

# Ion trap transducers for quantum electromechanical oscillators

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An enduring challenge for contemporary physics is to experimentally observe and control quantum behavior in macroscopic systems. We show that a single trapped atomic ion could be used to probe the quantum nature of a mesoscopic mechanical oscillator precooled to 4K, and furthermore, to cool the oscillator with high efficiency to its quantum ground state. The proposed experiment could be performed using currently available technology.

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A quantum electromechanical system (QEMS) is a device where the quantum nature of either the electronic or mechanical degrees of freedom becomes important in the observable behavior [1, 2]. Potential applications of QEMS include single spin detection [3], single molecule mass spectrometry [4], and readout for quantum information devices [5]. An excellent example of such a system is a rf single electron-transistor integrated with a low loss, high frequency nanomechanical resonator which demonstrated both continuous position detection approximately a factor of five away from the uncertainty principle limit, and direct observations of the mechanical mode occupation factor as low as  $N \approx 58$  [6]. However, this device suffers from both practical and fundamental limitations; picowatt levels of dissipation at the transistor is suspected to have blocked passive cooling of the mechanics to lower temperatures and occupation factors, as well as the fundamental sensitivity limitation due to coupling to an intrinsically non QND (quantum non-demolition) variable such as position. Other methods to perform measurements on quantum limited mechanical systems are clearly needed. Furthermore, coupling atomic systems to nano-electronic and nano-mechanical devices appears to be a very promising and exciting new frontier, where it is hoped one might combine the unmatched quantum coherence and detection efficiencies of atomic physics, with the sophisticated electronic and microstructures which are possible in the condensed matter realm [7, 12].

In this paper we investigate the use of a laser-cooled, trapped atomic ion to both monitor and manipulate the number-state of a QEMS oscillator, following earlier suggestions by Wineland and others [9, 10]. As the temperature of an ion's vibrational degree of freedom can be  $< 10^{-3}$  K, it is hoped that such a transducer could generate far less thermal power than low temperature single-electron devices. Furthermore, due to the excellent optical readout achieved in ion trap systems via fluorescence shelving [11], the ion trap transducer is expected

to possess sensitivity of QEMS energy at the level of a single quanta, a detection which is very difficult with a simple linear coupling to displacement. Finally, we show that the ion-QEMS system can be configured to provide a very effective cooling mechanism for the mechanical oscillators [9] and estimate that it should be possible to cool a QEMS cantilever, precooled only to 4 Kelvin, to its ground state with very high efficiency.

Here we describe a quantum model of a single trapped atomic ion which is electrostatically coupled to a very small doubly clamped cantilever, Fig. 1. This coupling can be switched on and off using an external bias voltage at an electrode on the oscillator. The ion is held in a mesoscopic micro-fabricated quadrupole Paul trap [12], and laser cooled using resolved sideband cooling [13]. External lasers are used to couple the internal electronic states of the ion to its vibrational degree of freedom. The

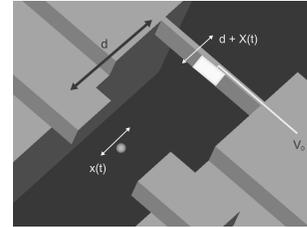


FIG. 1: Schematic representation of a single trapped atomic ion coupled to a nanoelectromechanical oscillator.

bias gate on the oscillator carries a charge  $Q = C_o V_o(t)$  where  $C_o$  is the capacitance of the gate. We allow for the possibility for the bias gate voltage  $V_o(t)$  to be time dependent so that it may be set to zero to turn off the electrostatic coupling to the trapped ion. The ion carries charge  $+e$ . In the geometry of Fig. 1, the interaction energy between the ion and the oscillator is given by  $V_c = keV_o C_o / |d + \hat{X}(t) - \hat{x}(t)|$  where  $d$  is the equilibrium separation of the oscillator centre of mass position and the ion. We set the equilibrium position of the ion at the

origin for simplicity. In that case  $\hat{X}(t)$  and  $\hat{x}(t)$  represent the small oscillations of the QEMS oscillator and the ion around their equilibrium positions, respectively. We now assume that the deviations from equilibrium are small compared to the equilibrium separation  $d$  and expand to second order in  $(\hat{X}(t) - \hat{x}(t))/d$ . The interaction energy is then given by  $V_c = (keV_oC_o/d) \left[ 1 - (\hat{X}(t) - \hat{x}(t))/d + (\hat{X}(t) - \hat{x}(t))^2/d^2 \right]$ . The linear term may be absorbed into the definition of the equilibrium positions. The quadratic term includes a coupling of the oscillators and a renormalisation of the oscillation frequency for both the ion and the oscillator. This frequency shift is a small perturbation of the bare frequencies and is neglected. We then see that the interaction Hamiltonian coupling the ion to the oscillator is  $H_c = -\chi\hat{X}(t)\hat{x}(t)$  where  $\chi = 2keV_oC_o/d^3$ . A similar ion-oscillator system is described in [14].

In ion trap technology a laser can be directed onto the ion to induce transitions between the internal electronic states [13]. The electric field seen by the ion depends on its motion in the trap, thus the external laser can couple the internal electronic state to the vibrational degree of freedom. In this way information on the vibrational motion can be transferred to the electronic degree of freedom where it can be efficiently measured using the technique of fluorescence shelving [11]. Such measurements are very nearly perfect projective measurements onto one of two electronic states,  $\{|g\rangle, |e\rangle\}$ . If the laser is detuned to the first red (blue) motional sideband, the ion can make a transition from the ground to the excited state absorbing a laser photon and a single quanta of vibrational energy is simultaneously added (deducted) in the process [13]. The probability of excitation depends on the phonon distribution in the trap, so subsequent readout of the electronic state then reveals information on the phonon number in the trap.

The overall scheme for using the trapped ion as a transducer of QEMS motion is as follows. The ion is cooled to the vibrational ground state and prepared in some appropriate electronic state. Next, a voltage is applied to the oscillator gate, coupling the motion of the QEMS to the vibrational state of the ion. This transfers phonons from the oscillator to the ion. In the next step an external laser couples the trap phonons onto the electronic degree of freedom which is then readout using fluorescence shelving.

We define dimensionless annihilation and creation operators for the ionic vibration and the QEMS vibration using the ground state standard deviation of position of the oscillator as a convenient length scale in each case,  $\hat{X} = (\hbar/(2M\omega))^{1/2} (a+a^\dagger)$  and  $\hat{x} = (\hbar/(2m\nu))^{1/2} (b+b^\dagger)$  where  $\nu$  is the vibrational frequency of the ion in the trap (also called the *secular* frequency),  $\omega$  is the resonant frequency of the QEMS oscillator, and  $M, m$  are the masses of the QEMS and the ion respectively. The total Hamil-

tonian for the three coupled systems may be written as  $H = \hbar\omega a^\dagger a + \hbar\nu b^\dagger b - \hbar\kappa(a+a^\dagger)(b+b^\dagger) + \hbar\frac{\omega_A}{2}\sigma_z + H_e$  where  $H_e$  describes the interaction between the external laser and the electronic states of the ion (see below). The coupling constant  $\kappa$  is given by  $\kappa = (mM\nu\omega)^{-1/2}keV_oC_o/d^3$ . The electronic transition frequency is  $\omega_A$  and  $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$  is a Pauli matrix. The interaction between the control laser and the electronic transition is given in the dipole approximation by  $H_e = 2\hbar\Omega\sigma_x \sin(k_l\hat{x} - \omega_l t)$  where  $\omega_l$  and  $k_l$  are the frequency and wave vector of the laser,  $\Omega$  is the effective Rabi frequency for the transition and  $\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$ . We will work in the Lamb-Dicke limit in which the amplitude of the ion's motion in the direction of radiation is much less than the wave length of the laser  $\sqrt{n_b + 1}\eta = \sqrt{n_b + 1}k_l(\hbar/(2m\nu))^{1/2} \ll 1$  in which case we may take  $H_e = 2\hbar\eta\Omega\sigma_x(b + b^\dagger)\sin(\omega_l t)$ . We now move to the interaction picture and make the rotating wave approximation for both the QEMS-ion coupling and the electronic coupling, and assume that  $\omega_l = \omega_A - \nu$  or  $\omega_l = \omega_A + \nu$  so that we are resolving the first red or blue sideband. The total Hamiltonian is  $H_I = \hbar\Delta a^\dagger a - \hbar\kappa(a^\dagger b + ab^\dagger) + H_{sb}$  with  $\Delta = \omega - \nu$  and  $H_{sb} = \hbar g(b\sigma_+ + b^\dagger\sigma_+^\dagger)$  for the red sideband and  $H_{sb} = \hbar g(b^\dagger\sigma_+ + b\sigma_+^\dagger)$  for the blue sideband where  $g = \eta\Omega$  and  $\sigma_+ = |e\rangle\langle g|$ . In our model both  $\kappa$  and  $g$  can be turned on and off. The measurement protocol has two stages. In stage I, the oscillator and cantilever are coupled for a time  $\tau$  by setting  $\kappa \neq 0, g = 0$ . In stage II,  $\kappa$  is turned off and the electronic and vibrational degrees of freedom of the ion are coupled by red and blue sideband excitation. At the end of this stage, the ionic electronic state is measured.

Using the ion as an ultrasensitive sensor for the cantilever motion is experimentally realistic. Assuming a 19.7 MHz cantilever, as reported recently by LaHaye et al. [6], a radio-frequency microfabricated ion trap [12] could be used to confine the ion at a distance  $\beta$  of  $50\mu m$  from the cantilever at a secular frequency of 19.7 MHz. Applying a static voltage of 7.5V one would obtain a coupling frequency  $\Omega_d \approx 2\pi \times 52.5$  kHz for a cadmium ion. Assuming the cantilever at a temperature at 4 K (liquid Helium), the cantilever contains on the order of 1000 quanta of motion. Therefore one would only require an interaction time of  $5 \mu s$  for the ion to undergo, on average, a single phonon excitation. Anharmonicity in the cantilever motion is included in the measured Q (too small to observe at 4K) and will not measurably distort the cantilever lineshape for any reasonable temperature [15]. An upper bound for the width of the secular motion frequency due to the anharmonicity of the ion motion at  $\bar{n}_b = 4000$  is found to be  $\Delta\nu \approx \nu(z/\beta)^2 \sim$  where  $z = (\hbar\bar{n}_b/(2m\nu))^{1/2}$  is the extent of the atomic wave function and  $\beta$  is the dimension of the ion trap. For the parameters considered here,  $\Delta\nu \ll \kappa$ , so ion trap anharmonicities are not expected to degrade the ion-cantilever

coupling. The QEMS oscillator is always coupled to a thermalising reservoir and before the coupling to the ion is turned on we assume that the oscillator is in a thermal equilibrium with mean vibrational quantum number  $\bar{n}_{a0} = (e^{\hbar\omega/k_B T} - 1)^{-1} \approx k_B T / (\hbar\omega)$ .

When the coupling between the oscillator and the ion is turned on, the ion becomes indirectly coupled to the phonon reservoir of the QEMS oscillator. This may be modelled using a quantum optics master equation for the oscillator (if the frequency of the oscillators is not too low) that describes the joint density matrix operator  $R(t)$  for the QEMS oscillator and the ionic vibration motion during stage I,

$$\begin{aligned} \frac{dR}{dt} = & -i\Delta[a^\dagger a, R] + i\kappa[a^\dagger b + ab^\dagger, R] \\ & + \gamma_a(\bar{n}_{a0} + 1)\mathcal{D}[a]R + \gamma_a\bar{n}_{a0}\mathcal{D}[a^\dagger]R \\ & + \mu_1\mathcal{D}[b]R + \mu_2\mathcal{D}[b^\dagger]R, \end{aligned} \quad (1)$$

where  $\mathcal{D}[\mathcal{O}]\rho = \mathcal{O}\rho\mathcal{O}^\dagger - (\mathcal{O}^\dagger\mathcal{O}\rho + \rho\mathcal{O}^\dagger\mathcal{O})/2$  is defined for arbitrary operators  $\mathcal{O}$  and  $\rho$ , and the damping rate  $\gamma_a$  is related to the quality factor  $Q$  of the QEMS oscillator by the expression  $\gamma_a = \omega/Q$ . Note that choosing the damping rates  $\mu_1 = \gamma_b(\bar{n}_{b0} + 1)$ ,  $\mu_2 = \gamma_b\bar{n}_{b0}$  corresponds to the dynamics for the ionic vibrational motion induced by interacting with a thermal bath, as in the case for the QEMS oscillator. However, if the main noise source coupled directly to the vibrational modes of the trapped ion is the fluctuating electric field generated in the trap electrodes [16], we can model it as a classical stochastic electric field. In this case, we take  $\mu_1 = \mu_2 = (1/\tau_1)$ , where  $\tau_1$  is the characteristic heating time [17]. Extrapolated from recent experiments and associated theory for characteristic heating times [16, 18], we obtain an expected approximate heating rate on the order of 0.06 quanta per ms for the above geometry for the cadmium ion. Comparing this heating rate with the coupling constant  $\kappa$  from our example above it is reasonable to neglect ion heating during the cantilever-ion interaction. We are interested in the mean phonon number  $\bar{n}_b(\tau) = \langle b^\dagger(\tau)b(\tau) \rangle$  of the thermal state of the ionic vibrational modes at the end of stage one. Using Eq. (1) the equations of motion for quadratic moments form a closed set and do not involve higher-order moments.

Neglecting heating due to a classical stochastic field during the ion-cantilever interaction (as justified above) and assuming the secular ion frequency equal to the resonance frequency of the cantilever one can derive a simple analytical expression ( $\Delta = \mu_1 = \mu_2 = 0$ ):

$$\begin{aligned} \bar{n}_b(\tau) = & \bar{n}_{a0} - \frac{(\bar{n}_{a0} - \bar{n}_{b0})}{8\Omega_\gamma^2} e^{-\gamma_a\tau/2} \\ & \times [(4\Omega_\gamma^2 - 2\kappa^2 + i\gamma_a\Omega_\gamma) e^{-i2\Omega_\gamma\tau} + 4\kappa^2 \\ & + (4\Omega_\gamma^2 - 2\kappa^2 - i\gamma_a\Omega_\gamma) e^{2i\Omega_\gamma\tau}], \end{aligned} \quad (2)$$

where  $\Omega_\gamma = \sqrt{\kappa^2 - (\gamma_a/4)^2}$ . The initial conditions used to obtain Eq. (2) assume that at the start of

stage one the QEMS oscillator and ionic vibrational mode are in thermal states with mean phonon number  $\bar{n}_{a0}$  and  $\bar{n}_{b0}$ , respectively. For  $\gamma_a \ll \kappa$  one obtains  $\bar{n}_b(\tau) = \bar{n}_{a0} - (\bar{n}_{a0} - \bar{n}_{b0})e^{-\gamma_a\tau/2}\cos^2(\kappa\tau)$ . The mean oscillator phonon number can also be found to be  $\bar{n}_a(\tau) = \bar{n}_{a0} - (\bar{n}_{a0} - \bar{n}_{b0})e^{-\gamma_a\tau/2}\kappa^2\sin^2(\Omega_\gamma\tau)/\Omega_\gamma^2$ . In Fig. 2, we plot  $\bar{n}_b(\tau)$  (obtained from Eq. (2)) using realistic parameters corresponding to the mechanical cantilever of Ref. [6]. Assuming  $\bar{n}_{a0} = 4000$ ,  $\bar{n}_{b0} = 0$ ,  $\Delta = 0$ ,  $\kappa = 2\pi \times 52.5$  kHz and assuming a Q factor for the cantilever of 30000 we find that the effect of the interaction of the cantilever with the thermal bath is approximately negligible during the first quarter coupling period, however, becomes significant on longer timescales. In the limit  $\gamma_a\tau \ll 1$  we find that  $\bar{n}_b(\tau) \approx \bar{n}_{a0}\sin^2(\kappa\tau)$ .

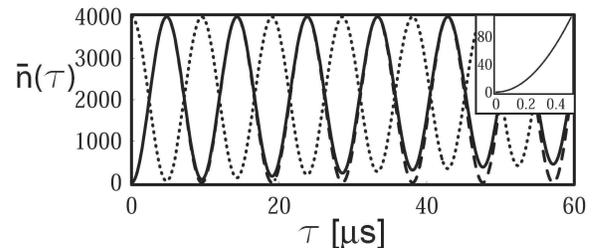


FIG. 2: The mean phonon number of the ion  $\bar{n}_b(\tau)$  as a function of time  $\tau$  obtained from Eq. (2) is plotted (solid line) and compared to the case where the coupling of cantilever and thermal reservoir is neglected during the cantilever-ion interaction (dashed line). The mean phonon number of the cantilever  $\bar{n}_a(\tau)$  is plotted as dotted line. The inset shows quadratic short time behaviour of the ion.

From Fig. 2 we see that eventually the ionic vibrational degree of freedom comes into thermal equilibrium with the cantilever. However due to the very strong coupling between the two, the dynamics of this process is certainly not exponential. The oscillations in Fig. 2 are thus evidence of non-exponential relaxation. The experiment proposed here provides a convenient way to study the transition from exponential relaxation to non-exponential short time behavior by varying the coupling strength  $\kappa$  with respect to the damping rate of the cantilever,  $\gamma_a$ .

The readout process of stage II is similar to the experiments described in Ref. [13]. In stage II we couple the electronic state of the ion to its vibrational motion for a time  $T$  using the first red and blue sideband transitions. If we write the probability for the atom to found in the excited state after time  $T$  as  $P_e^R(T)$  and  $P_e^B(T)$  for red and blue sideband excitation respectively it can be shown [16, 19] that the mean phonon number  $\bar{n}_b(\tau)$  is given by  $\bar{n}_b(\tau)/(1 + \bar{n}_b(\tau)) = P_e^R(T)/P_e^B(T) \equiv R_e$ . As this result is independent of the coupling time  $T$ , noise due to heating during stage two can be kept small by minimizing  $T$ . Indeed such experiments are routinely performed [18, 19, 20] and a phonon number smaller than unity

can be determined. As the parameters  $\kappa, \tau$  are assumed to be known this gives  $\bar{n}_{a0}$  directly. In particular, for the short time region of Fig. 2 (and  $\gamma_a \tau \ll 1$ ) we find that  $P_e^R(T)/P_e^B(T) \approx \bar{n}_{a0} \kappa^2 \tau^2$ . Thus measurement of the ratio of excitation probability on the first red and blue sideband yields  $\bar{n}_{a0}$  directly. Note that  $\bar{n}_b(\tau)$  should be on the order or smaller than  $n_{max} = 20$  for reliable measurement [10, 13, 16].

From Fig. 2 we see that it is possible to almost completely exchange the mean phonon number of the cantilever and the ionic vibration. Since the ion begins in the vibrational ground state, the result after one exchange time is to transfer the thermal energy of the cantilever rapidly to the ionic vibration, leaving the cantilever near its ground state.

A number of cooling procedures appear possible. One could simply dump the resulting hot ion or one could implement two ion traps that are located adjacent to the cantilever: one ion trap to be used to cool the mechanics to the ground state, while the other ion trap is used to perform the quantum measurement of the cantilever. Alternately, an iterative cooling mechanism could be accomplished by decoupling the resulting ion from the cold cantilever (instead of dumping it) by detuning the ion trap secular frequency and then laser cooling the ion. One could also accomplish cooling of the cantilever and obtain a reasonable temperature estimate using just a single ion provided  $\bar{n}_{a0} \pi \gamma_a / \kappa < n_{max}$  (meaning that the ion carries less than  $n_{max}$  phonons after one full coupling period). In this scheme one waits for a full coupling period to map out the minimum of  $\bar{n}_b(\tau)$ . Using this revival one could deduce the optimum cooling interaction time (half the coupling period) and the associated temperature of the cantilever. Although the cold state is metastable for all of the schemes above, with a lifetime determined by  $\gamma_a$ , we expect a significantly lower resonator temperature compared to what has been achieved with continuous refrigeration of the cantilever bath,  $\bar{n}_{a0} \sim 50$  [6]. Finally, one could couple the ion continuously resulting in sympathetic cooling and obtain a stable low temperature of the cantilever. This can achieve a mean phonon number of the cantilever less than unity [14].

Approaching the mechanical ground state may be facilitated by starting with a much higher frequency resonator (eg. a 1 Ghz cantilever [21]) and dilution refrigeration. While it may be difficult to fabricate an ion trap with such high secular frequency, it is possible to couple the cantilever to the micromotion [10, 22] in the ion trap that occurs at  $\Omega_{rf} + \nu$  and  $\Omega_{rf} - \nu$  where  $\Omega_{rf}$  is the radio-frequency driving frequency of the ponderomotive ion trapping potential. Micromotion could then be coupled via laser excitation [23] into the electronic state of the ion.

This scheme is not a single shot readout of phonon number in the oscillator, rather it enables a statistical inference of the mean phonon number of the oscillator. Can it be used to measure a weak classical force? The sensitivity to changes in  $\bar{n}_b(\tau)$  is best for values of  $\bar{n}_b(\tau)$  less than unity. This can be achieved by ensuring  $\bar{n}_{a0} \kappa^2 \tau^2 \leq 1$ . It should then be possible to detect a change in the inferred mean phonon number of the oscillator of the order of one quanta. If a weak classical force is continuously applied to the cantilever after it has been prepared close to the ground state (eg. via sympathetic cooling via the ion), its equilibrium phonon distribution will shift by an amount proportional to the square of the ratio of applied force to the energy damping rate. Thus we should be able to infer the size of a classical force so weak as to shift the mean phonon number of the oscillator by one quanta. This corresponds to a displacement sensitivity at the standard quantum limit  $\Delta x_{SQL} = (\hbar/2m\omega)^{1/2}$ .

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