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Is Baryon Number Violated when Electroweak Strings Intercommute?

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Abstract

We reexamine the self-helicity and the intercommutation of electroweak strings. A plausible argument for baryon number conservation when electroweak strings intercommute is presented. The connection between a segment of electroweak string and a sphaleron is also discussed.

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One of the most basic observational facts in cosmology is the predominance of matter over antimatter. In 1967 Sakharov[1] considered the possibility that the Universe began in a baryon-symmetric state but that particle interactions produced a net asymmetry. In addition to baryon-number violation, this requires C and CP violation as well as a departure from thermal equilibrium.[2] In the standard electroweak theory all three conditions are satisfied. Baryogenesis during the weak phase transition[3] is particularly interesting as it may eventually be experimentally verifiable.

Recently, there has been a revival of interest in the study of classical solutions in the standard model of the electroweak interactions.[4] It has been conjectured[5,6,7] that a segment of electroweak string, which connects a monopole to an antimonopole is a kind of "stretched" sphaleron. Since the sphaleron[8,9] is of crucial interest to the study of baryon number violation processes, one may contemplate the role electroweak strings in baryogenesis shortly after the cosmological electroweak phase transition.[10]

In a recent Letter,[6] it is pointed out that the baryon number anomaly equation may be interpreted as a conservation law for baryon number minus helicity. Since the helicity is a sum of link and twist numbers, linked and twisted loops of electroweak string necessarily carry baryon number. It is also claimed that helicity and hence baryon number is conserved when electroweak strings intercommute.[11] However, two things put the argument given by those authors for baryon number conservation during intercommutation in doubt. First, the configuration after intercommutation given in Ref. 6 is implausible. Second, self-helicity is not properly accounted for in that Letter.

The main point of this Letter is to give a careful calculation of self-helicity and present a plausible electroweak string configuration after intercommutation. A proper calculation of helicity with our field configuration after intercommutation suggests that helicity is conserved when electroweak strings intercommute. We also clarify the connection between the sphaleron and a segment of Z string connecting a monopole and an antimonopole.

We will briefly review the concept of helicity of electroweak strings.[6,7] Our starting point is the Adler-Bell-Jackiw anomaly equation

$$\partial_\mu J^\mu_B = \frac{N_F}{32\pi^2}[g^2 W^{\mu\nu}_a \tilde{W}^a_{\mu\nu} - g'^2 Y_{\mu\nu}\tilde{Y}^{\mu\nu}], \tag{1}$$

where N_F is the number of families, $W^{\mu\nu}_a$ $a = 1, 2, 3$ and $Y_{\mu\nu}$ are the $SU(2)$ and $U_Y(1)$ field strengths respectively. Note that the right hand side can be expressed as a total divergence. We define the Chern-Simons numbers N_{CS} and n_{CS}

$$\begin{aligned} N_{CS} &\equiv \frac{g^2}{32\pi^2}\int d^3x \epsilon^{ijk}[W_{aij}W^a_k - \frac{g}{3}\epsilon_{abc}W^a_i W^b_j W^c_k] \\ n_{CS} &\equiv \frac{g'^2}{32\pi^2}\int d^3x \epsilon^{ijk} Y_{ij} Y_k, \end{aligned} \tag{2}$$

where W^a_k and Y_k are gauge potentials and obtain by integrating Eq. (1) over a spacetime volume that the change in baryon number is related to the changes in the Chern-Simons numbers:

$$\Delta B = N_F(\Delta N_{CS} - \Delta n_{CS}). \tag{3}$$

The Chern-Simons number is not a meaningful physical quantity as it changes by an integer upon a large gauge transformation. However, the *change* in the Chern-Simons number in any physical process is guage invariant. We will be interested in Z string configurations in which $W^1_\mu = W^2_\mu = 0$.[12] With this simplification and the transformation

$$\begin{aligned} Z_j &= \cos\theta_w W^3_j - \sin\theta_w Y_j \\ A_j &= \sin\theta_w W^3_j + \cos\theta_w Y_j. \end{aligned} \tag{4}$$

Eq. (2) gives[13]

$$N_{CS} - n_{CS} = \frac{\alpha^2}{16\pi^2}\int d^3x \left[\cos(2\theta_w) B_Z \cdot Z + \frac{1}{2}\sin(2\theta_w)(B_Z \cdot A + B_A \cdot Z)\right] \tag{5}$$

where $\alpha = \sqrt{g^2 + g'^2}$, $\tan\theta_w = g'/g$, B denotes the magnetic field, and the subscripts denote the gauge field for which the magnetic field is to be evaluated. As discussed by Vachaspati and Field, the first term on the r.h.s. has a simple interpretation in terms of the helicity[14,6,7] associated with the Z field

$$H_Z = \int d^3x B_Z \cdot Z. \tag{6}$$

The helicity is a measure of the linkage of the magnetic field. If space is divided into a collection of flux tubes, magnetic helicity arises from the internal structure within each flux tube, such as twist and kinking, and external relations between flux tubes, i.e. linking and knotting.[15] For two (untwisted) closed flux tubes linked once, a simple integration of Eq. (6) gives

$$H = \pm 2\Phi_1\Phi_2 \tag{7}$$

where Φ_1 and Φ_2 measure the magnetic fluxes of the tubes and the sign of H depends on the sense of linkage. This ends our review of the discussion of helicity made by Vachaspati and Field.

Consider a Z string loop of flux Φ that is twisted by an angle $2p\pi$. To compute its (internal) helicity,[16] let us divide the tube up into m "subtubes" each with the flux Φ/m. The linking number of each pair of subtubes is the same as that of each pair of magnetic field lines in the loop. We observe that, for a pair of field lines in a uniformly twisted torus, one of the field lines can always be deformed to the axis of the tube without intersecting the second field line. Thus, the linking number is just p. The total helicity is the sum of these self-helicities plus the interactive helicities arising from the linkage of the flux tubes. Therefore,

$$H = mH_m + 2\sum_{i<j} p\Phi_i\Phi_j, \tag{8}$$

with $\Phi_i = \Phi/m$ $(i = 1, 2, \ldots, m)$. Since self-helicities scale as Φ^2,

$$H = \frac{H}{m} + 2p\frac{m(m-1)}{2}\left(\frac{\Phi}{m}\right)^2 \tag{9}$$

i.e.[17]

$$H = p\Phi^2. \tag{10}$$

Let us turn to the main subject of our investigation: does intercommutation of electroweak strings violate helicity (and hence baryon number)? It is well known to fluid dynamicists that helicity is approximately conserved in the magnetohydrodynamics of fluids with negligible viscosities and large magnetic Reynolds numbers.[18] However, a network of electroweak string is far from a superconducting fluid. If viscous effects are negligible, a fluid can support no tangential stress. On the other hand, a string has string tension.

Generally speaking, naive reconnections of flux tubes after intercommutation violate helicity. Conservation of helicity would require (additional local) twisting (and/or writhing) of flux tubes after reconnections.[19]

The question is: what happens in general when two electroweak strings intersect and intercommute? Consider a pair of antiparallel electroweak strings approaching each other. The magnetic field goes through zero. It breaks and reconnects immediately. Such a matching of field lines leads to helicity conservation during intercommutation.[20]. Pfister and Gekelman have proposed the following visual demonstration of helicity conservation with a simple Christmas ribbon.[21] Let us begin with two singly linked loops of untwisted ribbons as shown in Fig. 1(a). We arrange both ribbons such that they touch at one point and the Z fields are antiparallel there (as required for reconnection). We staple the ribbons together to the left and right of the contact point and cut the ribbons in between the staples (Fig. 1(b)). What we have obtained is one loop that has two complete (360°) twists as shown in Fig. 1(c). From equation (7) and (10), we find that both the initial and final configurations have a helicity of $2\Phi^2$. Therefore, helicity is conserved. For comparison, we also show the intercommutation of two unlinked string loops in Fig. 2. They intercommute to form an untwisted loop of string. Even though we have only discussed the case of two antiparallel electroweak strings, we conjecture that helicity conservation is valid for electroweak strings intersecting at any angle. The magnetic field always goes through zero and reconnect immediately as in the case of perfect MHD.

A segment of Z string can also exist, but it has to connect a monopole (m) to an antimonopole ($\bar{m}$). The asymptotic field configurations of m and $\bar{m}$ (each connected to a semi-infinite Z string) have been written by Nambu[22]

$$\Phi_m = \begin{pmatrix} \cos(\theta_m/2) \\ \sin(\theta_m/2)e^{i\phi} \end{pmatrix}, \qquad \Phi_{\bar{m}} = \begin{pmatrix} \sin(\theta_{\bar{m}}/2) \\ \cos(\theta_{\bar{m}}/2)e^{i\phi} \end{pmatrix}, \tag{11}$$

where θ_m and ϕ are spherical coordinates centered on m (and similarly for $\bar{m}$) and the gauge field satisfies $D_i\Phi = 0$.[23] It has been suggested[5] that a segment of Z string is a kind of "extended" sphaleron. Here we are interested in a "family" of extended sphaleron by which we mean a set of field configurations each with a Chern-Simons number of $1/2$ parametrized by a deformation variable, d, such that the $d = 0$ element is a sphaleron. However, as discussed by M. Hindmarsh,[24] a simple symmetry argument shows that an untwisted segment of Z string has a Chern-Simons number zero rather than $1/2$. More recently, Vachaspati and Field[6] have considered the Higgs field configuration

$$\Phi_{m\bar{m}}(\gamma) = \begin{pmatrix} \sin(\theta_m/2)\sin(\theta_{\bar{m}}/2)e^{i\gamma} + \cos(\theta_m/2)\cos(\theta_{\bar{m}}/2) \\ \sin(\theta_m/2)\cos(\theta_{\bar{m}}/2)e^{i\phi} - \cos(\theta_m/2)\sin(\theta_{\bar{m}}/2)e^{i(\phi-\gamma)} \end{pmatrix}, \tag{12}$$

where θ_m and $\theta_{\bar{m}}$ are the polar angles and ϕ is the aximuth angle. Eq. (12) reduces to Φ_m when $\theta_{\bar{m}} \to 0$ and to $e^{i\gamma}\Phi_{\bar{m}}$ when $\theta_m \to \pi$ and, in addition, we perform the rotation $\phi \to \phi + \gamma$. Thus, we see that the antimonopole is rotated and globally transformed by $U(1)$. When $\theta_m \to \pi$ and $\theta_{\bar{m}} \to 0$, Eq. (12) reduces to the configuration of an *untwisted* Z string. Therefore, Eq. (12) describes an untwisted segment of Z string connected to a monopole and a transformed antimonopole. See Fig. 1e of Ref. 6.

In Refs. 6 and 7 this configuration was also loosely interpreted as "a monopole and antimonopole connected by a Z string that is twisted through an angle γ". We disagree with such an interpretation. Their reinterpreted configuration is, in fact,

Fig. 1d of Ref. 6. Starting with the field configuration of Fig. 1e, we can obtain the configuration of Fig. 1d by the following process. Divide the space into three regions, $x < -a$, $-a < x < a$ and $x > a$. In the first region, the two field configurations are the same. In the third region, Fig. 1d is obtained by rotating Fig. 1e by γ. (We are considering the general case where the twist is γ.) In the second region, the string is twisted. In other words, at each x, the configuration of Fig. 1d is locally the result of rotating Fig. 1e by an angle , ψ, a function of x such that $\psi(x = -a) = 0$ and $\psi(x = a) = \gamma$. It is easy to see that the contributions from regions 1 and 3 to the Chern-Simons numbers of Fig. 1d and 1e are the same. In the second region, the untwisted string of Fig. 1e is replaced by one with a twist by γ. To compute the change of Chern-Simons number, we must not forget that since B_A is non-zero in the presence of monopoles, there are contributions from the last two terms on the right hand side of Eq. (5).[25] These two terms can be interpreted as the cross linkage of B_A with B_Z. Ignoring these terms for a moment, from Eq. (6) and (10) (with $\Phi = 4\pi/\alpha$), the first term of Eq. (5) alone will give a difference of the Chern-Simons numbers $\frac{\alpha^2}{16\pi^2}\cos(2\theta_w)\frac{\gamma}{2\pi}\left(\frac{4\pi}{\alpha}\right)^2 = \frac{\gamma}{2\pi}\cos(2\theta_w)$. Let us now return to the cross linkage piece. Conservation of the hypercharge flux implies that[26] the B_A flux flowing across a plane $x = cst$ (for $-a < x < a$) is $\frac{4\pi}{\alpha}\tan\theta_w$. Hence, from Eqs. (5), (6) and (7), the contribution to the change of C-S number from cross linkage is $2\frac{\alpha^2}{16\pi^2}(\frac{1}{2})\sin(2\theta_w)\frac{\gamma}{2\pi}2(\frac{4\pi}{\alpha})(\frac{4\pi}{\alpha})\tan\theta_w = \frac{2\gamma}{\pi}\sin^2\theta_w$. Summing the two contributions gives $\frac{\gamma}{2\pi}(1 + 2\sin^2\theta_w)$. Note that when the monopole and antimonopole at the two ends of a twisted segment of Z string are brought together and annihilate each other, a closed loop is formed and we expect B_A to go away.

The authors of Refs. 6 and 7 attempted to compute the Chern-Simons number of Fig. 1e. Unfortunately, they wrongly believed that the Chern-Simons number of Fig. 1e is the same as that of Fig. 1d. So, they argued as follows. If $\gamma = 2\pi n/m$, where n and m are integers, one can join together m of these twisted segments and form a loop of Z string that is twisted by an angle $2\pi n$. They concluded that the Chern-Simon number of the configuration in Eq. (12) is $\frac{\gamma}{2\pi}\cos(2\theta_w)$ and for $\gamma = \pi/\cos(2\theta_w)$ it has a Chern-Simons number of 1/2. They conjectured that it is a "stretched" sphaleron.

This conjecture is not well motivated because as discussed in the last paragraph, the two configurations shown in Figs. 1d and 1e have different Chern-Simons numbers. What they have computed is actually the *contribution of the first term* of Eq. (5) to the *difference* of the Chern-Simons numbers of these two configurations rather than that of Fig. 1e.

One can obtain an extended sphaleron by setting $\gamma = \pi$ and considering parity odd field configurations. In fact, there is a general theorem by Axenides and Johansen[27] which states that any parity odd configuration which as $r \to \infty$ approaches the field configuration of the sphaleron (at $\theta_w = 0$) have (1/2) as its Chern-Simons number.

The connection between a segment of electroweak string and a sphaleron is considered. Besides, we have discussed a plausible final field configuration of electroweak strings when they intersect and intercommute and presented a careful calculation of self-helicities. Taken together, they suggest that baryon number is conserved when electroweak strings intercommute. However, other processes in the course of evolution of electroweak strings may still lead to baryon number violation. An estimate for helicity density fluctuations produced during the electroweak phase transition has been made in Ref. 6. Suppose that a network of Z string was produced at the electroweak phase transition and it survives long enough to fall out of thermal equilibrium. Since the full electroweak Lagrangian, which governs the evolution of the system, is CP violating, a change of helicity in one direction is favored over the other and hence net change in baryon number may occur.

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11. Baryon number violation may still occur in other processes in the course of evolution of electroweak strings.

12. Such a gauge choice is consistent with the Lorentz gauge condition $\partial^\mu W^a_\mu = 0$ and is valid for *dynamical* Z strings.

13. In Refs. 6 and 7, the factor 16 in the denominator was mistaken to be 32. We thank T. Vachaspati for pointing this out. Because of another factor of 2 error of self-helicity to be discussed below (See 17 below), their calculation of the Chern-Simons number of a twisted string remains valid.

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15. With $W^1_\mu = W^2_\mu = 0$, the $SU(2)$ gauge field is Abelianized and the Z flux does not terminate. Later in this paper we will introduce monopoles and antimonopoles which rotate Z flux to A flux. Configurations with monopoles and antimonopoles, however, do not satisfy $W^1_\mu = W^2_\mu = 0$.

16. See e.g. R. I. Ricca and H. K. Moffatt, in *Topological Aspects of the Dynamics of Fluids and Plasma*, edited by H. K. Moffatt *et al*, NATO ASI series E Vol. 218 (Kluwer Academic Publishers, 1992).

17. Refs. 6 and 7 erroneously gave $2p\Phi^2$ as the answer.

18. The magnetic Reynolds number R_m is defined to be $\mu_0 \sigma u_0 l_0$ where σ is the electric conductivity of the fluid, u_0 is a typical scale for the velocity field and l_0 is a typical length scale over which it varies.

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20. The mathematical framework of "framed bordisms" is introduced in Ref. 19 in the discussion of the reconnection process. Note that, owing to their erroneous calculation of self-helicities, the final field configuration suggested by Vachaspati and Field actually violates helicity and is different from ours.

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23. This involves a gauge choice and assumes that there is no A flux in the semi-infinite flux tube.

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Figure Captions:

FIG. 1. In the reconnection of two singly linked Christmas ribbons, local twists are formed and helicity is conserved.

FIG. 2. Two unlinked Christmas ribbons intercommute to form an untwisted ribbon.

This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9409319v2