

# Prevalence of Earth-size planets orbiting Sun-like stars

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**Determining whether Earth-like planets are common or rare looms as a touchstone in the question of life in the universe. We searched for Earth-size planets that cross in front of their host stars by examining the brightness measurements of 42,000 stars from National Aeronautics and Space Administration's *Kepler* mission. We found 603 planets, including 10 that are Earth size ( $1-2 R_{\oplus}$ ) and receive comparable levels of stellar energy to that of Earth ( $0.25-4 F_{\oplus}$ ). We account for *Kepler*'s imperfect detectability of such planets by injecting synthetic planet-caused dimmings into the *Kepler* brightness measurements and recording the fraction detected. We find that  $11 \pm 4\%$  of Sun-like stars harbor an Earth-size planet receiving between one and four times the stellar intensity as Earth. We also find that the occurrence of Earth-size planets is constant with increasing orbital period ( $P$ ), within equal intervals of  $\log P$  up to  $\sim 200$  d. Extrapolating, one finds  $5.7^{+1.7}_{-2.2}\%$  of Sun-like stars harbor an Earth-size planet with orbital periods of 200–400 d.**

extrasolar planets | astrobiology

The National Aeronautics and Space Administration's (NASA's) *Kepler* mission was launched in 2009 to search for planets that transit (cross in front of) their host stars (1–4). The resulting dimming of the host stars is detectable by measuring their brightness, and *Kepler* monitored the brightness of 150,000 stars every 30 min for 4 y. To date, this exoplanet survey has detected more than 3,000 planet candidates (4).

The most easily detectable planets in the *Kepler* survey are those that are relatively large and orbit close to their host stars, especially those stars having lower intrinsic brightness fluctuations (noise). These large, close-in worlds dominate the list of known exoplanets. However, the *Kepler* brightness measurements can be analyzed and debiased to reveal the diversity of planets, including smaller ones, in our Milky Way Galaxy (5–7). These previous studies showed that small planets approaching Earth size are the most common, but only for planets orbiting close to their host stars. Here, we extend the planet survey to *Kepler*'s most important domain: Earth-size planets orbiting far enough from Sun-like stars to receive a similar intensity of light energy as Earth.

## Planet Survey

We performed an independent search of *Kepler* photometry for transiting planets with the goal of measuring the underlying occurrence distribution of planets as a function of orbital period,  $P$ , and planet radius,  $R_p$ . We restricted our survey to a set of Sun-like stars (GK type) that are the most amenable to the detection of Earth-size planets. We define GK-type stars as those with surface temperatures  $T_{\text{eff}} = 4,100\text{--}6,100$  K and gravities  $\log g = 4.0\text{--}4.9$  ( $\log g$  is the base 10 logarithm of a star's surface gravity measured in  $\text{cm s}^{-2}$ ) (8). Our search for planets was further restricted to the brightest Sun-like stars observed by *Kepler* ( $Kp = 10\text{--}15$  mag). These 42,557 stars (Best42k) have the lowest photometric noise, making them amenable to the detection of Earth-size planets. When a planet crosses in front of its star, it causes a fractional dimming that is proportional to the fraction of the stellar disk blocked,  $\delta F = (R_p/R_*)^2$ , where  $R_*$  is the radius of the star. As viewed by a distant observer, the Earth dims the Sun by  $\sim 100$  parts per million (ppm) lasting 12 h every 365 d.

We searched for transiting planets in *Kepler* brightness measurements using our custom-built TERRA software package described in previous works (6, 9) and in *SI Appendix*. In brief, TERRA conditions *Kepler* photometry in the time domain, removing outliers, long timescale variability ( $>10$  d), and systematic errors common to a large number of stars. TERRA then searches for transit signals by evaluating the signal-to-noise ratio (SNR) of prospective transits over a finely spaced 3D grid of orbital period,  $P$ , time of transit,  $t_0$ , and transit duration,  $\Delta T$ . This grid-based search extends over the orbital period range of 0.5–400 d.

TERRA produced a list of “threshold crossing events” (TCEs) that meet the key criterion of a photometric dimming SNR ratio  $\text{SNR} > 12$ . Unfortunately, an unwieldy 16,227 TCEs met this criterion, many of which are inconsistent with the periodic dimming profile from a true transiting planet. Further vetting was performed by automatically assessing which light curves were consistent with theoretical models of transiting planets (10). We also visually inspected each TCE light curve, retaining only those exhibiting a consistent, periodic, box-shaped dimming, and rejecting those caused by single epoch outliers, correlated noise, and other data anomalies. The vetting process was applied homogeneously to all TCEs and is described in further detail in *SI Appendix*.

To assess our vetting accuracy, we evaluated the 235 *Kepler* objects of interest (KOIs) among Best42k stars having  $P > 50$  d, which had been found by the *Kepler* Project and identified as planet candidates in the official Exoplanet Archive ([exoplanetarchive.ipac.caltech.edu](http://exoplanetarchive.ipac.caltech.edu); accessed 19 September 2013). Among them, we found four whose light curves are not consistent with being planets. These four KOIs (364.01, 2,224.02, 2,311.01, and 2,474.01) have long periods and small radii (*SI Appendix*). This exercise suggests that our vetting process is robust and that careful scrutiny of the light curves of small planets in long period orbits is useful to identify false positives.

## Significance

**A major question is whether planets suitable for biochemistry are common or rare in the universe. Small rocky planets with liquid water enjoy key ingredients for biology. We used the National Aeronautics and Space Administration *Kepler* telescope to survey 42,000 Sun-like stars for periodic dimmings that occur when a planet crosses in front of its host star. We found 603 planets, 10 of which are Earth size and orbit in the habitable zone, where conditions permit surface liquid water. We measured the detectability of these planets by injecting synthetic planet-caused dimmings into *Kepler* brightness measurements. We find that 22% of Sun-like stars harbor Earth-size planets orbiting in their habitable zones. The nearest such planet may be within 12 light-years.**

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Data deposition: The *Kepler* photometry is available at the Mikulski Archive for Space Telescopes ([archive.stsci.edu](http://archive.stsci.edu)). All spectra are available to the public on the Community Follow-up Program website ([cfop.ipac.caltech.edu](http://cfop.ipac.caltech.edu)).

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Vetting of our TCEs produced a list of 836 eKOIs, which are analogous to KOIs produced by the *Kepler* Project. Each light curve is consistent with an astrophysical transit but could be due to an eclipsing binary (EB), either in the background or gravitationally bound, instead of a transiting planet. If an EB resides within the software aperture of a *Kepler* target star (within  $\sim 10$  arcsec), the dimming of the EB can masquerade as a planet transit when diluted by the bright target star. We rejected as likely EBs any eKOIs with these characteristics: radii larger than  $20 R_{\oplus}$ , observed secondary eclipse, or astrometric motion of the target star in and out of transit (*SI Appendix*). This rejection of EBs left 603 eKOIs in our catalog.

*Kepler* photometry can be used to measure  $R_p/R_*$  with high precision, but the extraction of planet radii is compromised by poorly known radii of the host stars (11). To determine  $R_*$  and  $T_{\text{eff}}$ , we acquired high-resolution spectra of 274 eKOIs using the HIRES spectrometer on the 10-m Keck I telescope. Notably, we obtained spectra of all 62 eKOIs that have  $P > 100$  d. For these stars, the  $\sim 35\%$  errors in  $R_*$  were reduced to  $\sim 10\%$  by matching spectra to standards.

To measure planet occurrence, one must not only detect planets but also assess what fraction of planets were missed. Missed planets are of two types: those whose orbital planes are so tilted as to avoid dimming the star and those whose transits were not detected in the photometry by TERRA. Both effects can be quantified to establish a statistical correction factor. The first correction can be computed as the geometrical probability that an orbital plane is viewed edge-on enough (from Earth) that the planet transits the star. This probability is  $P_T = R_*/a$ , where  $a$  is the semimajor axis of the orbit.

The second correction is computed by the injection and recovery of synthetic (mock) planet-caused dimmings into real *Kepler* photometry. We injected 40,000 transit-like synthetic dimmings having randomly selected planetary and orbital properties into the actual photometry of our Best42k star sample, with stars selected at random. We measured survey completeness,  $C(P, R_p)$ , in small bins of  $(P, R_p)$ , determining the fraction of injected synthetic planets that were discovered by TERRA (*SI Appendix*). Fig. 1 shows the 603 detected planets and the survey completeness,  $C$ , color-coded as a function of  $P$  and  $R_p$ .

The survey completeness for small planets is a complicated function of  $P$  and  $R_p$ . It decreases with increasing  $P$  and decreasing

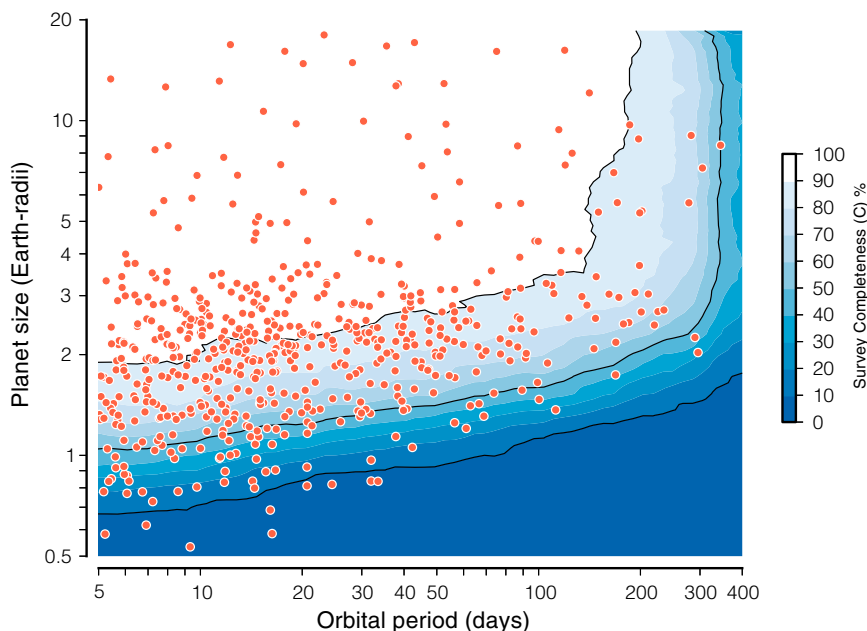
$R_p$  as expected due to fewer transits and less dimming, respectively. It is dangerous to replace this injection and recovery assessment with noise models to determine  $C$ . Such models are not sensitive to the absolute normalization of  $C$  and only provide relative completeness. Models also may not capture the complexities of a multistage transit-finding pipeline that is challenged by correlated, nonstationary, and non-Gaussian noise. Measuring the occurrence of small planets with long periods requires injection and recovery of synthetic transits to determine the absolute detectability of the small signals buried in noise.

## Planet Occurrence

We define planet occurrence,  $f$ , to be the fraction of stars having a planet within a specified range of orbital period, size, and perhaps other criteria. We report planet occurrence as a function of planet size and orbital period,  $f(P, R_p)$  and as a function of planet size and the stellar light intensity (flux) incident on the planet,  $f(F_p, R_p)$ .

**Planet Occurrence and Orbital Period.** We computed  $f(P, R_p)$  in a  $6 \times 4$  grid of  $P$  and  $R_p$  shown in Fig. 2. We start by first counting the number of detected planets,  $n_{\text{cell}}$ , in each  $P$ - $R_p$  cell. Then we computed  $f(P, R_p)$  by making statistical corrections for planets missed because of nontransiting orbital inclinations and because of the completeness factor,  $C$ . The first correction augments each detected transiting planet by  $1/P_T = a/R_*$ , where  $P_T$  is the geometric transit probability, to account for planets missed in inclined orbits. Accounting for the completeness,  $C$ , the occurrence in a cell is  $f(P, R_p) = 1/n_* \sum_i a_i / (R_{*i} C_i)$ , where  $n_* = 42,557$  stars, and the sum is over all detected planets within that cell. Uncertainties in the statistical corrections for  $a/R_*$  and for completeness may cause errors in the final occurrence rates of  $\sim 10\%$ . Such errors will be smaller than the Poisson uncertainties in the occurrence of Earth-size planets in long period orbits.

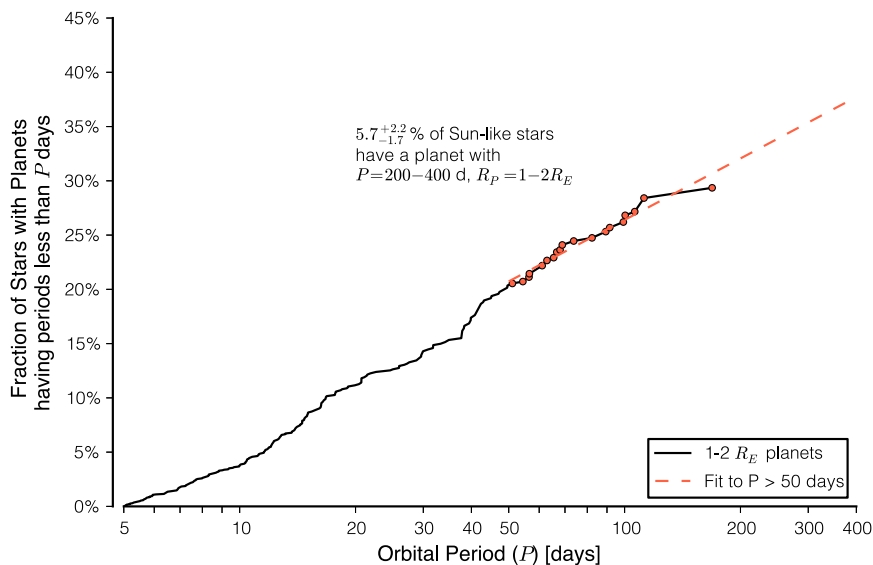
Fig. 2 shows the occurrence of planets,  $f(P, R_p)$ , within the  $P$ - $R_p$  plane. Each cell is color-coded to indicate the final planet occurrence: the fraction of stars having a planet with radius and orbital period corresponding to that cell (after correction for both completeness factors). For example,  $7.7 \pm 1.3\%$  of Sun-like stars have a planet with periods between 25 and 50 d and sizes between 1 and  $2 R_{\oplus}$ .



**Fig. 1.** 2D domain of orbital period and planet size, on a logarithmic scale. Red circles show the 603 detected planets in our survey of 42,557 bright Sun-like stars ( $K_p = 10$ –15 mag, GK spectral type). The color scale shows survey completeness measured by injection and recovery of synthetic planets into real photometry. Dark regions represent  $(P, R_p)$  with low completeness,  $C$ , where significant corrections for missed planets must be made to compute occurrence. The most common planets detected have orbital  $P < 20$  d and  $R_p \approx 1$ – $3 R_{\oplus}$  (at middle left of graph). However, their detectability is favored by orbital tilt and detection completeness,  $C$ , that favors detection of such close-in, large planets.







**Fig. 5.** The fraction of stars having nearly Earth-size planets ( $1-2 R_{\oplus}$ ) with any orbital period up to a maximum period,  $P$ , on the horizontal axis. Only planets of nearly Earth size ( $1-2 R_{\oplus}$ ) are included. This cumulative distribution reaches 20.2% at  $P = 50$  d, meaning 20.4% of Sun-like stars harbor a  $1-2 R_{\oplus}$  planet with an orbital period,  $P < 50$  d. Similarly, 26.2% of Sun-like stars harbor a  $1-2 R_{\oplus}$  planet with a period of  $P < 100$  d. The linear increase in this cumulative quantity corresponds to planet occurrence that is constant in equal intervals of  $\log P$ . One may perform a modest extrapolation into the  $P = 200-400$  d range, equivalent to assuming constant occurrence per  $\log P$  interval, using all planets with  $P > 50$  d. Such an extrapolation predicts that  $5.7^{+2.2}_{-1.7}$  % of Sun-like stars have a planet with size  $1-2 R_{\oplus}$ , with an orbital period between  $P = 200$  and  $400$  d.

analog planets beyond what is reported here. Indeed, low survey sensitivity to Earth analogs was the primary reason behind a 4-y extension to the *Kepler* mission. Modest extrapolation is required to understand the prevalence of Earth-size planets with Earth-like orbits around Sun-like stars.

We offer empirical and theoretical justification for extrapolation out to 400 d. As shown in Fig. 2, the prevalence of small planets as a function of  $\log P$  is remarkably uniform. To test the reliability of our extrapolation into a region of low completeness, we used the same technique to estimate occurrence in more complete regions of phase space and compared the results with our measured occurrence values.

Again, assuming uniform occurrence per  $\log P$  interval, extrapolating the occurrence of  $2-4 R_{\oplus}$  planets from 50 to 200 d orbits out to 400 d predicts  $6.4^{+0.5}_{-1.2}$  % occurrence of planets in the domain of  $R_p = 2-4 R_{\oplus}$  and  $P = 200-400$  d. The extrapolation is consistent with the measured value of  $5.0 \pm 2.1$  % to within  $1\sigma$  uncertainty. Furthermore, extrapolation based on  $1-2 R_{\oplus}$  planets with  $P = 12.5-50$  d predicts  $6.5^{+0.9}_{-1.7}$  % occurrence within the  $P = 50-100$  d bin. Again, the measured value of  $5.8 \pm 1.6$  % agrees to better than  $1\sigma$ .

Although planet size is governed by nonlinear processes such as runaway gas accretion (13), which favors certain planet sizes over others, no such nonlinear processes occur as a function of orbital period in the range of 200–400 d. Extrapolation out to orbital periods of 200–400 d, although dangerous, seems unlikely to be unrealistic by more than a factor of 2.

**Earth-Size Planets in the Habitable Zone.** Although the details of planetary habitability are debated and depend on planet-specific properties as well as the stochastic nature of planet formation (14), the habitable zone (HZ) is traditionally defined as the set of planetary orbits that permit liquid water on the surface. The precise inner and outer edges of the HZ depend on details of the model (15–18). For solar analog stars, Zsom et al. (17) estimated that the inner edge of the HZ could reside as close as 0.38 astronomical unit (AU) for planets having either a reduced greenhouse effect due to low humidity or a high reflectivity. One AU is the average distance between the Earth and Sun and is equal to  $1.50 \times 10^{11}$  m. Pierrehumbert and Gaidos (18) estimated that the outer edge of

the HZ may extend up to 10 AU for planets that are kept warm by efficient greenhouse warming with an  $H_2$  atmosphere.

A planet’s ability to retain surface liquid water depends, in large part, on the energy received from its host star. We consider a planet to reside in the HZ if it is bathed in a similar level of starlight as Earth. One may adopt  $F_p = 0.25-4 F_{\oplus}$  as a simple definition of the HZ, which corresponds to orbital separations of 0.5–2.0 AU for solar analog stars. This definition is more conservative than the range of published HZ boundaries that extend from 0.38 to 10 AU (17, 18). This HZ includes Venus (0.7 AU) and Mars (1.5 AU), which do not currently have surface liquid water. However, Venus may have had liquid water in its past, and there is strong geochemical and geomorphological evidence of liquid water earlier in Mars’ history (14).

Previously, we showed that  $11 \pm 4$  % of stars harbor a planet having an  $R_p = 1-2 R_{\oplus}$  and  $F_p = 1-4 F_{\oplus}$ . Using the definition of  $F_p$  and Kepler’s third law,  $F_p$  is proportional to  $P^{-4/3}$ . Therefore, uniform occurrence in  $\log P$  translates to uniform occurrence in  $\log F_p$ . We find that the occurrence of  $1-2 R_{\oplus}$  planets is constant per  $\log F_p$  interval for  $F_p = 100-1 F_{\oplus}$ . If one was to adopt  $F_p = 0.25-4 F_{\oplus}$  as the HZ and extrapolate from the  $F_p = 1-4 F_{\oplus}$  domain, then the occurrence of Earth-size planets in the HZ is  $22 \pm 8$  % for Sun-like stars.

**Table 1. Occurrence of small planets in the habitable zone**

HZ definition	$a_{inner}$	$a_{outer}$	$F_{p,inner}$	$F_{p,outer}$	$f_{HZ}$ (%)
Simple	0.5	2	4	0.25	22
Kasting (1993)	0.95	1.37	1.11	0.53	5.8
Kopparapu et al. (2013)	0.99	1.70	1.02	0.35	8.6
Zsom et al. (2013)	0.38		6.92		26*
Pierrehumbert and Gaidos (2011)		10		0.01	$\sim 50^{\dagger}$

\*Zsom et al. (17) studied the inner edge of the habitable zone. Here we adopt an outer edge of 2 AU from the Simple model.

<sup>†</sup>Pierrehumbert and Gaidos (18) studied the outer edge of the habitable zone. Here we adopt an inner edge of 0.5 AU from the Simple model. Extrapolation out to 10 AU is severely underconstrained. This estimate is highly uncertain and is included for completeness.

One may adopt alternative definitions of both the properties of Earth-size planets and the domain of the HZ. We showed previously that the occurrence of planets is approximately constant as a function of both  $R_P$  (for  $R_P < 2.8 R_\oplus$ ) and  $P$  (in logarithmic intervals). Thus, the occurrence of planets in this domain is proportional to logarithmic area in the  $R_P$ - $F_P$  parameter space being considered. For example, the occurrence of planets of size  $1.0$ – $1.4 R_\oplus$  in orbits that receive  $0.25$ – $1.0 F_\oplus$  in stellar flux is  $22\%/4 = 5.5\%$ . We offer a number of estimates for the prevalence of Earth-size planets in the HZ based on different published definitions of the HZ in Table 1.

Cooler, M dwarf stars also have a high occurrence of Earth-size planets. Based on the *Kepler* planet catalog, Dressing et al. (19) found that  $15^{+15}_{-6}\%$  of early M dwarfs have an Earth-size planet ( $0.5$ – $1.4 R_\oplus$ ) in the HZ using a conservative definition of  $0.5$ – $1.1 F_\oplus$  (15) and three times that value when the HZ is expanded to  $0.25$ – $1.5 F_\oplus$  (20). This result is consistent with a Doppler survey that found that  $41^{+54}_{-13}\%$  of nearby M dwarfs have planets with masses  $1$ – $10$  Earth masses ( $M_\oplus$ ) in the HZ (21). Thus, Earth-size planets appear to be common in the HZs of a range of stellar types.

## Conclusions

Using *Kepler* photometry of Sun-like stars (GK-type), we measured the prevalence of planets having different orbital periods and sizes, down to the size of the Earth and out to orbital periods of 1 y. We gathered Keck spectra of all host stars of planets having periods greater than 100 d to accurately determine their radii. The detection of planets with periods longer than 100 d is challenging, and we characterized our sensitivity to such planets by using injection and recovery of synthetic planets in the photometry. After correcting for orbital tilt and detection completeness, we find that  $26 \pm 3\%$  of Sun-like stars have an Earth-size ( $1$ – $2 R_\oplus$ ) planet with  $P = 5$ – $100$  d. We also find that  $11 \pm 4\%$  of Sun-like stars harbor an Earth-size planet that receives nearly Earth levels of stellar energy ( $F_P = 1$ – $4 F_\oplus$ ).

We showed that small planets far outnumber large ones. Only  $1.6 \pm 0.4\%$  of Sun-like stars harbor a Jupiter-size ( $8$ – $16 R_\oplus$ ) planet with  $P = 5$ – $100$  d compared with a  $23 \pm 3\%$  occurrence of Earth-size planets. This pattern supports the core accretion scenario in which planets form by the accumulation of solids first and gas later in the protoplanetary disk (13, 22–24). The details of this family of models are hotly debated, including the movement of material within the disk, the timescale for planet formation, and the amount of gas accretion in small planets.

Our measurement of a constant occurrence of  $1$ – $2.8 R_\oplus$  planets per  $\log P$  interval establishes an important observational constraint for these models.

The occurrence of Earth-size planets is constant with decreasing stellar light intensity from  $100 F_\oplus$  down to  $1 F_\oplus$ . If one was to assume that this pattern continues down to  $0.25 F_\oplus$ , then the occurrence of planets having flux levels of  $1$ – $0.25 F_\oplus$  is also  $11 \pm 4\%$ .

Earth-size planets are common in the *Kepler* field. If the stars in the *Kepler* field are representative of stars in the solar neighborhood, then Earth-size planets are common around nearby Sun-like stars. If one were to adopt a  $22\%$  occurrence rate of Earth-size planets in habitable zones of Sun-like stars, then the nearest such planet is expected to orbit a star that is less than 12 light-years from Earth and can be seen by the unaided eye. Future instrumentation to image and take spectra of these Earths need only observe a few dozen nearby stars to detect a sample of Earth-size planets residing in the HZs of their host stars.

**Note Added in Proof.** Estimates of the occurrence of Earth analog planets appear in several previous works including Catanzarite and Shao (25), Traub (26), and Dong and Zhu (27). These estimates, which range from  $1\%$  to  $34\%$ , were built upon early catalogs of *Kepler* planet candidates (based on less than 1.3 years of photometry). These estimates did not address survey completeness with injection and recovery or uncertain stellar radii with spectroscopy.

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