Supporting Information

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Gating System

The phononic transistor shown in Movie S1 can be represented by the schematic diagram in Fig. S1. A metamaterial, consisting of an array of seven spiral-spring resonators with embedded magnets (not shown in the schematic), connects the source (i.e., a mechanical shaker) to the drain. A control stage, which contains seven magnets corresponding to the number of unit cells in the metamaterial, is positioned 7 mm below the metamaterial. The control stage also contains two other magnets, one on each side (left and right in the schematic diagram), which couple to two magnetic, resonant cantilevers.

The ON/OFF switching occurs when a low-amplitude mechanical signal of frequency (f_0) excites the magnetic cantilever, causing it to resonate. This resonance disturbs the energy potential of the control stage due to its magnetic coupling with the cantilever and induces a shift of the control stage from left to right (Fig. S2). When the stage moves, the magnetic field between the control stage magnets and the metamaterial magnets causes a shape change in the spiral-spring geometry from a flat (2D) shape to a helical (3D) shape (Fig. 1*B*). The shape change, in turn, shifts the band-gap frequency. This allows a mechanical signal with the same frequency (f_0) and higher amplitude to flow from the source to the drain through the metamaterial.

Bistable Potential

The system in Fig. S1 can be described analytically with a discrete mass-spring model. We assume the two cantilevers to be identical, having a stiffness k_1 and mass m_1. Each cantilever is coupled with the control stage by a magnet, which is represented by a nonlinear spring with stiffness k_2. The control stage has a mass m_2 and has two hard-end stops represented by the nonlinear spring k_3 in both direction (Fig. S2). The side-to-side distance between the two resting positions of the control stage is 6 mm, which is the required displacement to switch the transistor ON and OFF.

The forces acting on the control stage are emerging only from the magnetic cantilevers (k_2) and the end stops of the stage (k_3). Their potential can be approximated as

$$V_{Tot} = V_S(x - x_{end}) + V_S(-x - x_{end})$$

$$+ \int (F_{Mag}(r_{S,L}, p_L, p_S) + F_{Mag}(r_{S,R}, p_R, p_S))\partial x,$$

where V_S represents the displacement of the control stage, with x_{end} = 3 mm. The magnetic potential is obtained by integrating F_{Mag} over ∂x, where p is the polarization vector and $r_{i,j}$ is the vector connecting the two magnets i and j. The hard-end stops are represented by the nonlinear springs k_3 that are one sided:

$$V_S(\Delta x) = \frac{1}{2}k[\Delta x]_+^2.$$

The magnetic dipole–dipole interaction force is calculated using ref. 29,

$$F_{Mag}(r, p_L, p_S) := \frac{3\mu}{4\pi \parallel r \parallel^5}((r \times p_L) \times p_S + (r \times p_S) \times p_L$$

$$-2r(p_L \cdot p_S) + 5\frac{r}{\parallel r \parallel^2}((r \times p_L) \cdot (r \times p_S))),$$

where μ is the magnetic permeability of air, $\approx 4^*\pi 10^{-7}(\mathrm{N/A}^2)$. An approximation of the magnetic momenta p_L, p_R, and p_S is obtained using the forces indicated by the manufacturer (www.supermagnete.ch), using F_{Mag} for touching magnets. The vectors $r_{S,L}$ and $r_{S,R}$ connect the stage to the cantilever magnets.

$$p_L = p_R = [2.24 \times 10^{-2}, 0, 0]^\intercal \ \mathrm{A^*m^2},$$

$$p_S = [-8.22 \times 10^{-2}, 0, 0]^\intercal \ \mathrm{A^*m^2},$$

$$r_{S,L} = [-x_{0,mags} - x, d_0 + d_{p_L}, 0]^\intercal \ \mathrm{m},$$

$$\text{and } r_{S,R} = [x_{0,mags} - x, d_0 + d_{p_R}, 0]^\intercal \ \mathrm{m},$$

where

$$x_{0,mags} = x_{end} + 1.7 \times 10^{-3},$$

$$\text{and } d_0 = 1.5 \times 10^{-3} \ \mathrm{m},$$

are the x and y components of the equilibrium position of the cantilever magnets relative to the centered stage. The parameters d_{p_L} and d_{p_R} represent the deflection of the gate (i.e., of the cantilever) having a spring (k_1). For a given deflection of the left- or right-hand side magnets that happens as a result of the harmonic excitation at the gate in the physical system, the potential becomes asymmetric and unstable, which induces a shift in the control stage position (Fig. S2). The energy potential of the control stage when it is exactly in the center between the two cantilevers is defined as the reference (i.e., equilibrium) state. The potential of the control stage moving from left to right or from right to left is plotted against the reference case (Fig. S2). The amount of energy required to switch a single transistor from one state to the other is estimated to be $\approx$70 μJ, corresponding to the required shift to modify the energy potential.

To demonstrate the stability of the proposed concept, we present the measured output signal in a NOT gate to alternating inputs of ones and zeros (Fig. S3). The output signal is consistent between multiple cycles.

Band-Structure Analysis for the Metamaterial

We calculate the band structure of an infinite array of the spiral-spring unit cells, using the finite-element method. We solve the elastic wave equations in three dimensions and apply the Bloch wave formulation in plane (i.e., Bloch boundary conditions) (30). We consider the wave propagation along the direction of periodicity, in the Γ-X direction (Fig. S4, *Left*). We then construct a finite system, corresponding to the experimental setup, consisting of an array of seven unit cells. We fix one end of the structure and apply a dynamic load to the other end. We obtain the frequency response function of the metamaterial in Fig. S4, *Right*. Both finite and infinite predictions agree well, particularly in the band gap frequency range, highlighted in gray. The band-gap range also agrees well with the experimental results (Fig. 1*C*).

Different Realizations of Logic Gates

The unified logic platform (Fig. 2), designed to perform all of the mechanical logic operations, is composed of four interconnected transistors. However, some of these operations could also be performed with a lower number of transistors, in different configurations. For example, the NOT operation requires only a single transistor (Fig. S5*A*). The OR and NAND operations can be achieved using only two transistors connected in parallel (Fig. S5*B*). Similarly, the AND and NOR operations can be realized with two transistors connected in series (Fig. S5*C*). The only

two gates that require four interconnected transistors are the XOR and XNOR (Fig. S5*D*).

Metamaterial Quality Factor

We characterize the quality factor, Q, of the spiral-spring resonators in the metamaterial by measuring the resonance amplitude of a single spring, using a mechanical shaker as a harmonic excitation source and a laser Doppler vibrometer to detect the velocity of the central mass of the spring. The calculated $Q = f/\Delta f$ is $\approx$20 (Fig. S6), where f is the resonance frequency and Δf is the width of the frequency range for which the amplitude is $1/\sqrt{2}$ of its peak value.

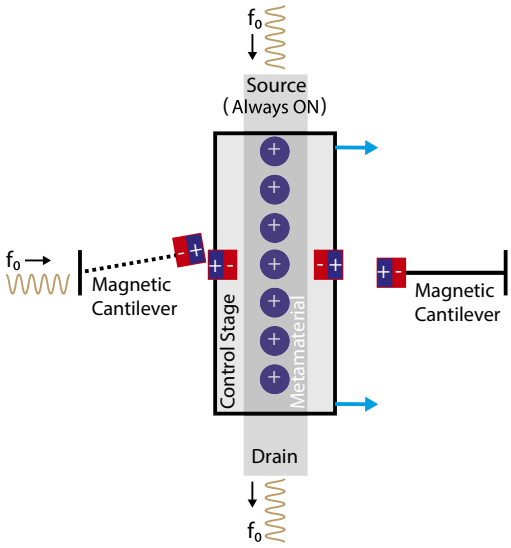


Fig. S1. Schematic of the phononic transistor-like device and its components.

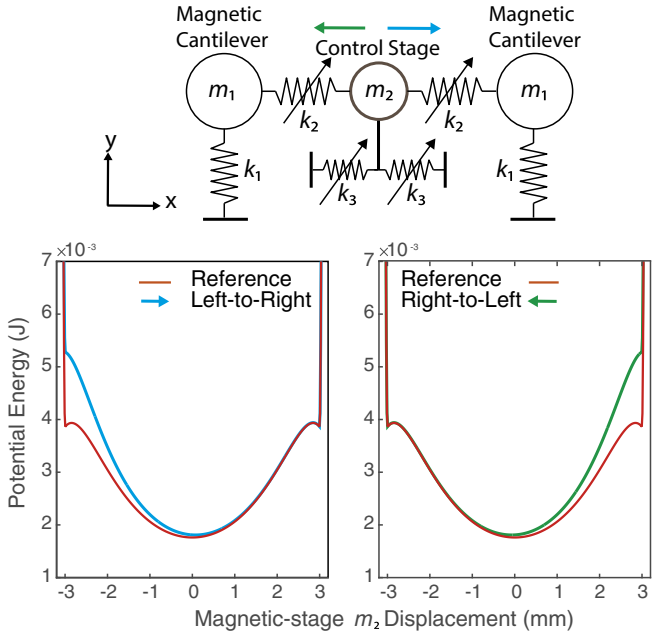


Fig. S2. Schematic of the dynamically bistable switching mechanism approximated into masses and springs. Shown is the energy potential of the control stage when it moved from one of its hard-end stops (k_3) to the other end (blue and green lines). The red line represents the equilibrium state of the control stage when at equal distance from the magnetic cantilevers.

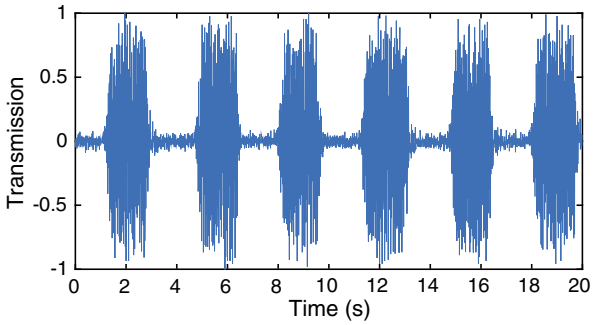


Fig. S3. Experimental time signal acquired for a NOT gate with 12 cycles of alternating input starting with a logical 1 at time $t = 0$.

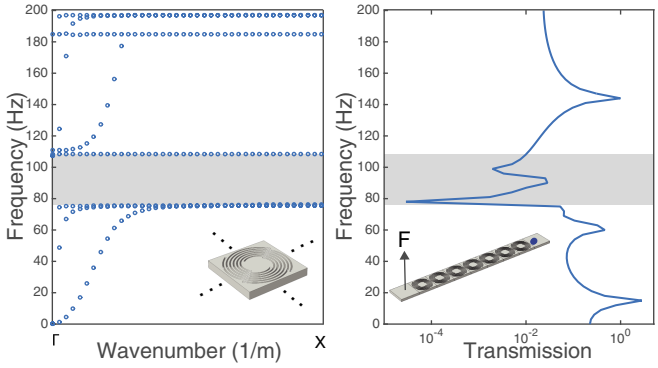


Fig. S4. (*Left*) Infinite dispersion curves in the Γ-*X* direction of the metamaterial, based on Bloch theorem. (*Right*) Finite frequency response function of a metamaterial consisting of seven unit cells. The band-gap region is highlighted in gray in both panels.

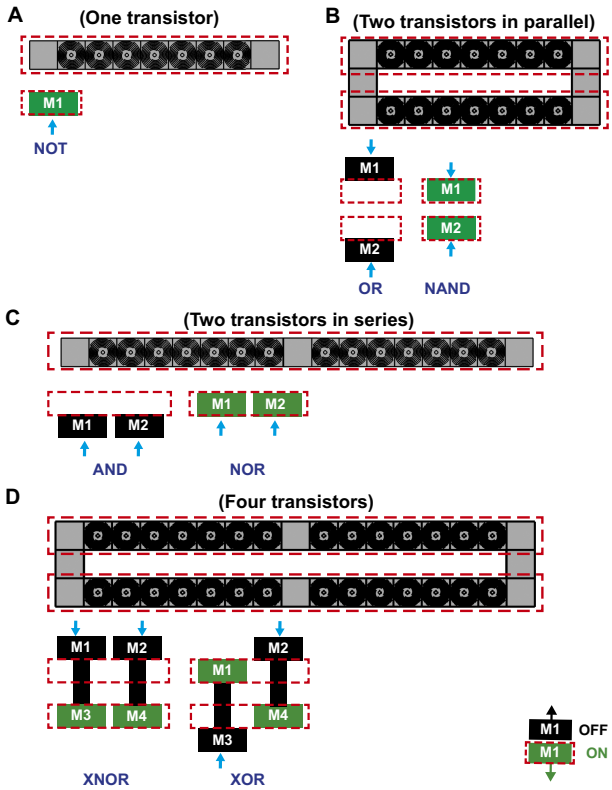


Fig. S5. Schematic representation of the minimum number of transistors necessary to realize the various logic gates. (*A*) the NOT gate with one transistor. (*B*) Both OR and NAND gates with two transistors in parallel. (*C*) Both AND and NOR gates with two transistors in series. (*D*) Both XNOR and XOR gates with four interconnected transistors. Similar to Fig. 2, the red, dashed lines are the reference frames for the magnetic control stages M_{1-4}. The green/ON and black/OFF represent the position of the stage, relative to the metamaterial. The magnetic stage is either underneath the spiral springs (ON) or displaced laterally by 6 mm (OFF). The cyan arrows represent the input signal.

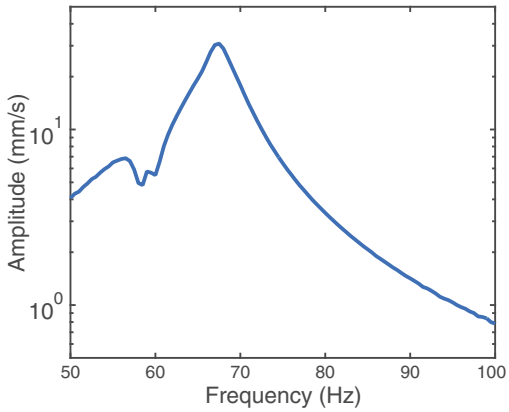
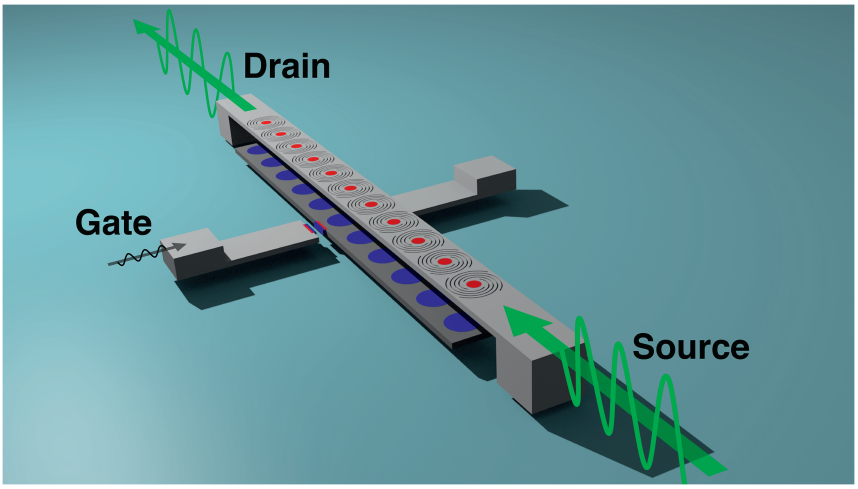


Fig. S6. Experimental characterization of the quality factor of a single spiral-spring resonator.



Movie S1. The experimental realization of the transistor's bistable gating system. By exciting the magnetic cantilever at one side of the metamaterial the energy potential becomes asymmetric, causing the magnetic control stage to flip to the opposite side and vice versa.

Movie S1