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Editorial

Fractionalized quantum excitations in correlated two-dimensional topological phases



The research field of topological phases and materials is one of the most exciting frontiers in modern condensed matter physics [1–7]. In contrast to the classification of conventional condensed matter phases according to their symmetries, different topological states of matter generally do not involve broken symmetries. Rather, they are associated with certain fundamental physical properties that are insensitive to smooth changes in materials parameters. Therefore, a topological state cannot change into another unless the system undergoes a quantum phase transition [6,7].

The quantum Hall effect (QHE) discovered in 1980 is the first known example of distinct topological orders that are associated with different quantized values of the Hall conductance in two-dimensional electron gas without breaking any symmetry [8]. The subsequent discovery of fractional quantum Hall effect (FQHE) in 1982 [9] and the accompanying theoretical studies [10–13] further opened up a new paradigm of topological orders with strong many-body interactions and fractionalized quantum excitations. These novel fractionalized topological excitations not only provide a rich ground for the studies of topological quantum field theory [2] but are also promising for the realization of topological quantum computation and quantum information technology [3,4,14]. Since 2008, various classes of new topological systems such as the quantum spin Hall insulators, [15,16] topological insulators, [17–22] topological superconductors, [7,23–26] Weyl semi-metals, [27,28] and topological crystalline insulators [29–33] have been theoretically predicted and experimentally verified, which further generated much excitement in the research field of topological phases and materials.

One of the most intriguing topics in the studies of topological states is the classification of fractionalized quantum excitations, known as the anyons, in gapped two-dimensional topological systems [3–5]. Such studies are not only of fundamental interest in condensed matter physics but also relevant to the development of quantum information science, because topologically fractionalized excitations can be used to store quantum information non-locally and to provide topologically protected quantum computation [3,4,14]. Moreover, studies of these fractionalized excitations from correlated topological phases have brought about new understanding of the patterns of long-range quantum entanglements in many-body systems [34,35]. In this context, the article entitled “Symmetry fractionalization in two-dimensional topological phases” by Xie Chen is a timely review of recent theoretical developments in the classification of symmetry fractionalization (SF) patterns under given topological orders and global symmetries [36]. Specifically, it is shown that a SF pattern can only be realized if it is consistent with the anion fusion rules. Furthermore, among all the possible SF patterns, some are “non-anomalous” and can be realized in strictly two-dimensional models, whereas others are “anomalous” and can only be realized on the surface of three-dimensional systems as a reflection of the nontrivial topological order in the bulk. Examples of such manifestations in three-dimensional topological insulators, topological superconductors and other symmetry-protected topological orders are reviewed. Methods that can be used to distinguish the anomalous patterns from the non-anomalous patterns are also discussed. Finally, important open questions such as identifying the SF patterns in three-dimensional topological phases and understanding how symmetry fractionalizes on gapless correlated topological systems are summarized.

Overall, the article by Chen provides an interesting perspective of recent theoretical developments in the research of interacting two-dimensional topological phases. Given the relevance of fractionalized topological excitations to topological quantum science and strongly interacting topological systems, eventual experimental detections of SF beyond the FQHE are of paramount importance to the advances of the field. In light of recent progress in the development of topological materials as the result of the ingenuity and diligence of researchers worldwide, we can be cautiously optimistic that new fractionalized topological excitations may become empirically realized in the near future.

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