SINGLE PHASED BURST ERROR CORRECTING ARRAY CODES

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Abstract

Array codes composed of row and column parities with a diagonally cyclic readout order are capable of correcting a single burst error along one diagonal. A new equation which defines permissible array sizes is presented. These codes have an optimal size which is shown to be a number theoretic problem. In addition, correction of approximate errors is presented; this can be generalized for many classes of error correcting codes.

Size Limits

An array code was introduced previously to correct phased burst errors. These codes are capable of correcting single phased burst errors and have the added benefits of high speed and efficiency. The code is constructed of a two-dimensional array of cells with row and column parity checks, including a check on checks to make the array a complete rectangle. Readout order is along diagonals taken cyclically [1][2][3].

These codes were also shown to have only certain sizes determined by the array dimensions through a geometric argument. In particular, the code was shown to correct a single phased burst given a shorter dimension n_1, $n_2 > n_1$, for all $n_2 \geq 2n_1$; in the range $n_1 > n_2 > 2n_1$, the code can correct a phased burst if and only if

$$n_2 \neq \frac{\alpha+1}{\alpha}(n_1 - \beta) \tag{1}$$

for all $\alpha \geq 1$ and $\beta \geq 1$ [4].

An algebraic approach provides another equation describing this same limitation: Given n_2, the largest value of n_1 which will still yield a code capable of correcting errors along a single diagonal of length n_1 is

$$n_1 \leq n_2\Big(1 - \frac{1}{p}\Big) \tag{2}$$

where p is the smallest prime factor of n_2. This condition is necessary and sufficient.

Optimal Sizes

From (1) and (2), it is easily shown that the Reiger bound can be met with equality when n_2 is prime. In this case, $n_1 = n_2 - 1$, and the code is capable of exceeding the speed while matching the error correcting performance of single error correcting Reed-Solomon codes of length n_2 with n_1 bits per symbol. Given a fixed n_1, however, finding the best (smallest) n_2 is a much more difficult problem. It can be shown that if there exists a prime between n_1 and $n_1 + \sqrt{n_1}$, then for all n_1, the best n_2 is the smallest prime number which is larger than n_1. This has been confirmed experimentally for all numbers $n_2 < 10,000,000$; since most applications require codewords of much smaller dimensions, this is true for all practical cases. To prove this in general, however, is a nontrivial and unproven number theoretic problem [5][6].

Approximate Errors

In some applications involving q-ary symbols, the most common error is not one where any random value is substituted for the actual one; rather, the erroneous value is often close to the true value. These types of errors will be called approximate errors here. Applications of these errors include high speed modems, multi-valued random access memories, and analog signal coding.

Phased burst approximate errors can be corrected easily with array codes. If the error is assumed to be no more than Δ away from the true value, then by taking the horizontal parities modulo $2\Delta + 1$ and the vertical parities modulo $\Delta + 1$, these errors can be corrected. These parity values can be combined—*i.e.* weighted and added to use fewer symbols—to obtain codes with yet higher rates.

This concept of taking some parity cells modulo $\Delta + 1$ and others modulo $2\Delta + 1$ and packing them into fewer symbols can be generalized to other codes. These will yield higher rate codes without the complexities of bit-sliced coding, another strategy by which approximate errors can be corrected.

Conclusion

We have presented a new equation describing size limitations of phased burst error correcting array codes. The optimal code size has a longer side which is the smallest prime which is larger than the shorter dimension for all practical applications; to prove this in general is a number theoretic problem. In addition, a means to correct approximate errors has been presented; this approach can be generalized for other codes.

References

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