Is baryon number violated when electroweak strings intercommute?

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(Received 31 October 1994)

We reexamine the self-helicity and the intercommutation of electroweak strings. A final field configuration when electroweak strings intersect and intercommute is proposed. It conserves helicity and hence baryon number. The connection between a segment of electroweak string and a sphaleron is also discussed.

PACS number(s): 11.27.+d, 11.30.Fs, 12.15.−y, 98.80.Cq

In 1967 Sakharov [1] considered the possibility that the Universe began in a baryon-symmetric state but that the dominance of matter over antimatter was produced by particle interactions. In addition to baryon-number violation, this requires C and CP violation as well as a departure from thermal equilibrium [2]. In the standard electroweak theory all three conditions are satisfied. Baryogenesis during the weak phase transition [3] is particularly interesting as it may eventually be experimentally verifiable. Recently, there has been a revival of interest in the study of classical solutions in standard electroweak interactions [4]. It has been conjectured [5–7] that a segment of electroweak string, which connects a monopole to an antimonopole, is a kind of "stretched" sphaleron. Since the sphaleron [8,9] is crucial to the study of baryon-number-violation processes, one may contemplate the role of electroweak strings in baryogenesis shortly after the cosmological electroweak phase transition [10].

It was further pointed out in a recent Letter [6] that the baryon number anomaly equation may be interpreted as a conservation law for baryon number minus helicity. Since the helicity is a sum of link and twist numbers, linked and twisted loops of electroweak string necessarily carry baryon number. It is also claimed that helicity (and baryon number) is conserved when electroweak strings intercommute. However, self-helicity is not properly accounted for in that Letter. With a correct calculation, the final field configuration used in Ref. [6] actually led to helicity violation when electroweak strings intercommute. Moreover, it was not totally convincing that the correct configuration after intercommutation is the one proposed in Ref. [6].

In this paper we give a careful calculation of self-helicity. An alternative electroweak string configuration after intercommutation is also proposed. A proper calculation of helicity with our alternative final field configuration suggests that helicity may still be conserved when electroweak strings intercommute. We also clarify the connection between the sphaleron and a segment of Z string connecting a monopole and an antimonopole.

We will briefly review the concept of helicity of electroweak strings [6,7]. Our starting point is the Adler-Bell-Jackiw anomaly equation

$$\partial_\mu J^\mu_B = \frac{N_F}{32\pi^2}[g^2 W^{\mu\nu}_a \tilde{W}^a_{\mu\nu} - g'^2 Y_{\mu\nu}\tilde{Y}^{\mu\nu}] \ , \qquad (1)$$

where N_F is the number of families, $W^{\mu\nu}_a$ $a=1,2,3$ and $Y_{\mu\nu}$ are the SU(2) and $\mathrm{U}_Y(1)$ field strengths respectively. We define the Chern-Simons numbers $N_{\rm CS}$ and $n_{\rm CS}$:

$$N_{\rm CS} \equiv \frac{g^2}{32\pi^2}\int d^3x \epsilon^{ijk}[W_{aij}W^a_k - \frac{g}{3}\epsilon_{abc}W^a_i W^b_j W^c_k]$$
$$n_{\rm CS} \equiv \frac{g'^2}{32\pi^2}\int d^3x \epsilon^{ijk} Y_{ij}Y_k \ , \qquad (2)$$

where W^a_k and Y_k are gauge potentials and obtain by integrating Eq. (1) that

$$\Delta B = N_F(\Delta N_{\rm CS} - \Delta n_{\rm CS}) \ . \qquad (3)$$

The Chern-Simons number is not a meaningful physical quantity as it changes by an integer upon a large gauge transformation. However, the *change* in the Chern-Simons number in any physical process is gauge invariant. We will be interested in Z string configurations in which $W^1_\mu = W^2_\mu = 0$ [11]. Consider the transformation

$$Z_j = \cos\theta_w W^3_j - \sin\theta_w Y_j \ ,$$
$$A_j = \sin\theta_w W^3_j + \cos\theta_w Y_j \ . \qquad (4)$$

Correcting a factor of 2 error in Refs. [6,7],[1] Eq. (2) gives

$$N_{\rm CS} - n_{\rm CS} = \frac{\alpha^2}{16\pi^2}\int d^3x[\cos(2\theta_w)B_Z\cdot Z + \tfrac{1}{2}\sin(2\theta_w)(B_Z\cdot A + B_A\cdot Z)] \ , \qquad (5)$$

where $\alpha = \sqrt{g^2+g'^2}$, $\tan\theta_w = g'/g$, B denotes the magnetic field, and the subscripts denote the gauge field for which the magnetic field is to be evaluated. As discussed by Vachaspati and Field, the first term on the right-hand side (RHS) has a simple interpretation in terms of the helicity [12,6,7] associated with the Z field

$$H_Z = \int d^3x\, B_Z\cdot Z \ . \qquad (6)$$

The helicity is a measure of the linkage of the magnetic field. If space is divided into a collection of flux tubes, magnetic helicity arises from the internal structure within each flux tube, such as twist and kinking, and external

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[1]We thank T. Vachaspati for pointing this out to us.

relations between flux tubes, i.e., linking and knotting [13]. For two (untwisted) closed flux tubes linked once, a simple integration of Eq. (6) gives

$$H = \pm 2\Phi_1\Phi_2 \ , \tag{7}$$

where Φ_1 and Φ_2 measure the magnetic fluxes of the tubes and the sign of H depends on the sense of linkage. This ends our review of the discussion of helicity made by Vachaspati and Field.

Consider a Z string loop of flux Φ that is twisted by an angle $2p\pi$. To compute its (internal) helicity [14], let us divide the tube up into m "subtubes" each with the flux Φ/m. The linking number of each pair of subtubes is the same as that of each pair of magnetic field lines in the loop. We observe that, for a pair of field lines in a uniformly twisted torus, one of the field lines can always be deformed to the axis of the tube without intersecting the second field line. Thus, the linking number is just p. The total helicity is the sum of these self-helicities plus the interactive helicities arising from the linkage of the flux tubes. Therefore,

$$H = mH_m + 2\sum_{i<j} p\Phi_i\Phi_j \ , \tag{8}$$

with $\Phi_i = \Phi/m$ $(i = 1, 2, \ldots, m)$. Since self-helicities scale as Φ^2,

$$H = \frac{H}{m} + 2p\frac{m(m-1)}{2}\left(\frac{\Phi}{m}\right)^2 \tag{9}$$

i.e. [15],

$$H = p\Phi^2 \ . \tag{10}$$

Does intercommutation of electroweak strings violate helicity (and hence baryon number)? It is well known to fluid dynamicists that helicity is conserved in perfect magnetohydrodynamics, but a network of electroweak string is far from a superconducting fluid. Naive reconnections of flux tubes after intercommutation is known to violate helicity. Conservation of helicity would require (additional local) twisting (and/or writhing) of flux tubes after reconnections [16]. Such a scheme for intercommutation does exist: Consider a pair of antiparallel electroweak strings approaching each other. The magnetic field goes through zero. It breaks and reconnects immediately. Such a matching of field lines leads to helicity conservation during intercommutation [17]. Pfister and Gekelman have proposed the following visual demonstration of helicity conservation with a simple Christmas ribbon [18]. Let us begin with two singly linked loops of untwisted ribbons as shown in Fig. 1(a). We arrange both ribbons such that they touch at one point and the Z fields are antiparallel there (as required for reconnection). We staple the ribbons together to the left and right of the contact point and cut the ribbons in between the staples [Fig. 1(b)]. What we have obtained is one loop that has two complete (360°) twists as shown in Fig. 1(c). From Eqs. (7) and (10), we find that both the initial and final configurations have a helicity of $2\Phi^2$. Therefore, helicity is conserved. For comparison, we also show the intercommutation of two unlinked string loops in Fig. 2. They intercommute to form an untwisted loop of string. Even though we have only discussed the case of two antiparallel electroweak strings, we conjecture that helicity conservation is valid for electroweak strings intersecting at any angle. We remark that schemes that violate helicity during intercommutation can also be constructed. For instance, the scheme proposed in Ref. [6] falls into this category. The question of whether helicity is conserved when electroweak strings intercommute is highly controversial. Only a rigorous analysis of the field dynamics beyond the scope of this paper will settle this issue. Nevertheless, it is interesting to see the construction of Pfister and Gekelman by which helicity can be conserved.

A segment of Z string can also exist, but it has to connect a monopole (m) to an antimonopole ($\bar{m}$). The asymptotic field configurations of m and $\bar{m}$ (each connected to a semi-infinite Z string) have been written by Nambu [19]:

$$\Phi_m = \begin{pmatrix} \cos(\theta_m/2) \\ \sin(\theta_m/2)e^{i\phi} \end{pmatrix}, \quad \Phi_{\bar{m}} = \begin{pmatrix} \sin(\theta_{\bar{m}}/2) \\ \cos(\theta_{\bar{m}}/2)e^{i\phi} \end{pmatrix} , \tag{11}$$

where θ_m and ϕ are spherical coordinates centered on m (and similarly for $\bar{m}$) and the gauge field satisfies $D_i\Phi = 0$ [20]. It has been suggested [5] that a segment of Z string is a kind of "extended" sphaleron. Here we are interested in a "family" of extended sphaleron by which we mean a set of field configurations each with a Chern-Simons number of 1/2 parametrized by a deformation variable d such that the $d = 0$ element is a sphaleron. However, a simple symmetry argument [21] shows that an untwisted segment of Z string has a Chern-Simons number zero rather than 1/2. More recently, Vachaspati and Field [6] have considered the Higgs field configuration

$$\Phi_{m\bar{m}}(\gamma) = \begin{pmatrix} \sin(\theta_m/2)\sin(\theta_{\bar{m}}/2)e^{i\gamma} + \cos(\theta_m/2)\cos(\theta_{\bar{m}}/2) \\ \sin(\theta_m/2)\cos(\theta_{\bar{m}}/2)e^{i\phi} - \cos(\theta_m/2)\sin(\theta_{\bar{m}}/2)e^{i(\phi-\gamma)} \end{pmatrix} , \tag{12}$$

where θ_m and $\theta_{\bar{m}}$ are the polar angles and ϕ is the aximuth angle. Equation (12) reduces to Φ_m when $\theta_{\bar{m}} \to 0$ and to $e^{i\gamma}\Phi_{\bar{m}}$ when $\theta_m \to \pi$ and, in addition, we perform the rotation $\phi \to \phi + \gamma$. Thus, we see that the antimonopole is rotated and globally transformed by U(1). When $\theta_m \to \pi$ and $\theta_{\bar{m}} \to 0$, Eq. (12) reduces to the configuration of an *untwisted* Z string. Therefore, Eq. (12) describes an untwisted segment of Z string connected to a monopole and a transformed antimonopole. See Fig. 1(e) of Ref. [6].

In Refs. [6,7] this configuration was also loosely interpreted as "a monopole and antimonopole connected by a Z string that is twisted through an angle γ." We disagree with such an interpretation. Their reinterpreted configuration is, in fact, Fig. 1(d) of Ref. [6]. Starting with the field configuration of Fig. 1(e), we can obtain the configuration of Fig. 1(d) by the following process. Divide the space into three regions, $x < -a$, $-a < x < a$, and

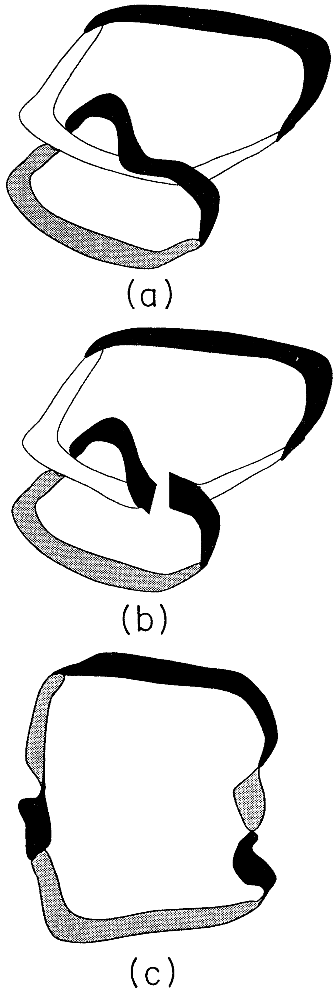

FIG. 1. In the reconnection of two singly linked Christmas ribbons, local twists are formed and helicity is conserved.

$x > a$. In the first region, the two field configurations are the same. In the third region, Fig. 1(d) is obtained by rotating Fig. 1(e) by γ. (We are considering the general case where the twist is γ.) In the second region, the string is twisted. In other words, at each x, the configuration of Fig. 1(d) is locally the result of rotating Fig. 1(e) by an angle ψ, a function of x such that $\psi(x=-a)=0$ and $\psi(x=a)=\gamma$. It is easy to see that the contributions from region 1, and similarly for region 3, to the Chern-Simons numbers of Fig. 1(d) and 1(e) are the same. In the second region, the untwisted string of Fig. 1(e) is replaced by one with a twist by γ. To compute the change of Chern-Simons number, we must not forget that, since B_A is nonzero in the presence of monopoles, there are contributions from the last two terms on the right-hand side of Eq. (5). These two terms can be interpreted as the cross linkage of B_A with B_Z. Ignoring these terms for a moment, from Eq. (6) and (10) (with $\Phi = 4\pi/\alpha$), the first term of Eq. (5) alone will give a difference of the Chern-Simons numbers

$$\frac{\alpha^2}{16\pi^2}\cos(2\theta_w)\frac{\gamma}{2\pi}\left(\frac{4\pi}{\alpha}\right)^2 = \frac{\gamma}{2\pi}\cos(2\theta_w) .$$

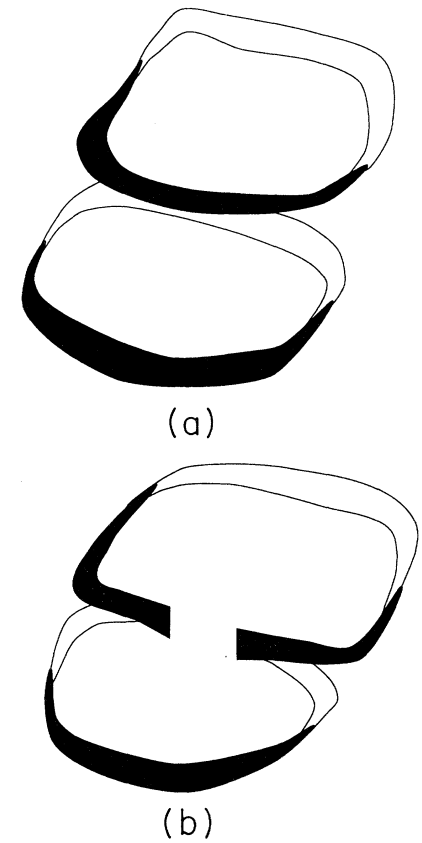

FIG. 2. Two unlinked Christmas ribbons intercommute to form an untwisted ribbon.

Let us now return to the cross linkage piece. Conservation of the hypercharge flux implies that [22] the B_A flux flowing across a plane $x = cst$ (for $-a < x < a$) is $(4\pi/\alpha)\tan\theta_w$. Hence, from Eqs. (5), (6), and (7), the contribution to the change of CS number from cross linkage is

$$2\frac{\alpha^2}{16\pi^2}\left(\frac{1}{2}\right)\sin(2\theta_w)\frac{\gamma}{2\pi}2\left(\frac{4\pi}{\alpha}\right)\left(\frac{4\pi}{\alpha}\right)\tan\theta_w$$
$$= \frac{2\gamma}{\pi}\sin^2\theta_w .$$

Summing the two contributions gives $(\gamma/2\pi)(1 + 2\sin^2\theta_w)$. Note that when the monopole and antimonopole at the two ends of a twisted segment of Z string are brought together and annihilate each other, a closed loop is formed and we expect B_A to go away.

The authors of Refs. [6] and [7] attempted to compute the Chern-Simons number of Fig. 1(e). Their calculation was incorrect because of two reasons. First, they wrongly believed that the Chern-Simons number of Fig. 1(e) is the same as that of Fig. 1(d). Second, even for Fig. 1(d), their argument of joining segments of electroweak strings into a loop and computing the Chern-Simons number of that loop was too naive to be trusted. More concretely, they argued as follows. If $\gamma = 2\pi n/m$, where n and m are integers, one can join together m of these twisted segments and form a loop of Z string that is twisted by an angle $2\pi n$. They concluded that the Chern-Simon number of the configuration in Eq. (12) is $(\gamma/2\pi)\cos(2\theta_w)$ and for $\gamma = \pi/\cos(2\theta_w)$ it has a Chern-Simons number of 1/2. They conjectured that it is a "stretched" sphaleron. What they have computed is actually the *contribution of the first term* of Eq. (5) to the *difference* of the Chern-Simons numbers of these two configurations rather than that of Fig. 1(e). It is not the Chern-Simons number of neither Fig. 1(e) nor Fig. 1(d). Therefore, their conjecture is not well motivated.

One can obtain an extended sphaleron by setting $\gamma = \pi$ and considering parity odd field configurations. In fact, there is a general theorem by Axenides and Johansen [23] which states that any parity odd configuration which as $r \to \infty$ approaches the field configuration of the sphaleron (at $\theta_w = 0$) has 1/2 as its Chern-Simons number.

We consider the connection between a segment of electroweak string and a sphaleron. A careful calculation of self-helicity is presented. We also suggest a final field configuration of electroweak strings when they intersect and intercommute. Our final field configuration conserves helicity and hence baryon number. Our result should be taken with a grain of salt. The field configuration after intercommutation remains a controversial subject. Various papers disagree [24]. Nonetheless, it is interesting to see a scheme in which intersecting strings can maintain their helicity. Finally, we remark that even if baryon number is conserved when electroweak strings intercommute, it may still be violated in other processes in the course of evolution of electroweak strings and baryogenesis due to electroweak strings remains an interesting subject.

We acknowledge numerous helpful discussions with U. Aglietti, M. Axenides, M. Bucher, A. Y. K. Chui, M. James, C. L. Y. Lee, and especially J. Preskill and T. Vachaspati. This work was supported by the U.S. Department of Energy under Grant No. DE-FG03-92-ER40701.

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