mu = 3.4 \times 10^8 \) cm²/V·s, while sample QW2, when gated as in Fig. 2, has \( \mu = 1.6 \times 10^8 \) cm²/V·s. Although the mobility is not necessarily the best measure of the disorder relevant to the QHE, the observed \( \nu = 1 \) gap magnitiude is systematically smaller in the samples with lower mobility. But, as already noted, the tilted-field behavior displayed in Figs. 2 and 3 is the same from one sample to the next. This is strong evidence that disorder is not playing a qualitatively important role. In particular, it argues against the suggestion [2] that the large slope at \( \nu = 1 \) is due to incomplete spin polarization of the \( \nu = 1 \) ground state resulting from the overlap of disorder-broadened spin branches of the lowest Landau level. (Indeed, were such a mechanism operative, one would expect large slopes at \( \nu = 3 \) and 5 as well as \( \nu = 1 \).)

While neither finite thickness nor disorder effects appear to explain the large \( \partial \Delta / \partial B_{\text{tot}} \) observed at \( \nu = 1 \), our results are in excellent qualitative agreement with recent theory [4,5], which predicts that the lowest-lying charged excitations at \( \nu = 1 \) are large-spin Skyrmionic quasiparticles. The spin \( s \) of a thermally excited Skyrmion–anti-Skyrmion pair depends upon the ratio \( \tilde{g} \) of Zeeman to Coulomb energies. For sufficiently large \( \tilde{g} \), \( s = 1 \) and Skyrmions are identical to single spin flips. In this
limit the predicted energy gap (in units of $e^2/\epsilon_0$) is $\tilde{\Delta} = \frac{\sqrt{2}}{2} \pi$. In the opposite limit, $\tilde{\tau} \rightarrow 0$, the Skyrmion spin and size diverge while the energy gap approaches $\tilde{\Delta} \approx \frac{1}{3\sqrt{2}}$, i.e., precisely one-half the energy required to flip a single spin [4]. The inset to Fig. 2 displays one calculation [5,14] of the $\nu = 1$ energy gap as a function of $\tilde{g}$ for an ideal, infinitely thin, 2DES. This Hartree-Fock calculation, which asymptotically approaches the single spin flip at large $g$, does not adhere to the expectation [15] that the spin of the Skyrmion–anti-Skyrmion pair is always an odd integer and that, as a result, the energy gap is actually a continuous sequence of straight line segments, each with slope $\partial \Delta/\partial \tilde{g} = s = 1, 3, 5, \ldots$. Nevertheless, it is apparent that the general shape of the theoretical curve in the inset is in good qualitative agreement with our experimental results. Furthermore, the spin size we infer, $s \approx 7$ at $\tilde{g} = 0.01$, is close to the theoretical estimate [5,14] of $s = 9$. On the other hand, the magnitude of the measured gap itself is only about 25% of the theoretical value. There are, however, several possible sources of energy gap suppression, including disorder, finite-thickness effects, and Landau level mixing. While the effect of disorder is probably small in sample SI, which has a mobility in excess of $3 \times 10^6$ cm$^2$/V.s, estimates [16] suggest that the thickness-induced softening of the Coulomb interaction reduces the $\nu = 1$ gap by roughly 30%. Landau level mixing, however, may be the most important effect since for our samples, in which $\nu = 1$ occurs at $B_L \approx 2$ T, the Coulomb energy $e^2/\epsilon_0$ actually exceeds the cyclotron gap. Indeed, recent variational calculations [17] suggest that Landau level mixing can reduce the $\nu = 1$ energy gap very substantially (~50%) at low magnetic fields. In view of these considerations, we do not believe that the quantitative disagreement between Hartree-Fock theory and our experiment invalidates our fundamental conclusion that large-spin charged excitations dominate the $\nu = 1$ QHE gap.

Remarkably, recent theoretical work [18] predicts that within the integer QHE, Skyrmions are the lowest-lying charged excitations only for the case $\nu = 1$. Even without the Zeeman energy, conventional single spin flips are predicted to be lower in energy than Skyrmions for all $\nu \geq 3$. In spite of the great similarity between the higher odd-integer filling factors and $\nu = 1$, Skyrmions are apparently destabilized by the subtly different electron interactions in the higher Landau levels. Although theory [18] ignores the possibly important effects of Landau level mixing, our observation of only very small slopes $\partial \Delta/\partial B_{rot}$ at $\nu = 3$ and 5 appears to qualitatively verify their prediction.

In conclusion, we have used tilted-field studies of the energy gap for the $\nu = 1, 3, 5$ integer quantized Hall states to estimate the spin of thermally excited quasielectron-quasihole pairs. At $\nu = 1$ our results reveal unusual charged excitations in which typically 7 spins are reversed at $B_L \approx 2.3$ T. In contrast, at $\nu = 3$ and 5 our findings are consistent with ordinary single spin-flip excitations. Both of these results are in excellent qualitative agreement with recent theory on Skyrmionic excitations in the quantized Hall effect.

It is a pleasure to thank Luis Brey, Song He, Allan MacDonald, Shivaji Sondhi, and Pär Ömling for useful discussions. We also thank Song He for performing informative small system exact diagonalization calculations at our request.

[8] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. 37, 1044 (1974). In this paper the $g$ factor was taken to be 2, as appropriate to bulk silicon. For GaAs the $g$ factor is smaller: $|g| = 0.44$.
[9] This well-established value is, in principle, only appropriate for bulk GaAs [C. Weisbuch and C. Hermann, Phys. Rev. B 15, 816 (1977)]. Quantum confinement effects tend to reduce the $g$ factor [M. Dobers, K. v. Klitzing, and G. Weimann, Phys. Rev. B 38, 5453 (1988)] and to introduce an anisotropy with respect to the growth axis [E.L. Ivchenko and A.A. Kiselev, Sov. Phys. Semic. 26, 827 (1992)]. For our samples these effects are minor, and their inclusion would only further increase the deduced Skyrmion spin values.
[11] Sample QW1 was fitted with both front and back gates. It was thus possible to obtain different mobilities at the same density by forcing the 2DES wave function toward the upper (cleaner) or lower (dirtier) interface of the quantum well.
[14] Allan MacDonald (private communication).
[16] This estimate is based upon finite-size exact diagonalization calculations of the gap; Song He (unpublished).