Colliding Regge Poles and Cuts*

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(Received 29 December 1969)

A class of models describing the collision of Regge poles and their associated Mandelstam branch cuts is described, from which one concludes that Regge trajectories generally develop left-hand branch cuts as a result of such collisions, and in terms of which the singularities in the l plane of the partial-wave amplitude and the asymptotic behavior of the scattering amplitude can be discussed.

I. INTRODUCTION

It is believed to be true that Regge cuts as well as Regge poles occur in physical scattering amplitudes.[1] In particular, it is believed that at least some of these cuts can be thought of as generated by Regge poles, and that the branch point $\alpha_c(t)$ of such a cut in the l plane can often be written

$$\alpha_c(t)=\alpha_1(\tfrac{1}{4}t)+\alpha_2(\tfrac{1}{4}t)-1$$

in terms of two poles α_1 and α_2 which cause it.[1] Therefore, if the Pomeranchuk trajectory α_P exists, and if $\alpha_P(0)=1$, then the cut caused by the Pomeranchon and any other Regge pole $\alpha(t)$ has the property that $\alpha(t)$ and the cut intersect at $t=0$. Consequently, it is natural to expect that the pole will develop some sort of singularity in t at $t=0$, and it has been suggested in fact that all Regge poles will have left-hand branch cuts in t from $t=-\infty$ to $t=0$.[2]

We wish, therefore, to explore the behavior of Regge poles, Regge cuts, and scattering amplitudes containing these poles and cuts, near $t=0$. In order to do this, a model is needed imitating the dynamics producing these singularities, because the detailed behavior near $t=0$ is evidently determined by dynamics.

We shall not attempt a full-blown dynamical calculation here; instead, we shall construct, by analogy with potential theory, a fairly general form which hopefully is characteristic of a wide class of dynamical situations. In particular, the few model dynamical worlds we have which incorporate (at least partially) both poles and cuts do fall in this class.[3] The construction is given in Sec. II.

In terms of our form, some general features of the collision of a cut and pole become evident and we shall describe these in Sec. III. More detailed behavior can be studied in various simple special cases, and we do this too, in Sec. IV; finally, we mention the expected asymptotic form of the scattering for some of these cases in Sec. V.

One interesting possibility created by the existence of left-hand cuts in trajectories is that the Pomeranchuk trajectory has a flat real part equal to unity for $t<0$ so that nonshrinking diffraction peaks become possible.[4] We comment on the special case of such a trajectory in Sec. VI.

Before we launch into all this detail, however, it may be useful for us to summarize our basic conclusions here.

We find that when a Regge pole collides with a Mandelstam branch cut, the pole always develops a branch point in t,[5] and one of two things happens naturally. Either the pole passes through the cut and disappears from the physical angular momentum sheet, or the cut "expels" a pole and the original pole and the expelled pole form a complex conjugate pair in the physical complex l plane, after meeting at a square-root branch point. Which of these two things happens depends on the sign and nature of the "coupling" of the pole to the cut. In the case of baryon trajectories, this quite reasonably leads to the removal of MacDowell partners, if the coupling is strong.

II. CONSTRUCTION OF MODEL

When two Regge poles $\alpha(t)$ and $\alpha_1(t)$ collide, it is well known that both develop square-root branch points in t. This is trivially argued from the analytic properties of $D(t,l)$ in the neighborhood of the collision, which must be a quadratic form in l, with coefficients which

* Work supported in part by the U. S. Atomic Energy Commission. Prepared under Contract Nos. AT(11-1)-68 and UCR-34P107-111 for the San Francisco Operations Office, U. S. Atomic Energy Commission.

[1] S. Mandelstam, Nuovo Cimento **30**, 1127 (1963); **30**, 1148 (1963); V. N. Gribov, I. Ya. Pomeranchuk, and K. A. Ter-Martirosyan, Phys. Rev. **139**, B184 (1965).

[2] J. S. Ball and F. Zachariasen, Phys. Rev. Letters **23**, 346 (1969).

[3] W. R. Frazer and C. H. Mehta, Phys. Rev. Letters **23**, 258 (1969).

[4] See Ref. 3. The suggestion of a flat Pomeranchon is also made by P. G. O. Freund and R. Oehme [Phys. Rev. Letters **10**, 450 (1963)], in which the Mandelstam cut is not invoked as the cause of a branch point in $\alpha_p(t)$, but a new cut, joining two complex conjugate points in the l plane, is invented for this purpose.

[5] In contrast to what has been alleged [R. Oehme, Phys. Letters **20B**, 414 (1969)], we find that in any realistic dynamical model, a Regge pole *must always* develop a branch point upon collision with the cut.

are analytic in t.[6] A simple model of D is

$$\begin{aligned}D(t,l)&=d[l-\alpha^0(t)][l-\alpha_1{}^0(t)]+c^2(t)\\&=[l-\alpha(t)][l-\alpha_1(t)]\\&=\det\begin{vmatrix}l-\alpha^0(t) & c(t)\\ c(t) & d[l-\alpha_1{}^0(t)]\end{vmatrix},\end{aligned}\tag{2.1}$$

where $\alpha^0(t)$, $\alpha_1{}^0(t)$, and $c(t)$ are polynomials in t, and $\alpha^0(t)$ and $\alpha_1{}^0(t)$ can be thought of as the unperturbed trajectories coupled by the function $c(t)$. Because of this coupling the trajectories $\alpha^0(t)$ and $\alpha_1{}^0(t)$ become modified to $\alpha(t)$ and $\alpha_1(t)$, and these modified trajectories develop branch points in t. We have, specifically,

$$\begin{aligned}\alpha(t)&=\tfrac{1}{2}\{\alpha^0(t)+\alpha_1{}^0(t)\\&\qquad+[(\alpha^0(t)-\alpha_1{}^0(t))^2-(4/d)c^2(t)]^{1/2}\},\\\alpha_1(t)&=\tfrac{1}{2}\{\alpha^0(t)+\alpha_1{}^0(t)\\&\qquad-[(\alpha^0(t)-\alpha_1{}^0(t))^2-(4/d)c^2(t)]^{1/2}\},\end{aligned}\tag{2.2}$$

and branch points are at t_b, given by

$$\alpha^0(t_b)-\alpha_1{}^0(t_b)=\pm(4/d)^{1/2}c(t_b).\tag{2.3}$$

[If we wish a branch point to be at $t=0$, we can let $\alpha^0(0)=\alpha_1{}^0(0)+(4/d)^{1/2}c(0)$.]

If a trajectory $\alpha(t)$ collides with a set of trajectories $\alpha_i(t)$, we may expect a reasonable model of the D function to be

$$D(t,l)=\det\begin{vmatrix}l-\alpha^0(t) & c_1(t) & c_2(t) & \cdots\\ c_1(t) & d_1(t)[l-\alpha_1{}^0(t)] & 0 & \cdots\\ c_2(t) & 0 & d_2(t)[l-\alpha_2{}^0(t)] & \cdots\\ \vdots & \vdots & \vdots & \cdots\end{vmatrix}.\tag{2.4}$$

This form couples the trajectory α^0 to each of the others, but neglects coupling between the other trajectories $\alpha_i{}^0(t)$.

The determinant is readily evaluated to give

$$\begin{aligned}D(t,l)&=[l-\alpha^0(t)]\prod_{j=1}^{\infty}[l-\alpha_j{}^0(t)]d_j(t)\\&\qquad-\sum_{i=1}^{\infty}C_i{}^2(t)\prod_{j\neq 1}^{\infty}[l-\alpha_j{}^0(t)]d_j(t)\\&=\left[l-\alpha^0(t)+\sum_{i=1}^{\infty}\frac{C_i(t)}{l-\alpha_i{}^0(t)}\right]\\&\qquad\times\exp\{\sum_{j=1}^{\infty}\ln[l-\alpha_j{}^0(t)]d_j(t)\},\end{aligned}\tag{2.5}$$

where $C_i(t)=-c_i{}^2(t)/d_i(t)$.

Now let us suppose all the trajectories $\alpha_i{}^0(t)$ are parallel and differ only a little:

$$\alpha_j{}^0(t)=\alpha_c{}^0(t)-(j-1)\epsilon,\quad j=1,2,\ldots.$$

Here, ϵ is a small number, which we will eventually allow to approach zero. Then

$$\begin{aligned}D(t,l)&=\left[l-\alpha^0(t)+\sum_{j=1}^{\infty}\frac{C_i(t)}{l-\alpha_c{}^0(t)+(j-1)\epsilon}\right]\\&\qquad\times\exp(\sum_{j=1}^{\infty}\{\ln[l-\alpha_c{}^0(t)+(j+1)\epsilon]d_j(t)\}).\end{aligned}\tag{2.6}$$

We note that every zero of the exponential part of D is matched by a pole in the bracket and is therefore not a zero of D itself. The zeros of D are then contained solely in the first bracket. To see where these zeros are, we set

$$\bar{D}(t,l)\equiv l-\alpha^0(t)+\sum_{j=1}^{\infty}\frac{C_i(t)}{l-\alpha_c{}^0(t)+(j-1)\epsilon}\tag{2.7}$$

equal to zero. Schematically, the equation

$$\alpha^0(t)-\alpha(t)=\sum_{j=1}^{\infty}\frac{C_i(t)}{\alpha(t)-\alpha_c{}^0(t)+(j-1)\epsilon}\tag{2.8}$$

is shown in Fig. 1. Two cases are represented, corresponding to t values such that the singled-out trajectory $\alpha^0(t)$ is above the infinite series of trajectories $[\alpha^0(t)>\alpha_c(t)]$ and below the top one of these $[\alpha^0(t)<\alpha_c(t)]$. The unperturbed solutions are marked by circles and the perturbed solutions by solid dots. In the figure we are in a regime where all solutions are real. Consider now decreasing ϵ, keeping everything else the same. The pole terms start crowding together, and on that particular pole term on which there are three solutions, two will approach each other, meet, and go into the complex plane. Since each pole term must be cut once by the line $\alpha^0(t)-l$, we form one and only one complex conjugate pair for some critical value of ϵ. Once the slope of the pole terms is larger than the slope of $\alpha^0(t)-l$, that is, once ϵ is small enough, t can be moved around and the complex conjugate pair will move with it, corresponding to the trajectory $\alpha(t)$ and one pole taken from the series $\alpha_c{}^0(t)-(j-1)\epsilon$, but it cannot be identified as belonging to one particular j.

Taking the case where the unperturbed trajectories are, respectively, one linear trajectory and infinitely many flat ones, we can now represent the sequence in terms of increasing the couplings $C_j(t)$, which is essentially equivalent to decreasing the spacing. This is shown in Fig. 2. When a critical coupling is reached,

[6] Hung Cheng, Phys. Rev. **130**, 1283 (1963). For simplicity, we shall ignore signature in what follows, since it is basically irrelevant for our purposes.

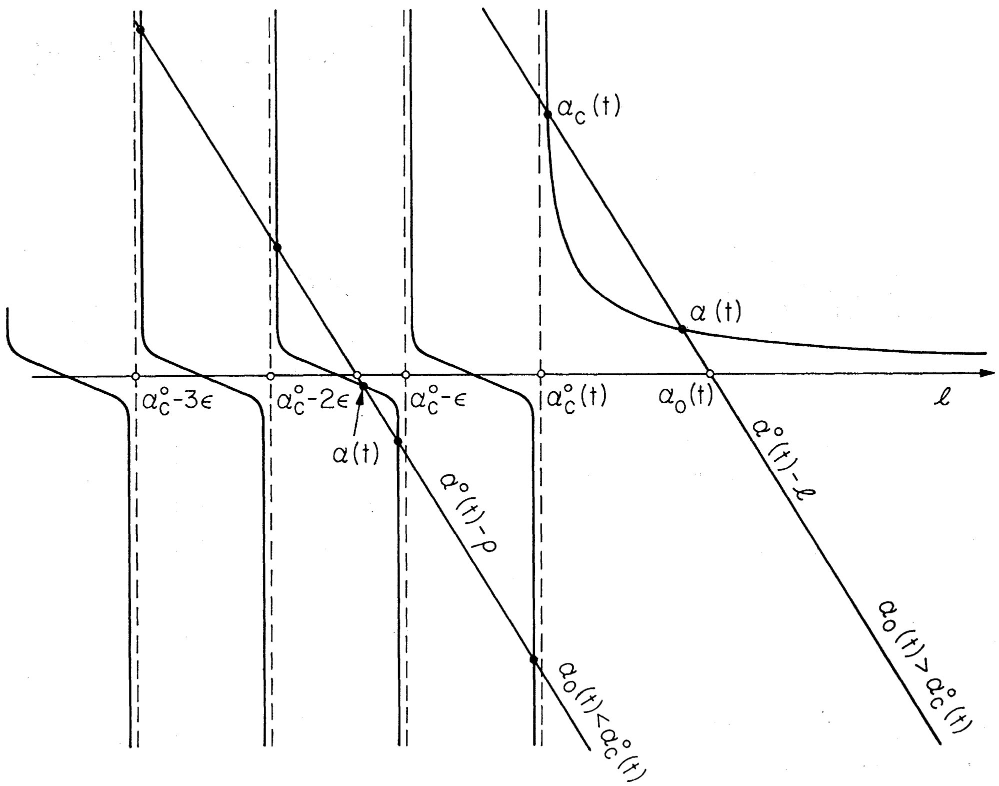

FIG. 1. Schematic representation of solutions to Eq. (2.8). Two cases are shown corresponding to t, such that $\alpha_0(t)>\alpha_c^0(t)$ and $\alpha_0(t)<\alpha_c^0(t)$, respectively. In each case, the unperturbed (perturbed) solutions correspond to the l values given by circles (dots).

a complex conjugate pair is formed near the intersection of $\alpha^0(t)$ and $\alpha_c^0(t)$, which keeps moving as a pair for negative t.

It is evident that in order to generate a cut in l, along with a moving pole, we can let the spacing between the trajectories shrink to zero. In the limit $\epsilon\rightarrow 0$, we obtain

$$D(t,l)=\left[l-\alpha^0(t)-\int_{-\infty}^{\alpha_c^0(t)}\frac{C(t',l)}{l'\times l}dl'\right]\times\exp\left[\int_{-\infty}^{\alpha_c^0(t)}\ln(l-l')d(t,l')dl'\right]. \quad (2.9)$$

The case of interest to us is the collision of the trajectory $\alpha^0(t)$ with the Mandelstam[1] branch cut. In its first iteration, the branch point is located at $\alpha_c^0(t)=2\alpha^0(\frac{1}{4})-1$; however, successive iterations progressively flatten out the cut until it eventually appears to be flat. Therefore, let us first consider the collision of the pole $\alpha^0(t)$ with a *flat* cut, located at $\alpha_c^0(t)=1$. [The point 1 is, obviously, arbitrary; but if we are thinking of $\alpha^0(t)$ as the Pomeranchon, the value 1 is the natural choice. We will also choose $\alpha^0(0)=1$.] The case of a moving cut is not appreciably different; we shall mention it briefly later.

Our model for the D function is then

$$D(t,l)=\left[l-\alpha^0(t)-\int_{-\infty}^{1}\frac{C(t,l')}{l'-l}dl'\right]\times\exp\left[\int_{-\infty}^{1}\ln(l-l')d(t,l')dl'\right] \quad (2.10)$$

$$\equiv\bar{D}(t,l)\exp\left[\int_{-\infty}^{1}\ln(l-l')d(t,l')dl'\right].$$

III. GENERAL FEATURES

Equation (2.5) may be used to describe, qualitatively, the broad features to be expected when a Regge pole collides with a Regge cut.

Regge poles are found from the equation

$$D(t,\alpha(t))=0. \quad (3.1)$$

The poles of interest to us are the solutions to the equation

$$\alpha(t)-\alpha^0(t)-\int_{-\infty}^{1}\frac{C(t,l')}{l'-\alpha(t)}dl'=0. \quad (3.2)$$

In particular, results obtained in the multi-Regge bootstrap (MRBS) are of the form of Eq. (3.2) with the integral replaced by $\sim\ln[\alpha(t)-1]$.[3] In this case, however, the trajectory on the physical sheet never does cross the cut and become complex, but starting from $\alpha^0(t)$ for large positive t, it approaches the branch point from above for large negative t.

More generally, the integral in Eq. (2.10) simply represents an analytic function of l with a cut from $l=-\infty$ to $l=1$, the discontinuity given by $2\pi i C(t,l)$. The integral in Eq. (3.2) represents the same function of $\alpha(t)$. Furthermore, we want $\alpha(0)=1$; thus in general $\alpha(t)$ will cross the cut and will have a branch point at $t=0$. The precise form of the branch point depends on the choice of the coupling $C(t,l)$, but whatever its form (apart from the logarithmic case), it always exists.

IV. SPECIAL EXAMPLES

As a special example of Eq. (3.2) which we can explore in some detail, and which is typical of what can happen generally, let us suppose $C(t,l)$ is such that

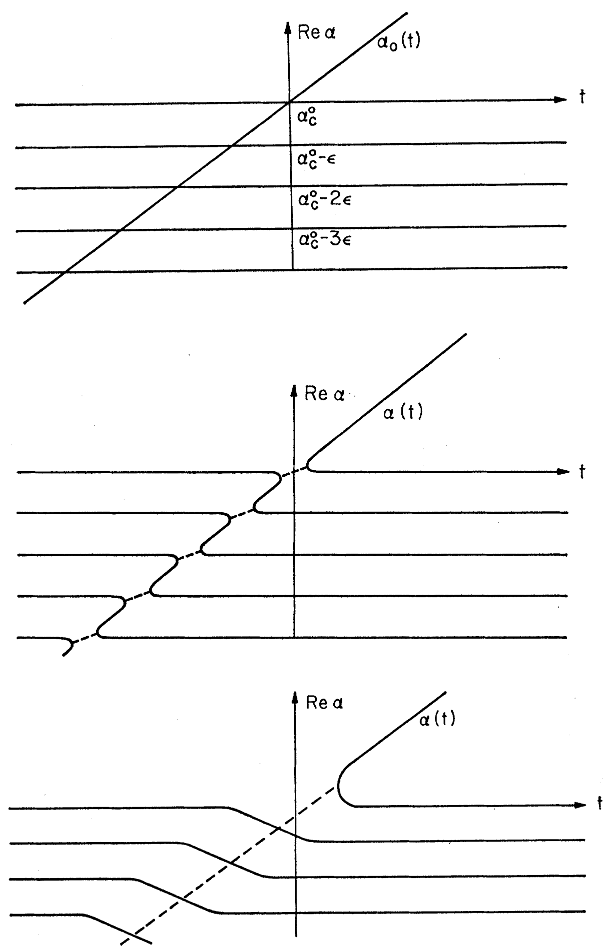

FIG. 2. Schematic diagram of solutions to Eq. (2.8), corresponding to increasingly stronger coupling or smaller spacing, ϵ. The dashed lines represent the real part of a complex conjugate pair of trajectories.

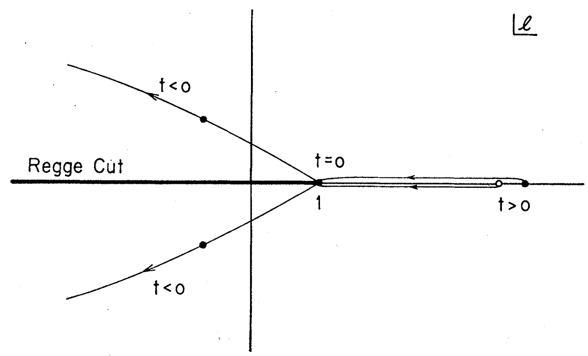

FIG. 3. Movement of poles in the cut l plane from $t>0$ to $t<0$ in the model described by Eq. (4.3). Poles in the physical (unphysical) l plane are shown as dots (circles). The case shown corresponds to the cut extruding a pole.

(3.2) reduces to

$$\alpha(t)-\alpha_0(t)-\beta_0(t)[\alpha(t)-1]^{1/2}=0\,, \tag{4.1}$$

where α_0 and β_0 are polynomials in t.[7]

There are two possible solutions $[\alpha(t)-1]^{1/2}$ given by

$$[\alpha(t)-1]^{1/2}=\tfrac{1}{2}\beta_0(t)\pm\tfrac{1}{2}\{\beta_0(t)^2+4[\alpha_0(t)-1]\}^{1/2}. \tag{4.2}$$

We define the right half $[\alpha(t)-1]^{1/2}$ plane as corresponding to the physical l plane.

The trajectories themselves are given by

$$\alpha_\pm(t)=\alpha_0(t)+\tfrac{1}{2}\beta_0(t)^2\left[1\pm\left(1+\frac{4[\alpha_0(t)-1]}{\beta_0(t)^2}\right)^{1/2}\right]. \tag{4.3}$$

For large positive t, only one of these [say, $\alpha_+(t)$] is physical. When t is such that the square root in (4.3) vanishes, $\alpha_+(t)$ develops a branch point, and as t decreases further, becomes a complex conjugate pair. Both complex α's are either physical or unphysical, depending on whether $\beta(t_b)$ (where t_b is the branch point) is positive or negative. From the point of view of the l plane, the physical pair corresponds to the cut having "extruded" a pole to meet $\alpha_+(t)$ and make the pair. The unphysical pair just looks as if the cut has swallowed $\alpha_+(t)$. The situation is displayed graphically in Fig. 3.

As a specific example, let us take

$$\begin{aligned}\alpha_0(t)&=1+at\,,\\ \beta_0(t)&=2bt.\end{aligned} \tag{4.4}$$

(Of course, this is to be taken seriously only in the neighborhood of $t=0$.) We obtain

$$[\alpha_\pm(t)-1]^{1/2}=bt[1\pm(1+a/b^2t)^{1/2}]$$

and

$$\alpha_\pm(t)=1+at+2b^2t^2[1\pm(1+a/b^2t)^{1/2}]. \tag{4.5}$$

Near $t=0$ we may expand, and obtain

$$(\alpha_\pm(t)-1)^{1/2}=bt\pm a^{1/2}t^{1/2}\pm O(t^{3/2}) \tag{4.6}$$

[7] We are only interested in t near zero; if such a model were to be taken more seriously, then α_0 and β_0 would both be expected to have branch points at physical threshold in t.

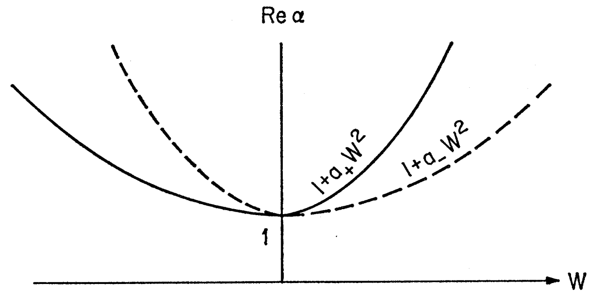

FIG. 4. Baryon trajectory model of Eq. (4.10). Poles in the physical (unphysical) l plane are drawn as full lines (dashed lines).

and

$$\alpha_\pm(t)=1+at\pm 2ba^{1/2}t^{3/2}+O(t^2)\,. \qquad (4.7)$$

For small t and positive b, the trajectory $\alpha_+(t)$ meets the branch point at $t=0$ and proceeds into the complex plane as a pair. For negative t, we then see two poles in the physical plane.

So far we have discussed only meson trajectories. However, the same considerations apply for baryon trajectories, except now we would expect the coupling to be a function of $W=\sqrt{t}$. Taking $\beta_0(W)=2bW$, and $\alpha_0=1+aW^2$ for simplicity, we obtain from (4.2)

$$[\alpha_\pm(W)-1]^{1/2}=[b\pm(b^2+a)^{1/2}]W \qquad (4.8)$$

and

$$\alpha_\pm=1+[a+2b^2+2b(b^2+a)^{1/2}]W^2\,. \qquad (4.9)$$

We have here two linear trajectories

$$\alpha_\pm=1+a_\pm W^2\,, \qquad (4.10)$$

with

$$a_\pm=a+2b^2\pm 2b(b^2+a)^{1/2}\,.$$

In the absence of coupling to the cut (i.e., if $b=0$), they are identical. However, α_+ is physical when $W>0$, and α_- when $W<0$, assuming $b>0$. As the coupling b is turned up, the slope of α_+ gets steeper and that of α_- flatter, thus avoiding MacDowell partners. This is shown in Fig. 4. If we simultaneously turn off a, we finally arrive at a situation where

$$\alpha_+=1+4b^2W^2\,, \qquad (4.11)$$

$$\alpha_-=1\,. \qquad (4.12)$$

Since

$$\bar{D}(l,W)=[(l-1)^{1/2}-(\alpha_+-1)^{1/2}][(l-1)^{1/2}-(\alpha_--1)^{1/2}],$$

we can write in this case, letting $1=\alpha(0)$,

$$A(l,W)\propto\frac{1}{\{[l-\alpha(0)]^{1/2}-(a_+W)^{1/2}\}\{[l-\alpha(0)]^{1/2}-(a_-W)^{1/2}\}}, \qquad (4.13)$$

and in the limit $a\to 0$,

$$A(l,W)\propto\frac{1}{[(l-\alpha_0)^{1/2}-2bW](l-\alpha_0)^{1/2}}. \qquad (4.14)$$

This is precisely the starting point of the model of Carlitz and Kislinger[8] for avoiding MacDowell partners. In this limit, the physical trajectory α_+ is entirely driven by the coupling to the cut. To what extent partners are avoided in any case depends on the strength of that coupling.

In case the Mandelstam cut with which the pole collides is moving, essentially everything we have said above still goes through. Equation (2.10) is now replaced by

$$D(t,l)=\left(l-\alpha^0(t)-\int_{-\infty}^{\alpha_c(t)}\frac{C(t,l')}{l'-l}dl'\right)\times\exp\left(\int_{-\infty}^{\alpha_c(t)}\ln(l-l')d(t,l')dl'\right), \qquad (4.15)$$

where, for example, we might have $\alpha_c(t)=2\alpha_0(\frac{1}{4}t)-1$, or, more generally, $\alpha_c(t)$ is anything such that $\alpha_c(0)=1$. For the case of a moving cut, we know that the discontinuity across the cut must vanish when l is a right-signature integer to avoid introducing false branch points in t into the physical partial-wave amplitudes. Therefore $C(t,l)$ should, strictly speaking, be proportional to $\sin\frac{1}{2}\pi l$ for an even-signature amplitude. For simplicity, however, we shall ignore this complication; it does not alter any of our results.

Equation (2.10) is then replaced by

$$\bar{D}(t,l)=l-1+[1-\alpha_0(t)]-2b(t)[l-\alpha_c(t)]^{1/2}, \qquad (4.16)$$

and the resulting trajectories are given by

$$\alpha_\pm(t)=\alpha_0(t)+\tfrac{1}{2}\beta_0(t)^2\times\left[1\pm\left(1+\frac{4[\alpha_0(t)-\alpha_c(t)]}{\beta_0(t)^2}\right)^{1/2}\right], \qquad (4.17)$$

in analogy to Eq. (4.3).

V. LARGE-s BEHAVIOR

What can be anticipated for the form of the scattering amplitude for large s and small t, if the pole-cut collision occurs? Evidently the partial-wave amplitude contains for positive t [that is, when $\alpha(t)$ is still real] just a pole and a fixed cut extending from $l=-\infty$ to $l=1$. Below the collision, however (that is, for negative t), there are two complex conjugate poles as well as the cut, comprising the original pole, the "extruded" pole and the remaining cut. Alternatively, the cut may swallow the pole, in which case only the cut remains at negative t.

In order to see the large-s behavior, we consider that in our model we have a partial-wave amplitude of the

[8] R. Carlitz and M. Kislinger, California Institute of Technology Report No. CALT-68-223, 1969 (unpublished).

form

$$A(l,t)=\frac{f(t)}{\{(l-1)^{1/2}-[\alpha_+(t)-1]^{1/2}\}\{(l-1)^{1/2}-[\alpha_-(t)-1]^{1/2}\}}. \tag{5.1}$$

In order to separate this clearly into two Regge poles and a cut, we note that we can add and subtract the missing poles on the other sheet and obtain

$$A(l,t)=\frac{\beta(\alpha_+,\alpha_-;\alpha_+)}{l-\alpha_+}+\frac{\beta(\alpha_+,\alpha_-;\alpha_-)}{l-\alpha_-}+\frac{1}{2}\frac{\beta(\alpha_+,\alpha_-;l)-\beta(\alpha_+,\alpha_-;\alpha_+)}{l-\alpha_+}+\frac{1}{2}\frac{\beta(\alpha_-,\alpha_+;l)-\beta(\alpha_-,\alpha_+;\alpha_-)}{l-\alpha_-}, \tag{5.2}$$

where

$$\beta(\alpha_1,\alpha_2;\alpha_3)=f(t)\frac{(\alpha_3-1)^{1/2}}{(\alpha_1-1)^{1/2}-(\alpha_2-1)^{1/2}}. \tag{5.3}$$

We note that the discontinuity across the l cut is then given by

$$\begin{aligned}\mathrm{disc}A(l,t)&=\frac{\beta(\alpha_+,\alpha_-;l)}{l-\alpha_+}+\frac{\beta(\alpha_-,\alpha_+;l)}{l-\alpha_-}\\&=\frac{f(t)}{(\alpha_+-1)^{1/2}-(\alpha_--1)^{1/2}}\\&\quad\times\left[\frac{(1-l)^{1/2}}{l-\alpha_+}-\frac{(1-l)^{1/2}}{l-\alpha_-}\right],\quad l<1.\end{aligned} \tag{5.4}$$

The large-s behavior from this contribution to the Regge cut is then given by

$$\begin{aligned}T_c(s,t)&\xrightarrow[s\to\infty]{}\frac{\bar{f}(t)}{(\alpha_+-1)^{1/2}-(\alpha_--1)^{1/2}}\\&\quad\times\int^1\left[\frac{(1-l)^{1/2}}{l-\alpha_+}-\frac{(1-l)^{1/2}}{l-\alpha_-}\right](s/s_0)^l dl,\\&\xrightarrow[s\to\infty]{}\tfrac{1}{2}\pi^{1/2}\bar{f}(t)\left[\frac{(\alpha_+-1)^{1/2}+(\alpha_--1)^{1/2}}{(\alpha_+-1)(\alpha_--1)}\right]\frac{s/s_0}{[\ln(s/s_0)]^{3/2}}.\end{aligned} \tag{5.5}$$

The large-s behavior we anticipate, then, is for $t<0$ where $\alpha_+(t)=\alpha_-^*(t)=\alpha(t)$,

$$\begin{aligned}T(s,t)&\xrightarrow[s\to\infty]{}\beta(t)(s/s_0)^\alpha\left(\frac{1\pm e^{-i\pi\alpha}}{2\sin\pi\alpha}\right)+\beta^*(t)(s/s_0)^{\alpha^*}\\&\quad\times\left(\frac{1\pm e^{-i\pi\alpha^*}}{2\sin\pi\alpha^*}\right)+\frac{\gamma(t)(s/s_0)}{[\ln(s/s_0)]^{3/2}}.\end{aligned} \tag{5.6}$$

For the cases we have been dealing with, where we select $\alpha(0)=1$, one of the two poles dominates the cut for $t>0$. For $t<0$, however, the cut dominates the poles, since the real part of the poles is now smaller than 1. At $t=0$, we would also expect the poles to dominate, since they both are at $\alpha_\pm=1$. Hence, except for the faster than usual attenuation of the cut contribution, very little experimentally noticeable happens as a result of the collision, at least in the case of weak coupling.

VI. FLAT POMERANCHONS

In principle, once the existence of left-hand cuts in Regge trajectories is admitted, trajectories of the form (near $t=0$)

$$\alpha(t)=1+t^{1/2}P(t), \tag{6.1}$$

where $P(t)$ is real, become possible. Thus a Pomeranchon with a constant (equal to unity) real part for negative t could exist, permitting us to have nonshrinking forward diffraction peaks.

The simple class of models discussed here does not, however, seem to permit solutions of this form. This is most easily seen from Eq. (4.17). A trajectory with a flat real part can result only if $\alpha_0+\frac{1}{2}\beta_0^2=1$, but in this case the argument of the square root is easily seen to be negative for $t>0$. [This can be avoided only if the cut $\alpha_c(t)$ is given a *negative* slope, which is manifestly undesirable physically.] Thus (6.1) is not possible.

Whether other kinds of models of the interaction of a Regge pole with its Mandelstam cut permit solutions like (6.1) we do not know; we suspect they do not. Consequently (6.1) could occur only if some new kind of l-plane cut is invented, in the existence of which there is no *a priori* reason to believe. The collision of a pole with this "other" cut might then yield (6.1).[4]