

# Intrinsic and extrinsic galaxy alignment

Paolo Catelan, Marc Kamionkowski<sup>★</sup> and Roger D. Blandford

*California Institute of Technology, Mail Code 130-33, Pasadena, CA 91125, USA*

Accepted 2000 October 11. Received 2000 October 2; in original form 2000 May 31

## ABSTRACT

We show with analytic models that the assumption of uncorrelated intrinsic ellipticities of target sources that is usually made in searches for weak gravitational lensing arising from large-scale mass inhomogeneities (‘field lensing’) is unwarranted. If the orientation of the galaxy image is determined either by the angular momentum or by the shape of the halo in which it forms, then the image should be aligned preferentially with the component of the tidal gravitational field perpendicular to the line of sight. Long-range correlations in the tidal field will thus lead to long-range ellipticity–ellipticity correlations that mimic the shear correlations arising from weak gravitational lensing. We calculate the ellipticity–ellipticity correlation expected if halo shapes determine the observed galaxy shape, and we discuss uncertainties (which are still considerable) in the predicted amplitude of this correlation. The ellipticity–ellipticity correlation induced by angular momenta should be smaller. We consider several methods for discriminating between the weak-lensing (extrinsic) and intrinsic correlations, including the use of redshift information. An ellipticity–tidal-field correlation also implies the existence of an alignment of images of galaxies near clusters. Although the intrinsic alignment may complicate the interpretation of field-lensing results, it is inherently interesting as it may shed light on galaxy formation as well as on structure formation.

**Key words:** gravitational lensing – cosmology: theory – large-scale structure of Universe.

## 1 INTRODUCTION

Searches for weak gravitational lensing arising from large-scale mass inhomogeneities are coming of age. Ellipticities of high-redshift sources are taken to be indicators of the shear field induced by weak gravitational lensing by mass inhomogeneities along the line of sight, and shear–shear correlations can be used as a probe of the lensing-mass distribution (Gunn 1967; Miralda-Escudé 1991; Blandford et al. 1991; Kaiser 1992; Bartelman & Schneider 1992, 1999). The advantage of weak lensing is that it determines the power spectrum (as well as higher order statistics: e.g. Bernardeau, van Waerbeke & Mellier 1997; Munshi & Jain 2000; Cooray & Hu 2000) for the *mass* rather than the light. In just the past few months, four groups have reported the first detections of such ‘field lensing’ (Bacon, Refregier & Ellis 2000; Kaiser, Wilson & Luppino 2000; Wittman et al. 2000; van Waerbeke et al. 2000).

Noise for the weak-lensing signal is provided by the intrinsic ellipticities of the sources. With a sufficiently large sample of sources, the random orientation of these sources can be overcome. One thus looks for an ellipticity correlation in excess of the Poisson noise provided by randomly oriented intrinsic ellipticities. The analysis always assumes that the intrinsic orientations of the sources are entirely random and isotropically distributed. The

point of this paper will be to demonstrate that this should not be the case.

To do so, we consider two *ansätze* for the origin of the ellipticity of the high-redshift sources. We first suppose that the ellipticity of the galaxy image may be determined primarily by the shape of the halo in which it forms; this might be expected if the sources are isolated ellipticals. In this case, a modification of the spherical top hat model for gravitational collapse in a tidal field suggests a preferential elongation of the galaxies along the direction of the tidal field. We show that, in this case, long-range correlations in the ellipticities of widely separated sources are proportional to long-range correlations in the tidal field, and thus to correlations in the mass distribution.

We then consider what happens if the orientation of the image is determined by the angular momentum of the halo in which it forms; this should be a good description if the sources are disc galaxies. The simplest hypothesis – adopted in nearly all disc formation models (e.g. Dalcanton, Spergel & Summers 1997; Mo, Mao & White 1998; Buchalter, Jimenez & Kamionkowski 2001) – is that the plane of the disc is perpendicular to the angular momentum vector of the galactic halo in which the disc forms. According to linear perturbation theory, a galactic halo acquires its angular momentum via torquing of the aspherical protogalaxy in the tidal gravitational field that arises from the large-scale mass distribution (Hoyle 1949; Peebles 1969; Doroshkevich 1970; White 1984; Heavens & Peacock 1988; Catalan & Theuns 1996).

<sup>★</sup>E-mail: kamion@tapir.caltech.edu

Averaging over all possible orientations of the protogalaxy, the disc orientations are correlated with the tidal field. In this case, long-range correlations in the ellipticities are expected to be smaller, as they will be proportional at lowest order to the square of the correlations in the tidal-field and/or mass distribution.

In the next section, we review briefly the statistics used to describe the weak-lensing signal. In Section 3, we explain how ellipticals should be preferentially elongated along the direction of the tidal field, and we present the calculation of the shear power spectrum for this case. Section 4 presents numerical results. In Section 5, we show how tidal torquing can align galaxies preferentially along the tidal gravitational field, and we explain why this should lead to smaller ellipticity correlations that are of higher order in the mass correlation. In Section 6 we put forth some ideas for disentangling the intrinsic and weak-lensing signals, including the use of redshift information, and we predict a corresponding alignment in the images of galaxies near clusters. We close with some concluding remarks in Section 7.

During preparation of this paper, we learned of related work (using numerical simulations) by Heavens, Refregier & Heymans (2000) and Croft & Metzler (2000). Our analytic approach should complement their numerical work and perhaps help to shed some light on the origin of their observed correlations. The analytic calculation should also be useful in determining the correlation at large angular separations, where it becomes increasingly difficult to measure in simulations. Our analytic approach also suggests some possible intrinsic/weak-lensing discriminators.

## 2 FORMALISM AND STATISTICS

As discussed e.g. in Kamionkowski et al. (1998), weak lensing induces a stretching of images on the sky at position  $\theta = (\theta_y, \theta_z)$  described by  $\epsilon_+$ , the stretching in the  $\hat{\theta}_y - \hat{\theta}_z$  directions, and  $\epsilon_\times$ , the stretching along axes rotated by  $45^\circ$  (we take the line of sight to be the  $\hat{x}$ -direction). Possible definitions for these ‘ellipticities’ are discussed, e.g. by Blandford et al. (1991), and strategies for averaging over the redshift distribution of the sources are described by Kaiser (1992). The weak-lensing shear,  $\gamma(\theta)$ , can be recovered through its Fourier transform (Stebbins 1996),

$$\tilde{\gamma}(\ell) = \frac{(\ell_y^2 - \ell_z^2)\tilde{\epsilon}_+(\ell) + 2\ell_y\ell_z\tilde{\epsilon}_\times(\ell)}{\ell_y^2 + \ell_z^2}, \quad (1)$$

where  $\tilde{\epsilon}_+(\ell) = \int d^2\theta \epsilon_+(\theta) e^{i\ell\cdot\theta}$  is the Fourier transform of the ellipticity (and similarly for the other quantities). Statistical homogeneity and isotropy guarantee that the Fourier coefficients,  $\tilde{\gamma}(\ell)$ , have zero mean and variances,

$$\langle \tilde{\gamma}^*(\ell)\tilde{\gamma}(\ell') \rangle = (2\pi)^2 \delta(\ell - \ell') C(\ell), \quad (2)$$

where  $C(\ell)$  is the weak-lensing power spectrum. The mean-square ellipticity (which is usually taken to be the mean-square shear) smoothed over some circular window of radius  $\theta_p$  is

$$\langle (\gamma_g^s)^2 \rangle = \int \frac{d^2\ell}{(2\pi)^2} C(\ell) |\tilde{W}(\ell)|^2, \quad (3)$$

where  $\tilde{W}(\ell) = 2J_1(x)/x$  is the Fourier-space window function,  $J_1(x)$  is a Bessel function, and  $x \equiv \ell\theta_p/\sqrt{\pi}$  (e.g. Mould et al. 1994).

Two non-zero two-point ellipticity–ellipticity correlation functions can be constructed from the two ellipticities,  $\epsilon_+$  and  $\epsilon_\times$ , determined by taking the coordinate axes to be aligned with the line connecting the two correlated points.

These correlation functions are  $C_1(\theta) = \langle \epsilon_+(\theta_0)\epsilon_+(\theta_0 + \theta) \rangle$  and  $C_2(\theta) = \langle \epsilon_\times(\theta_0)\epsilon_\times(\theta_0 + \theta) \rangle$ , and they are given in terms of the power spectra by

$$\begin{aligned} C_1(\theta) + C_2(\theta) &= \int_0^\infty \frac{\ell d\ell}{2\pi} C(\ell) J_0(\ell\theta), \\ C_1(\theta) - C_2(\theta) &= \int_0^\infty \frac{\ell d\ell}{2\pi} C(\ell) J_4(\ell\theta), \end{aligned} \quad (4)$$

where  $J_\nu(x)$  are Bessel functions. These are the correlation functions that have been found to be non-zero. The power spectrum can be recovered from these correlation functions through an inverse transform (see e.g. Kamionkowski et al. 1998).

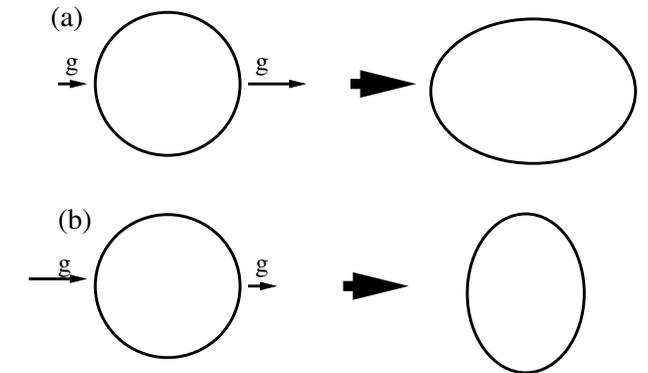
## 3 HALO SHAPE DISTORTIONS

The observed shape of a galaxy may be determined, at least in part, by the shape of its halo. Suppose that a spherical overdensity undergoes gravitational collapse to form a galaxy in a region of constant tidal gravitational field. Then the acceleration on one side of the galaxy will differ from that at the other side, and the resulting gravitational collapse, which would have otherwise been spherically symmetric, will be anisotropic. As illustrated in Fig. 1, the resulting ellipticity could be either positive or negative depending on whether the tidal field compresses the sphere (resulting in an oblate halo) or stretches the sphere (resulting in a prolate halo). Although overdensities are not all expected to be spherical, there should still be some net elongation of haloes that form in a given tidal field after averaging over all shapes. Moreover, the shape of the luminous galaxy should be determined, at least in part, by the shape of the halo in which it forms. For example, the stellar distribution of luminous ellipticals that form in isolation should trace the shape of the halo. Alternatively, a gaseous pancake could form in a plane perpendicular to the tidal field if the tidal-field axis undergoes gravitational collapse first.

The symmetry dictates that the mean ellipticity of a population of galaxies that form in a given tidal field will be related to the potential via

$$\begin{aligned} \epsilon_+ &= C(\partial_y^2 - \partial_z^2)\phi, \\ \epsilon_\times &= 2C\partial_y\partial_z\phi, \end{aligned} \quad (5)$$

where  $C$  is a constant to be discussed further below. To see that equation (5) is the lowest order term that encodes the expected



**Figure 1.** The two panels show how a tidal field can either elongate or compress a galactic halo. The arrows labelled ‘g’ indicate the size and direction of the gravitational field. In (a) the tidal field stretches the halo, and in (b) the tidal field compresses the halo.

qualitative behaviour, consider the motion of a sphere of test masses in a slowly varying (spatially) potential  $\phi(x)$ . If we Taylor expand  $\phi(x)$  about the origin, the zeroth order term has no physical effect. The linear term gives rise to a uniform translation of the sphere (constant gravitational field  $\mathbf{g} = \nabla\phi$ ), but does not affect its shape. The quadratic term (the tidal field) will change the shape of the sphere, as it gives rise to different accelerations at different points of the sphere, as shown in Fig. 1. To lowest order in  $\phi$ , the ellipticity induced in the sphere is given by equation (5).

We now proceed to show that long-range correlations in the tidal field should lead to ellipticity–ellipticity correlations that mimic the weak-lensing shear. The shear measured in direction  $\theta$  will be the integrated ellipticity along the line of sight,

$$\begin{aligned} \epsilon_+(\theta) &= \int_0^\infty dx g(x) \epsilon_+(x, x\theta_y, x\theta_z) \\ &= C \int_0^\infty dx g(x) (\partial_y^2 - \partial_z^2) \phi(x, x\theta_y, x\theta_z), \end{aligned} \quad (6)$$

where  $g(x)$  is the distribution of sources along the line of sight normalized to unity, and  $x$  is the comoving distance. We have assumed the Universe to be flat, as the cosmic microwave background seems to indicate (Kamionkowski, Spergel & Sugiyama 1994; Miller et al. 1999; de Bernardis et al. 2000; Hanany et al. 2000). Using the generalized Limber equation (Kaiser 1992), as well as the definition of  $\gamma$  in equation (1), we find that intrinsic alignments yield a shear power spectrum,

$$C(\ell) = C^2 \left(\frac{3}{2} \Omega_0 H_0^2\right)^2 \int dx \frac{g^2(x)}{x^2} P(\ell/x). \quad (7)$$

If the ellipticity–ellipticity correlation evolves with time, then the time dependence of the mass power spectrum should be taken into account in the integrand. However, we are supposing that the ellipticity correlations are fixed by some primordial density field, so the time evolution does not matter. The overall normalization of  $P(k)$  also does not matter, as the normalization of the shear power spectrum will be fixed below to match the observed source ellipticities.

In principle, a complete galactosynthesis model would allow us to tell how the ellipticity of the luminous galaxy is related to the shape of the dark halo in which it forms, but in practice we are far from being able to do this. We thus choose to estimate empirically the constant of proportionality  $C$  between the ellipticity and the tidal field.

To do so, we make the *ansatz* that the halo shape is the only factor that determines the ellipticity of the observed luminous galaxy. We can then use our model to calculate the expected rms ellipticity of individual galaxies and compare this prediction with the typical source ellipticity to fix  $C$ . We use  $\bar{\epsilon} \approx 0.15$  (Metzger, private communication), although it might be larger for some high-redshift populations. Doing so, we find

$$\begin{aligned} \langle \epsilon^2 \rangle &= \langle \epsilon_+^2 + \epsilon_-^2 \rangle \\ &= \frac{8C^2}{15} \langle (\nabla^2 \phi)^2 \rangle \\ &= \frac{8C^2}{15} \int \frac{d^3\mathbf{k}}{(2\pi)^3} k^4 P_\phi(k), \end{aligned} \quad (8)$$

where  $P_\phi(k)$  is the power spectrum for the gravitational potential which is related to the mass power spectrum  $P(k)$ , through the

Poisson equation. We thus obtain

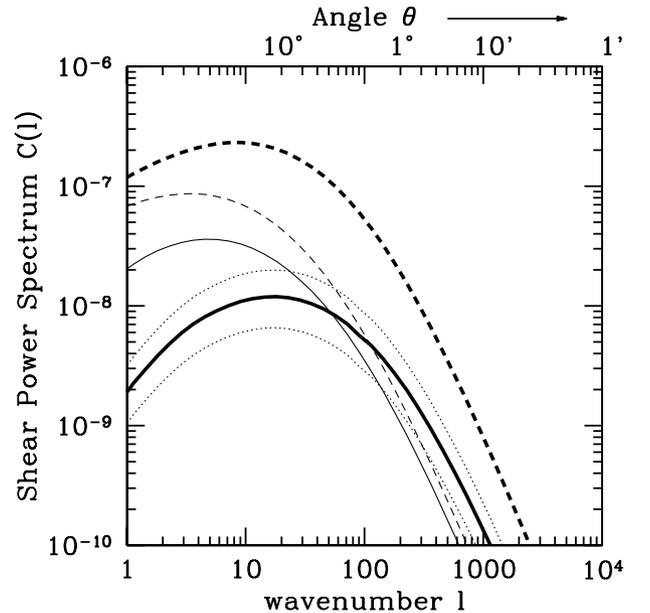
$$\langle \epsilon^2 \rangle = \frac{8}{15} C^2 \left(\frac{3}{2} \Omega_0 H_0^2\right)^2 \frac{1}{2\pi^2} \int k^2 dk P(k) \left[ \frac{3j_1(kR)}{kR} \right]^2. \quad (9)$$

This fixes the constant of proportionality  $C$  when we adopt  $\langle \epsilon^2 \rangle = \bar{\epsilon}^2 = (0.15)^2$ . We have inserted a top-hat window function and choose to smooth over a radius  $R = 1 h^{-1}$  Mpc, a characteristic scale over which a galaxy forms (tidal fields on smaller scales should not contribute to the torque). Below we will discuss how uncertainty in the smoothing scale will affect our final results.

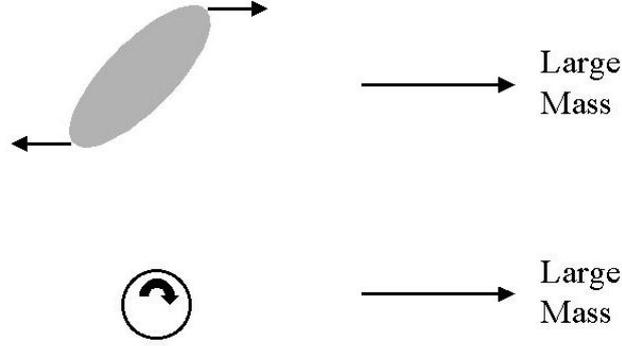
Changing the variable of integration in equation (7) from  $x$  to  $k$  and inserting our expression for  $C$ , we get the intrinsic shear power spectrum,

$$C(\ell) = \frac{15\pi^2 \bar{\epsilon}^2}{4} \frac{1}{\bar{\ell}} \frac{\int dk g^2(\ell/k) P(k)}{\int k^2 dk P(k) \left[ \frac{3j_1(kR)}{kR} \right]^2}. \quad (10)$$

Thus, if the observed shape of a galaxy is determined, even partially, by the tidal field in which it forms, then long-range correlations in the tidal field will lead to long-range correlations in the ellipticity, or equivalently in the induced shear. Equation (10) bears some resemblance to the power spectrum for weak gravitational lensing. The difference is that the  $g(x)$  here is replaced by the distribution of lenses along the line of sight, and the normalization differs. All this should come as no surprise, since the shear produced by weak lensing also depends on the perpendicular components of the tidal field. Thus, barring the small difference expected from the different line-of-sight distribution, the angular dependence of the intrinsic correlation function should look quite a bit like that for weak lensing.



**Figure 2.** The angular shear power spectra for weak lensing (dashed curves) and for intrinsic alignments (solid curves) for a high-redshift source population with median redshift  $z_m \sim 1$  (heavier curves) and a lower redshift source population with  $z_m \sim 0.3$  (lighter curves). The upper and lower dotted curves show the intrinsic power spectrum that would be obtained (for the high-redshift population) using a smoothing radius of  $R = 2 h^{-1}$  Mpc or  $0.5 h^{-1}$  Mpc, respectively, instead of the nominal value of  $R = 1 h^{-1}$  Mpc. The mean source ellipticity is assumed here to be  $\bar{\epsilon} = 0.15$ , and the amplitude of the intrinsic power spectrum scales with  $\bar{\epsilon}^2$ .



**Figure 3.** The upper panel shows a distant mass acting upon a prolate halo and causing it to start spinning as shown. The torque vanishes when  $\theta = 0, \pi/2$ , cf. equation (12). The lower panel shows that baryons in the potential well will fall in to form a disc lying in the plane of the figure. When this disc is viewed from the direction perpendicular to the figure, there will be no induced ellipticity; when viewed from the plane of the figure, the ellipticity will be  $\sim 1$ . If the mass is replaced by a void, the sense of rotation is reversed, but the shape remains unchanged. Averaging over all viewing directions gives a non-zero mean elongation of the galaxy image along the projected direction of the distant mass, and this will be correlated among neighbouring galaxies. More generally, tidal gravitational fields will be induced by the large-scale distribution of mass (rather than by a single large mass), and long-range correlations in these tidal fields will induce long-range correlations in the ellipticities of widely separated galaxies.

#### 4 NUMERICAL RESULTS

Fig. 2 shows results for the shear power spectrum induced by intrinsic alignments as well as the power spectrum expected from weak lensing. Results are shown for a population of sources with a mean redshift  $z_m \sim 1$  and a population with a mean redshift  $z_m \sim 0.3$ . The weak-lensing power spectrum is taken from Kamionkowski et al. (1998), and is for a cluster abundance normalized cold dark matter model.<sup>1</sup> The light dotted curves indicate how the intrinsic power spectrum (for the high-redshift population) would change if the smoothing scale for the normalization to the observed ellipticities were taken to be  $R = 0.5 h^{-1}$  Mpc (lower dotted curve) or  $R = 2 h^{-1}$  Mpc (upper dotted curve), instead of  $R = 1 h^{-1}$  Mpc.

The calculation indicates that, for high-redshift sources, the intrinsic correlation is unlikely to dominate that from weak lensing. However, given the uncertainty in the normalization, and the approximation inherent in our calculation, we cannot rule out the possibility that the intrinsic correlation might be larger than our calculation has indicated. There is also some chance that the correlation could be smaller than we have predicted. We have normalized our prediction by assuming that the observed ellipticities are due entirely to the halo shape. However, realistically, some fraction of the intrinsic ellipticity will be due to other effects (e.g. galactic spins: see below), and the amplitude of our predicted signal will be accordingly lowered. However, even if the intrinsic signal is small, it will not be zero, and will thus need to be considered for cosmological parameter estimation (e.g. Hu & Tegmark 1999) from future precise weak-lensing maps, as well as for studies of higher order weak-lensing statistics. The curves in Fig. 2 for the lower redshift population suggest that intrinsic alignment will be more important for weak-lensing searches with shallower surveys, such as the Sloan Digital Sky Survey and/or the Two-Degree Field.

<sup>1</sup> The amplitude of the weak-lensing power spectrum *does* depend on the amplitude of  $P(k)$ .

#### 5 TIDAL TORQUES AND IMAGE ORIENTATIONS

Another possibility is that the shapes of galaxies are determined by the angular momenta of the haloes in which they form. This is heuristically expected to account for the shapes of disc galaxies (while the shapes of ellipticals might be expected to be determined more by the shape of the halo, as discussed above). In this section, we will argue that the tidal-torque theory for the origin of galactic angular momenta also suggests that galaxy images should tend to be aligned with the component of the tidal gravitational field perpendicular to the line of sight.

When the mass that will eventually collapse to form a galaxy breaks away from the expansion, it will in general have an anisotropic moment-of-inertia tensor  $\mathcal{I}_{\alpha\beta}$  and reside in a tidal gravitational field  $\mathcal{D}_{\alpha\beta} = \partial_\alpha \partial_\beta \phi$  generated by the larger scale distribution of mass, where  $\phi$  is the gravitational potential. The moment-of-inertia eigenframe will most generally not be aligned with the that of the tidal field. A net torque will thus be applied to the protogalaxy, and the galactic halo that results will have an angular momentum  $L_\alpha \propto \varepsilon_{\alpha\beta\gamma} \mathcal{I}_{\beta\sigma} \mathcal{D}_{\gamma\sigma}$ , where  $\varepsilon_{\alpha\beta\gamma}$  is the Levi-Civita symbol (see Fig. 3). The relation between the ellipticity and angular momentum can ultimately be determined empirically by fitting to the observed ellipticities (see Section 6 below), so the absolute magnitude of  $\mathbf{L}$  is irrelevant. Thus, when we refer to the ‘angular momentum’  $\mathbf{L}$  in the following, we will really be discussing a vectorial spin parameter, an angular momentum that has been scaled by the galaxy mass and binding energy to be dimensionless.

Let us assume that, if a disc forms in a galactic halo, it will form in the plane perpendicular to the angular momentum vector. If viewed at an inclination angle  $\alpha$ , the ellipticity will be reduced by  $\cos \alpha$  from the value that it would have if viewed edge-on. Taking the line of sight to be the  $\hat{x}$ -axis, the observed ellipticity will be

$$\begin{aligned} \epsilon_+ &= f(L, L_x)(L_y^2 - L_z^2), \\ \epsilon_\times &= 2f(L, L_x)L_y L_z, \end{aligned} \quad (11)$$

where  $L = |\mathbf{L}|$ .

Let us now suppose that a number of galaxies form in the same tidal field. Since the angular momentum (and corresponding observed ellipticity) of a given galaxy will depend on the moment of inertia of its parent protogalaxy, the tidal field alone does not determine the orientation of a given disc. However, if the moment-of-inertia eigenframes are distributed isotropically, a net alignment of the galaxies with the tidal field remains after averaging over all initial moments of inertia.

To demonstrate this, we will consider a somewhat simplified model that should still capture the essential physics and leave out details. Rather than average over the entire distribution of moments of inertia – which can be obtained e.g. from Gaussian peak statistics (Bardeen et al. 1986; Heavens & Peacock 1988; Catelan & Theuns 1996) – we will suppose that each galaxy has the same eigenframe moment of inertia and that the eigenframes are distributed isotropically. Moreover, we will suppose that two of the three principal moments are equal, and the third is slightly larger. The unequal moment defines a symmetry axis  $\hat{n} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$ , and the contributing part of the moment-of-inertia tensor is  $\mathcal{I}_{\alpha\beta} \propto n_\alpha n_\beta$ , where  $\hat{n} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$ .

To simplify our illustration, we will take  $f(L) = C$ , where  $C$  is a constant (so the ellipticity is quadratic in the spin). Consider now a region of fixed tidal field, which we will take to be  $D_{\alpha\beta} = Ak_\alpha k_\beta$ , where  $A$  is a constant. The induced angular momentum is thus  $L = A(\hat{n} \cdot \mathbf{k})(\hat{n} \times \mathbf{k})$  (up to a constant of proportionality to be dealt with below).

If  $\mathbf{k}$  is perpendicular to the line of sight, say in the  $\hat{z}$ -direction, then the observed ellipticity of a galaxy that forms from a protogalaxy with symmetry axis in the  $\hat{n}$ -direction is

$$\epsilon_+ = CA^2 k^4 \sin^2 \theta \cos^2 \theta \cos^2 \varphi, \quad \epsilon_x = 0. \quad (12)$$

So, for example, if the symmetry axis is aligned with ( $\theta = 0$ ) or perpendicular to ( $\theta = \pi/2$ ) the tidal field, there will be no torque. If the symmetry axis is in the  $x$ - $z$  plane, then it will be torqued about the  $\hat{y}$ -direction and the resulting image will be elongated in the  $\hat{z}$ -direction. If the symmetry axis is in the  $y$ - $z$  plane, the galaxy will spin about the line of sight and produce a face-on (zero-ellipticity) disc. If the tidal field is reversed in direction (i.e.  $A \rightarrow -A$ ), then the same ellipticity arises.

We now assume that the orientations of the protogalaxy eigenframe are random. In principle, the location and shapes of peaks from which the protogalaxy forms depend on all Fourier modes in the initial density field, including those that determine the long-range tidal field. Although the tidal field may play some role in determining the halo shape, as discussed above and in Lee & Pen (2000a), it cannot exclusively determine the halo shape. We thus make the usual peak-background split and assume that the small-scale fluctuations that play the dominant role in determining the shape of the protogalaxy are statistically independent of larger scale fluctuations.

Integrating equation (12) over all angles, we find that the inferred shear, the mean ellipticity of galaxies formed in this tidal gravitational field, is  $\epsilon_+ = CA^2 k^4 / 15$ ,  $\epsilon_x = 0$ . Thus, although there is no one-to-one correspondence between the tidal gravitational field in which a galaxy forms and the orientation of the galaxy image, *on average, there will be a tendency for galaxy images to be aligned with the major axis of the tidal gravitational field.*

The tidal field,  $D_{\alpha\beta} = Ak_\alpha k_\beta$ , used in the derivation above is not the most general tidal field. For an arbitrary tidal field

perpendicular to the line of sight –  $D_{\alpha\beta} = \partial_\alpha \partial_\beta \phi$  with  $\partial_x \phi = 0$  – the *mean* ellipticity of sources that form will be (Kamionkowski, Mackey & White, in preparation)

$$\begin{aligned} \epsilon_+ &= \frac{C^2}{15} [(\partial_y^2 \phi)^2 - (\partial_z^2 \phi)^2], \\ \epsilon_x &= \frac{-2C^2}{15} (\partial_y \partial_z \phi)(\partial_y^2 \phi + \partial_z^2 \phi). \end{aligned} \quad (13)$$

Galaxies that form in a tidal field parallel to the line of sight will form edge-on discs, but their orientations will be isotropic, thus leading to no induced shear. Although the detailed form of equation (13) would be altered if we had taken some other form for  $f(L)$  (e.g. a constant or linear dependence of  $\epsilon$  on  $L$  instead of a quadratic relation), we would still have found some correlation between the induced ellipticity and the tidal field.

If the ellipticity of sources has a quadratic dependence on the tidal field, as in equation (13), then correlations in the ellipticity are higher order in tidal-field correlations. In this case, long-range ellipticity–ellipticity correlations should be smaller, and their power spectrum should be stronger at smaller scales. Moreover, the ellipticity correlations will have some curl component, rather than the pure curl-free power spectrum produced by weak lensing (Stebbins 1996; Kamionkowski et al. 1998) and by the correlations considered in Section 3. Detailed results for this case will be presented elsewhere (Kamionkowski et al., in preparation). We could have alternatively postulated that  $f(L) \propto L^{-1}$ : i.e. an ellipticity that is proportional to the angular momentum and thus to the tidal field. However, even in this case, the *correlations* would still be higher order in the tidal-field and/or mass correlations (Lee & Pen 2000a,b; Crittenden et al. 2000). For every mass distribution, there is another in which overdensities are replaced by underdensities and vice versa. Although these mass-reversed configurations will lead to angular momenta that differ in sign, they lead to the same ellipticity. In constructing the statistical ensemble from which the ellipticity correlations are measured, the contributions from the two angular momenta cancel, and this prohibits any ellipticity correlation that is linear in the mass or tidal-field correlations.

## 6 INTRINSIC VERSUS EXTRINSIC DISCRIMINATORS

### 6.1 Redshift information

Redshift information could be used to determine the relative contributions of the intrinsic and weak-lensing correlations. As Fig. 2 indicates, the weak-lensing signal is larger for more distant sources (as there is more line of sight for the signal to accrue), while the intrinsic correlation should actually increase for lower redshift sources. We should caution, however, that dynamical processes might dilute intrinsic ellipticity correlations with time; this should be amenable to further study by numerical simulations. It is thus plausible that low-redshift populations show no intrinsic correlation even though high-redshift populations do.

Another possibility would be to exploit the different dependence of the weak-lensing and intrinsic correlations on the redshift distribution of the sources. Since the weak-lensing signal accrues along the line of sight, the ellipticity correlation between two objects nearby on the sky but widely separated in redshift may be significant. On the other hand, the intrinsic ellipticity correlation should be larger for pairs of sources that are close in redshift. Some indication of this can be seen in equation (7). Consider a

distance distribution  $g(x)$  that is a top hat centred at  $x_0$  with width  $\Delta x$ . The shear power spectrum that results is inversely proportional to  $\Delta x$  [as long as  $\Delta x$  is not so small that the approximation used in deriving equation (7) breaks down].

Along similar lines, our model predicts the existence of correlations between ellipticities of spatially close pairs of galaxies. Redshift surveys should yield a good sample of close (in redshift as well as in celestial position) pairs of galaxies with which this correlation could be searched.

## 6.2 Cuts on intrinsic ellipticities

The strength of the shear induced by weak lensing does not depend on the intrinsic ellipticities of the sources. On the other hand, our calculation [cf. equation (10)] suggests that the magnitude of the intrinsic correlation is proportional to the square of the average ellipticity of the sources. Thus the intrinsic correlations could be reduced by using sources with smaller intrinsic ellipticities, or by choosing isophotes that yield images of the smallest intrinsic ellipticity. Alternatively, one could determine the ellipticity correlation functions for the same population of sources, but using several different isophotes. Since the weak-lensing signal should not depend on the intrinsic ellipticity while the intrinsic correlation should, comparison between these various correlation functions can be used to separate the intrinsic and weak-lensing signals in much the same way as multifrequency cosmic microwave background maps disentangle the cosmological signal from Galactic foregrounds.

## 6.3 Cross-correlation with density

Another possibility is cross-correlation between the shear and the convergence. In addition to distorting images, weak lensing will also affect the density of sources on the sky, and this might be used to isolate the weak-lensing signal. However, given that the tidal field is correlated with the mass (and thus the galaxy) distribution, there may be a similar cross-correlation between the density of sources and the intrinsic orientations which should be investigated further (Kamionkowski et al., in preparation).

## 6.4 Intrinsic correlations around clusters

Our model makes a precise prediction for the relation between the mean orientation of galaxies and the tidal field in which they form. Clusters are produced at very high-density peaks of the initial density distribution, so they should have been surrounded at early times by large primordial tidal fields (in the radial direction). There should thus be an alignment of nearby galaxies *at redshifts comparable to that of the cluster*. The algorithms that have been developed to look for alignment of distant background galaxies arising from weak lensing by the cluster could be applied to galaxies near the cluster to search for an alignment with the tidal field. If the alignment is too small to be detectable with the finite number of galaxies associated with one cluster, it may be possible to ‘stack’ several clusters to improve the sensitivity to this alignment. This ought to provide an alternative and possibly superior calibration of the intrinsic alignment.

## 6.5 Morphological distinctions

We have considered two possible mechanisms for aligning galaxy shapes – correlations with halo angular momenta and with halo

shapes. Heuristically, the shapes of disc galaxies are expected to be determined more by the spin of the halo, while the shapes of ellipticals should be determined more by the shapes of the haloes in which they form. These two mechanisms make considerably different predictions for long-range ellipticity correlations. Thus the weak-lensing shear could be disentangled from the intrinsic signal by using information about the morphology of the sources. Specifically, we expect the intrinsic alignment of ellipticals to have more power on larger scales relative to smaller scales than the intrinsic alignment of spirals.

## 7 CONCLUSIONS AND DISCUSSION

We have shown that the shapes and/or angular momenta of galactic haloes should depend to some extent on the tidal gravitational field in which they are produced. Long-range correlations in the gravitational field should thus lead to long-range correlations in the shear inferred from images of distant galaxies. Although uncertainties in the relation between the luminous galaxy shape and the halo shape prohibit us from carrying out a ‘first principles’ calculation of the correlation, we can estimate the magnitude of the correlation that arises if ellipticities are determined by halo shapes by calibrating to the observed distribution of ellipticities.

The amplitude of the intrinsic power spectrum is increased (decreased) with a larger (smaller) smoothing length  $R$ . Realistically, there will be factors in addition to those that we have considered that contribute to the observed orientation, and these could decrease the correlation. One example is the halo spin. As another example, major mergers could affect the halo shape as well as the orientation of the disc relative to that of the halo. All of these effects will tend to diminish the correlations predicted by our model. However, results from numerical simulations (Croft & Metzler 2000; Heavens et al. 2000) seem to indicate that the correlations in the haloes have not been much diluted by these effects. Although these simulations quantify the correlations of the parent haloes, there is still a considerable amount of physics relating the halo shape to the shape of the luminous galaxy that cannot yet be described properly with simulations. Heuristically, these effects should tend to diminish the intrinsic correlations even further. If future theoretical work determines that the degradation is considerable, then the effects that we are discussing will be unimportant for interpretation of recent field-lensing detections. However, even if the correlation is small, it should not be zero – we have indeed identified realistic physical effects that should play at least some role in aligning galaxy images. Thus the physical effects that we have discussed here will be important for interpretation of future more precise weak-lensing maps, as well as for understanding the implications of measurements of higher order weak-lensing statistics.

At first, this intrinsic correlation may be seen as a nuisance for field-lensing searches. However, the intrinsic correlation arises from the same long-range correlations in the density field as give rise to the weak-lensing correlation. Moreover, the galaxy formation physics that produces spins and shapes of galaxies is itself inherently interesting. Thus measurement of these intrinsic correlations would be of fundamental significance for structure formation and galaxy formation.

## ACKNOWLEDGMENTS

We thank D. Bacon, R. Croft, R. Ellis, A. Heavens, M. Metzger,

C. Metzler, A. Refregier, M. Santos and M. White for discussions, and we thank especially U.-L. Pen and M. Zaldarriaga for identifying an error in an earlier draft. This work was supported in part by NSF AST-9900866, AST-0096023, NASA NAG5-8506, and DoE DE-FG03-92-ER40701.

## REFERENCES

- Bacon D., Refregier A., Ellis R. S., 2000, *MNRAS*, 318, 625  
 Bardeen J. M., Bond J. R., Kaiser N., Szalay A. S., 1986, *ApJ*, 304, 15  
 Bartelmann M., Schneider P., 1992, *A&A*, 259, 413  
 Bartelmann M., Schneider P., 1999, *Phys. Rep.*, in press (astro-ph/9912508)  
 Bernardeau F., van Waerbeke L., Mellier Y., 1997, *A&A*, 322, 1  
 Blandford R. D., Saust A. B., Brainerd T. G., Villumsen J. V., 1991, *MNRAS*, 251, 600  
 Buchalter A., Jimenez R., Kamionkowski M., 2001, *MNRAS*, in press (astro-ph/0006032)  
 Catelan P., Theuns T., 1996, *MNRAS*, 282, 436  
 Cooray A., Hu W., 2000, astro-ph/0004151  
 Crittenden R. G., Natarajan P., Pen U.-L., Theuns T., 2000, astro-ph/0009052  
 Croft R., Metzler C., 2000, astro-ph/0005384  
 Dalcanton J., Spergel D. N., Summers F. J., 1997, *ApJ*, 482, 659  
 de Bernardis P. et al., 2000, *Nat*, 404, 955  
 Doroshkevich A. G., 1970, *Afz*, 6, 581  
 Gunn J. E., 1967, *ApJ*, 150, 737  
 Hanany S. et al., 2000, astro-ph/0005123  
 Heavens A. F., Peacock J. A., 1988, *MNRAS*, 243, 133  
 Heavens A., Refregier A., Heymans C., 2000, *MNRAS*, 319, 649 (astro-ph/0005269)  
 Hoyle F., 1949, in Burgers J. M., van de Hulst H. C., eds, *Problems of Cosmical Aerodynamics*. Central Air Documents, Dayton, OH, p. 195  
 Hu W., Tegmark M., 1999, *ApJ*, 514, L65  
 Kaiser N., 1992, *ApJ*, 388, 272  
 Kaiser N., Wilson G., Luppino G. A., 2000, astro-ph/0003338  
 Kamionkowski M., Spergel D. N., Sugiyama N., 1994, *ApJ*, 426, L57  
 Kamionkowski M., Babul A., Cress C., Refregier A., 1998, *MNRAS*, 301, 1064  
 Lee J., Pen U.-L., 2000a, *ApJ*, 532, L5  
 Lee J., Pen U.-L., 2000b, astro-ph/0008135  
 Miller A. D. et al., 1999, *ApJ*, 524, L1  
 Miralda-Escudé J., 1991, *ApJ*, 380, 1  
 Mo H. J., Mao S., White S. D. M., 1998, *MNRAS*, 297, L71  
 Mould J., Blandford R., Villumsen J., Brainerd T., Smail I., Small T., Kells W., 1994, *MNRAS*, 271, 31  
 Munshi D., Jain B., 2000, *MNRAS*, 318, 109  
 Peebles P. J. E., 1969, *ApJ*, 155, 393  
 Stebbins A., 1996, astro-ph/9609149  
 Sugerman B., Summers F. J., Kamionkowski M., 2000, *MNRAS*, 311, 762  
 van Waerbeke L. et al., 2000, *A&A*, 358, 30  
 Wittman D. M., Tyson J. A., Kirkman D., Dell'Antonio I., Bernstein G., 2000, *Nat*, 405, 143  
 White S. D. M., 1984, *ApJ*, 286, 38

This paper has been typeset from a  $\text{\TeX/L\AA\TeX}$  file prepared by the author.