Supplement–Directional limits on persistent gravitational waves from Advanced LIGO's first observing run

In this supplement we describe how we use the narrowband, directed radiometer search[?] to make a statement on the gravitational wave (GW) strain amplitude h_0 of a persistent source given some power described by our cross correlation statistic. We take into account the expected modulation of the quasi-monochromatic source frequency over the duration of the observation. We do this by combining individual search frequency bins into *combined bins* that cover the extent of the possible modulation.

Source Model.—We can relate GW frequency emitted in the source frame f_s to the observed frequency in the detector frame f_d using the relation

$$f_\mathrm{d} = [1 - A(t) - B(t) - C(t)] f_\mathrm{s} \tag{1}$$

where $A(t)$ takes into account the modulation of the signal due to Earth's motion with respect to the source, $B(t)$ takes into account the orbital modulation for a source in a binary orbit, and $C(t)$ takes into account any other modulation due to intrinsic properties of the source (for example any spin-down terms for isolated neutron stars).

The Earth modulation term is given by

$$A(t) = \frac{\vec{v}_\mathrm{E}(t) \cdot \hat{k}}{c} \tag{2}$$

where $\vec{v}_\mathrm{E}$ is the velocity of the Earth. In equatorial coordinates:

$$\vec{v}_\mathrm{E}(t) = \omega R \left[\sin\theta(t)\hat{u} - \cos\theta(t)\cos\phi\hat{v} - \cos\theta(t)\sin\phi\hat{w}\right], \tag{3}$$

in which R is the mean distance between Earth and the Sun, ω is the angular velocity of the Earth around the Sun and $\phi = 23°$, 26 min, 21.406 sec is the obliquity of the ecliptic. The time dependent phase angle $\theta(t)$ is given by $\theta(t) = 2\pi(t - T_\mathrm{VE})/T_\mathrm{year}$, where T_year is the number of seconds in a year and T_VE is the time at the Vernal equinox. The unit vector $\hat{k}$ pointing from the source to the earth is given by $\hat{k} = -\cos\delta\cos\alpha\hat{u} - \cos\delta\sin\alpha\hat{v} - \sin\delta\hat{w}$, where δ is the declination and α is the right ascension of the source on the sky.

In the case of a source in a binary system, the binary term (for a circular orbit) is given by

$$B(t) = \frac{2\pi}{P_\mathrm{orb}} a\sin i \times \cos\left(2\pi\frac{t - T_\mathrm{asc}}{P_\mathrm{orb}}\right) \tag{4}$$

where $a\sin i$ is the projection of the semi-major axis (in units of light seconds) of the binary orbit on the line of sight, T_asc is the time of the orbital ascending node and P_orb is the binary orbital period.

In the case of an isolated source we set $B(t) = 0$, while $C(t)$ can take into account any spin modulation expected to occur during an observation time. In the absence of a model for this behaviour, a statement can be made on the maximum allowable spin modulation that can be tolerated by our search.

Search.—The narrowband radiometer search is run with 192 s segments and 1/32 Hz frequency bins. For each 1/32 Hz frequency bin we combine the number of bins required to account for the extent of any signal frequency modulation The source frequency f_s is taken as the center of a frequency bin. We calculate the minimum and maximum detector frequency f_d over the time of the analysis corresponding to the respective *edges* of the bin in order to define our combined bins.

We combine the detection statistic Y_i and variance $\sigma_{Y,i}$ into a new combined statistic Y_c for each representative frequency bin via

$$Y_c = \sum_{i=-b}^{a} Y_i \text{ and } \sigma^2_{Y,c} = \sum_{i=-b}^{a} \sigma^2_{Y,i}, \tag{5}$$

where i represents the index for each of the frequency bins we want to combine. If we assign $i = 0$ to the bin where the source frequency falls, then a and b are the number of frequency bins we want to combine above and below the source frequency bin, respectively. The overlapping bins, which ensure we do not lose signal due to edge effects, create correlations between our *combined* bins.

Significance.—To establish significance, we assume that the strain power in each frequency bin is consistent with Gaussian noise and simulate > 1000 noise realizations. For each realization, we generate values of Y_i in each frequency bin i by drawing from a Gaussian distribution with $\sigma = \sigma_{Y,i}$. We then combine these bins into combined bins as we do in the actual analysis and calculate the maximum of the signal to noise ratio, SNR $= Y_c/\sigma_{Y,c}$, across all of the combined bins. We use the distribution of maximum SNR to establish the significance of our results.

Upper limits.—In the absence of a significant detection statistic, we set upper limits on the tensor strain amplitude h_0 of a gravitational wave source with frequency f_s. To take into account the unknown parameters of the system, such as the polarization ψ and inclination angle ι, and consider reduced sensitivity to signals that are not circularly polarized, we calculate a direction-dependent and time-averaged value $\mu_{\iota,\psi}$. This value represents a scaling between the true value of the amplitude h_0 and what we would measure with our search, and is given by

$$\mu_{\iota,\psi} = \frac{\sum_{j=1}^{M}\left[(A^+/h_0)^2 F^+_{\mathrm{d}j} + (A^\times/h_0)^2 F^\times_{\mathrm{d}j}\right]\left(F^+_{\mathrm{d}j} + F^\times_{\mathrm{d}j}\right)}{\sum_{j=1}^{M}\left(F^+_{\mathrm{d}j} + F^\times_{\mathrm{d}j}\right)^2} \tag{6}$$

for each time segment j. Here

$$A^{+} = \frac{1}{2} h_0 (1 + \cos^2 \iota) \;\; \text{and} \;\; A^{\times} = h_0 \cos \iota \,, \tag{7}$$

and ψ dependence is implicit in $F^{A}_{\mathrm{d}j} = F^{A}_{1j} F^{A}_{2j}$, where A indicates ($+$ or $\times$) polarization and the response functions F^{A}_{1j} and F^{A}_{2j} for the LIGO detectors are defined in [**?**] (see also [**?**]). We calculate $\mu_{\iota,\psi}$ many times for a uniform distribution of $\cos\iota$ and ψ, and then marginalize over it. We also marginalize over calibration uncertainty, where we assume (as in the past) that calibration uncertainty is manifest in a multiplicative factor $(l + 1) > 0$ where l is normally distributed around 0 with uncertainty given by the calibration uncertainty, $\sigma_l = 0.18$. The full expression for our posterior distribution given a measurement, Y and its uncertainty σ_Y in a single combined bin is given by

$$p(h_0 | Y, \sigma_Y) = \int_{-1}^{1} d(\cos\iota) \int_{-\pi/4}^{\pi/4} d\psi \int_{-1}^{\infty} dl \, e^{L(l)}, \tag{8}$$

where

$$L(l) = -\frac{1}{2} \left\{ \left(\frac{l}{\sigma_l} \right)^2 + \left[\frac{Y(l+1) - \mu_{\iota,\psi} h_0^2}{\sigma_Y (l+1)} \right]^2 \right\}. \tag{9}$$

We set upper limits on h_0 at a 90% credible level for frequencies within each combined bin that correspond to each source frequency f_{s}. Denoted h_0^{UL}, upper limits are calculated via $0.9 = \int_0^{h_0^{\mathrm{UL}}} dh_0 \, p(h_0|Y, \sigma_Y)$.

Frequency Notches.—For frequency bins flagged to be removed from the analysis due to instrumental artifacts, we set our statistic to zero, so they will not contribute to the combined statistics described in Eq **??**. We require that more than half of the frequency bins are still available when generating a combined bin. When setting upper limits, the noise $\sigma_{Y,c}$ in any combined bin that contained notched frequency bins is rescaled to account for the missing bins to provide a more accurate representation of the true sensitivity in that combined bin.