New Properties of Matter in (A)dS and their Consequences

S. Deser¹

Department of Physics, Brandeis University
Waltham, MA 02454, USA

Abstract: I review briefly, primarily for relativists, a series of recent results, obtained with A. Waldron, on the novel behavior of massive higher ($s > 1$) spin systems in constant curvature backgrounds. We find that the cosmological constant Λ , together with the mass parameter, define a “phase plane” in which partially massless gauge invariant lines separate allowed regions from forbidden, non-unitary, ones. These lines represent short multiplet systems, with missing lower helicities, removed by novel local gauge invariances, and (despite having $m \neq 0$) propagating on the light cone. In the limit of an infinite tower of these higher spin bosons and fermions, unitarity requires Λ to vanish.

The kinematical effects of gravity on matter (as against the well-known dynamical ones) have not received much attention in the past, nor would one intuitively expect any major surprises there. I will report here (primarily) on a series of very recent investigations by A. Waldron and myself [1] in which “massive” higher spin ($s > 1$) free bosons and fermions exhibit unexpected, qualitative, differences from flat space in the simplest curved backgrounds – constant curvature (deSitter) spaces – denoted collectively by (A)dS to cover both (negative)/positive cosmological constants Λ . Concepts synonymous in Minkowski geometry, such as masslessness, light cone propagation, maximal helicity modes only, and gauge invariance become nondegenerate, giving rise to such exotic effects as null propagation, partial (local) gauge invariances and shortened helicity multiplets, all with $m \neq 0$. Furthermore, entire ranges of mass become forbidden by unitarity. Perhaps most dramatically, accomodation of towers of excitations of all possible spins is only possible in the limit of vanishing Λ , thus providing a dramatic, if not quite yet physical, solution of the cosmological problem. All this happens because the dull flat space mass line is here replaced by a plane, parametrized by the dimensional duo (m^2, Λ) . I will also briefly mention the relation of all this to the old problem of the $m^2 \rightarrow 0$ limit discontinuity in matter-matter interactions [2] and its Newtonian counterpart [3].

Because my space is very limited, and most of the results are now available, I will only skim the highpoints just summarized. Indeed, let me devote a good fraction of this space to a single picture that summarizes many of these results.

¹Invited talk at Journées Relativistes, Dublin, 2001. Dedicated to the memory of Lochlain O’Raifeartaigh.
deser@brandeis.edu

Figure 1: The top/bottom halves of the half-plane represent dS/AdS (and also bosons/fermions) respectively. The $m^2 = 0$ vertical is the familiar massless helicity $\pm s$ system, while the other lines in dS represent truncated (bosonic) multiplets of partial gauge invariance: the lowest has no helicity zero, the next no helicities $(0, \pm 1)$, etc. Apart from these discrete lines, bosonic unitarity is preserved only in the region below the lowest line, namely that including flat space (the horizontal) and all of AdS. In the AdS sector, it is the topmost line that represents the pure gauge helicity $\pm s$ fermion, while the whole region below it, including the partially massless lines, is non-unitary. Thus, for fermions, only the region above the top line, including the flat space horizontal and all of dS, is allowed. Hence the overlap between permitted regions straddles the $\Lambda = 0$ horizontal and shrinks down to it as the spins in the tower of spinning particles grow; only $\Lambda = 0$ is allowed for generic (m^2 not growing as s^2) infinite towers.

In the remaining pages, I motivate these results in terms of a typical – and most familiar to relativists – system, namely spin 2, followed by an even briefer excursion into the fermion case. Consider then, the massive Pauli–Fierz system in a cosmological background, whose equations are

$$\mathcal{G}_{\mu\nu}(\phi) + m^2(\phi_{\mu\nu} - \bar{g}_{\mu\nu}\phi^\alpha_\alpha) = 0 . \quad (1)$$

Here all metrics, indices and covariant derivatives are those of the cosmological background, and $\phi_{\mu\nu}$ the spin 2 field. Thus $\mathcal{G}_{\mu\nu}$ is just the linearized deviation about (A)dS of the Einstein metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \phi_{\mu\nu}$; the minor ($[D_\mu, D_\nu] \sim \Lambda$) ambiguity in the order of the two covariant derivatives in $\mathcal{G}_{\mu\nu}$ is determined by requiring the order in which the Bianchi identity $\bar{D}_\mu \mathcal{G}^{\mu\nu}(\phi) \equiv 0$ holds.

Now let us proceed as in flat space and determine the consequences of taking successive divergences eq. (1). It follows directly that (of course)

$$\bar{D}_\mu \mathcal{G}^{\mu\nu} = -m^2 \bar{D}_\mu (\phi^{\mu\nu} - \bar{g}^{\mu\nu} \phi^\alpha_\alpha) , \quad (2)$$

but then comes the surprise

$$\bar{D}_\mu \bar{D}_\nu \mathcal{G}^{\mu\nu} + \frac{1}{2} m^2 \mathcal{G}^\mu_\mu = 3/2 m^2 (m^2 - 2/3 \Lambda) \phi^\alpha_\alpha . \quad (3)$$

[For higher spins, further divergences are possible and generate additional terms of the form $(m^2 - \alpha_i \Lambda)$ on the right side.] The first step, (2), is super-familiar: If $m^2 = 0$, the theory possesses the invariance $\delta\phi_{\mu\nu} = \bar{D}_\mu \phi_\nu + \bar{D}_\nu \phi_\mu$ and hence has the usual two degrees of freedom (DoF) corresponding to helicity ± 2 (a valid concept in (A)dS). If $m^2 \neq 0$ we expect the loss of gauge invariance to permit the 5 helicity states $(\pm 2, \pm 1, 0)$. But now look at (3): while it teaches us nothing further for $m^2 \equiv 0$, there appears a new zero, at $m^2 = 2\Lambda/3$. This is the first appearance of partial masslessness/gauge invariance. Partial masslessness because in fact this $m^2 \neq 0$ system nevertheless has null propagation just like the $m^2 = 0$ one (remember that (A)dS, being conformally flat, shares the usual Minkowski light cone). Gauge invariance because it is clear from (3) that the action leading to (1), namely $\frac{1}{2} \int \phi_{\mu\nu} [\mathcal{G}^{\mu\nu}(\phi) + m^2(\phi_{\mu\nu} - \bar{g}_{\mu\nu}\phi^\alpha_\alpha)]$ is invariant under the reduced, but still local gauge invariance

$$\delta\phi_{\mu\nu} = (\bar{D}_\mu \bar{D}_\nu + \bar{D}_\nu \bar{D}_\mu + 2\Lambda/3 \bar{g}_{\mu\nu}) \xi(x) , \quad (4)$$

since $\delta I[\phi] = \int \delta\phi_{\mu\nu} [\mathcal{G}^{\mu\nu} + m^2(\phi^{\mu\nu} - \bar{g}^{\mu\nu} \phi^\alpha_\alpha)]$. But a gauge invariance removes a degree of freedom, in our case that of helicity zero. This system was actually discovered earlier [4], as was its DoF content [5]. The actual mechanism of this amputation in a Hamiltonian analysis of the system makes for a very amusing (if messy) calculation, but the real surprise here is that, as one varies m^2 in the (m^2, Λ) plane in a given (Λ fixed) geometry, this excitation turns from being a normal one to vanishing, then reemerging as a ghost – thereby generating a unitarily forbidden region as shown in the top (dS) part of Fig. 1. Let me just write the relevant part of the helicity zero Hamiltonian to show how this works:

$$H \approx \frac{1}{2}[\nu^{-2}p^2 + \nu^2(\nabla q)^2] . \quad (5)$$

Clearly, if $\nu^2 \equiv (m^2 - 2\Lambda/3)$ is positive (as it is for $\Lambda = 0$) an obvious rescaling $(p, q) \rightarrow (\nu p, \nu^{-1}q)$ will give a normal action. However, in the region $\nu^2 < 0$, the other side of the (partial) gauge line, this can only be accomplished at the price of imaginary variables or negative energy. Thus, the $\nu^2 = 0$ line (where it is easy to see from a previous step that the whole helicity 0 action vanishes) serves as a divider between the good and forbidden regions of the (m^2, Λ) plane.

While the above interesting behavior for bosons takes place in dS ($\Lambda > 0$), fermions prefer AdS for reasons we do not yet understand. The simplest case is $s = 3/2$, which actually does not display the novel behavior; that starts with the tensor-spinor $\psi_{\mu\nu}$ of $s = 5/2$. Nevertheless, it illustrates another deep fact (known from the birth of supergravity), that when $\Lambda \neq 0$, gauge invariance for $s = 3/2$ is NOT displayed at $m=0$ (as it is for bosons) but rather at a finite mass parameter, and only in AdS ($\Lambda < 0$). Recall that the field equation reads

$$R^\mu \equiv \gamma^{\mu\nu\alpha} \mathcal{D}_\nu \psi_\alpha = 0 , \quad \mathcal{D}_\nu \equiv D_\nu + \frac{1}{2} \gamma_\nu m \quad (6)$$

and that the gauge invariant – hence truly “massless” – system occurs at $\mathcal{D}_\mu R^\mu = 0$, namely for $m^2 + \frac{1}{3} \Lambda = 0$, where $[\mathcal{D}_\mu, \mathcal{D}_\nu] \psi_\alpha \equiv 0$. The helicity $\pm 1/2$ component of the system mimics the behavior of helicity 0 for spin 2, but in the “inverted”, AdS, sector: Above the gauge line it is present and unitary, below it it is non-unitary and is of course absent entirely at $m^2 + \frac{1}{3} \Lambda = 0$. Put another way, the additional difference is that, unlike bosons, where $s = 1$ has only one index and so no novel behavior, already at $s = 3/2$ (and beyond) “true” masslessness, *i.e.*, complete (not partial) gauge invariance occurs not at $m = 0$, but rather, generically, at $m^2 = -|\alpha|\Lambda$. This is due to the spinor part of the spinor-vector (or -tensor) field, and was already discovered by Dirac very long ago for $s = 1/2$. Essentially, when one squares the Dirac equation, $(\gamma \cdot D + m)(\gamma \cdot D - m)\psi = 0$, there is a residue proportional to $(m^2 + |\alpha|\Lambda)$ due to the famous identity $(\gamma \cdot D)^2 \equiv D^2 + R/4$. That one is totally separate from and just adds to the partial mass effect from the world indices.

To complete our navigation of Fig. 1, we note first that partial gauge systems defined by the transition lines correspond to truncated multiplets from which one or more of the lowest helicities are missing, but that in each case, the remaining (higher than 0 or $\pm 1/2$) helicities all propagate on-cone. Secondly, the gauge lines for generic spin are governed respectively by $m_B^2 \sim \frac{1}{3} \Lambda s(s-1)$ and $m_F^2 \sim -\frac{1}{3} \Lambda(s - \frac{1}{2})^2$. Hence if one considers towers of rising spin particles of both statistics, and if their masses are not tuned to rise with spin (or at least not as fast as s^2) then since the allowed, unitary, region common to bosons and fermions spans a cone around $\Lambda = 0$, our infinite tower (such as found in zero slope string expansions) is only permitted at $\Lambda \rightarrow 0$ in the limit. This mechanism, while simple and appealing, is not necessarily robust under more physical circumstances, such as presence of dynamical gravity and of other interactions. In this connection, we mention that consistent coupling of (even massive) higher spin fields to gravity, is hard to achieve beyond supergravity. This and the additional problems for charged as well as gravitating systems has recently been investigated systematically in the last paper of [1].

Finally, we turn to the question that triggered some of our studies, namely the so-called vDVZ discontinuities of linearized massive gravity and spin 3/2 interacting with – necessarily prescribed – conserved matter sources [2]. Spin 2, in flat space, for example, when coupled to a conserved external stress tensor differs from its spin 1 counterpart in that the helicity 0 mode fails to decouple from the source in the massless limit, with the result that there is a large finite difference in the prediction of light bending in the $m^2 \rightarrow 0$ and $m^2 \equiv 0$ cases once they are both fixed to give the proper Newtonian gravity limit, *i.e.*, the effective coupling differ in the ratio of $t_\mu^\mu T_\nu^\nu$ to $t_{\mu\nu} T^{\mu\nu}$ couplings between two sources, being 1/3 vs 1/2 respectively. When $\Lambda \neq 0$, however, these ratios depend on both (m^2, Λ) parameters and can have almost any value depending on the path taken to (0,0). It is also instructive [3] to discuss what happens in the Newtonian ($c \rightarrow \infty$) limit. Here matter essentially consists of only T_0^0 and $T_\mu^\mu \sim T_0^0 \neq 0$; there should be no discontinuity since there is no light whose bending is to be explained. Indeed, one obtains a result that correctly reflects this physical expectation, but only after realizing that apparent wrong – repulsive or vanishing or infinite Newtonian couplings are excluded either by the nonunitarity of the offending “gravitons”, or by the truncated multiplet case which, by the invariance (4), can only couple to traceless conserved $T^{\mu\nu}$ sources.

I thank A. Waldron and B. Tekin for stimulating collaborations. This work was supported by NSF Grant PHY99-73935.

References

- [1] S. Deser and A. Waldron, Phys. Rev. Lett. **87**, 031601 (2001); S. Deser and A. Waldron, Nucl. Phys. **B607**, 577–604 (2001); S. Deser and A. Waldron, Phys. Lett. **B508**, 347–353 (2001); S. Deser and A. Waldron, Phys. Lett. **B513**, 137–141 (2001).
- [2] S. Deser and A. Waldron, Phys. Lett. Phys.Lett. **B501**, 134–139 (2001) for a history.
- [3] S. Deser and B. Tekin, Class. Quant. Grav. **18**, L171 (2001).
- [4] S. Deser and R. Nepomechie, Phys. Lett. **B132**, 321 (1983); Ann. Phys. **154**, 396 (1984).
- [5] A. Higuchi, Nucl. Phys. **B282**, 397 (1987); *ibid.* **325**, 745 (1989); J. Math. Phys. **28**, 1553 (1987).