Inferences about supernova physics from gravitational-wave measurements: GW151226 spin misalignment as an indicator of strong black-hole natal kicks

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The inferred parameters of the binary black hole GW151226 are consistent with nonzero spin for the most massive black hole, misaligned from the binary's orbital angular momentum. If the black holes formed through isolated binary evolution from an initially aligned binary star, this misalignment would then arise from a kick imparted to the first-born black hole at its birth during stellar collapse. We use simple kinematic arguments to constrain the characteristic magnitude of this kick, and find that a natal kick $v_k \gtrsim 50$ km/s must be imparted to the black hole at birth to produce misalignments consistent with GW151226. This large natal kick would be difficult to explain within conventional supernova theory. Primordial spin misalignment may be necessary to explain current and future gravitational wave observations.

Introduction– The Laser Interferometer Gravitational Wave Observatory (LIGO) has reported the confident discovery of two binary black holes (BHs): GW150914 and GW151226 [1]. The masses and inferred birthrate of these events are surprisingly consistent with prior predictions [2–6], derived by assuming these objects form from the evolution of isolated pairs of stars; see, e.g., [7]. At this early stage, observations cannot firmly distinguish between this formation channel and other proposed alternatives, such as the formation of binary BHs in densely interacting clusters [8] or as primordial BHs [9]. If, however, binary BHs do form from isolated binary evolution, then precise measurements of their properties will provide unique clues into how BHs and massive stars evolve.

Assuming BH binaries form from initially aligned binary stars (i.e., all angular momenta are parallel), the most likely processes that can misalign their spin angular momenta are the linear momentum recoils imparted when a BH's progenitor star ends its life in a supernova (SN) [10, 11]. Observations strongly suggest asymmetries in the SN process can indeed impart strong kicks to newly formed compact objects. Based on the proper motion measurements of pulsars in the Milky Way, it is believed SNe can impart velocities as high as $v_k \sim 450$ km/s to neutron stars [12]. Conversely, the occurrence of natal kicks onto BHs is less clear. On the one hand, observations of galactic X-ray binaries suggest BH natal kicks may be as large as hundreds of km/s [13–16]. On the other hand, kicks onto heavier BHs could be significantly reduced, as their very massive progenitor stars are expected to undergo prompt collapse and not eject enough material to enable strong recoils (see, e.g., [17] and references therein). Measurements of natal kicks through electromagnetic observations have already been proved crucial to understand the physics of SNe. For instance, if BH kicks are indeed as large as those imparted to neutron stars, this would require large-scale asymmetries of the SN ejecta, which in turn significantly delay BH formation [18, 19].

Gravitational wave (GW) measurements of merging binary BHs have the potential to provide crucial insights on this issue. SN kicks can reach (or even exceed) the expected orbital velocities of the stellar binary from which binary BHs formed with dramatic effects on its formation and evolution. Strong natal kicks disrupt many potential compact binary progenitors (thus affecting the expected GW rates [2, 20]) and drastically tilt the orbital plane of the few that survive (which greatly affects the spin precession dynamics by the time the source becomes visible in LIGO [10, 11]). Several previous studies have demonstrated that the GW signature of BH spin-orbit misalignments can be efficiently identified [21–24] and used to distinguish between formation channels [10, 25, 26]. We point out two notable examples. First, LIGO provides strong constraints on a quantity that is both nearly conserved on astrophysical timescales [27–29] and of key astrophysical interest: the effective spin $\chi_{\rm eff} = \hat{\mathbf{L}} \cdot (\mathbf{S}_1/m_1 + \mathbf{S}_2/m_2)/(m_1 + m_2)$, where $m_{1,2}$ and $\mathbf{S}_{1,2}$ are the masses and spins of the component BHs, and $\mathbf{L}$ is the binary's orbital angular momentum (we used natural units $G = c = 1$). BH binaries assembled in densely interacting environments have random spin orientations and thus $\chi_{\rm eff}$ is frequently negative, while binaries formed in isolation from initially aligned stellar progenitors are expected to be found with positive effective spin [30]. Second, for binaries formed in isolation, the azimuthal projection of the BH spins onto the orbital plane $\Delta\Phi$ was found to directly track the occurrence of mass transfer and tidal spin alignment between the stellar progenitors [10, 24, 31].

In this Letter, we use simple kinematic arguments to draw conclusions about the strength of SN kicks from the

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reported observation of GW151226. This is the less massive of the two confirmed GW detections, where nonzero natal kicks are more likely. Leaving complicated binary evolution physics aside, we show how to translate the spin misalignments reported by LIGO into concrete constraints on the strength of the first SN kick.

Observations of GW151226– The LIGO and Virgo Collaborations characterized GW151226 as a binary BH, with component masses $14.2^{+8.3}_{-3.7}M_\odot$ and $7.5^{+2.3}_{-2.3}M_\odot$ [32]. The right panel of their Figure 4 provides a posterior distribution on the magnitude and orientation of the two BH spins, relative to the orbital angular momentum. Their analysis suggests both that the more massive BH likely had nonzero spin and, critically, that this spin was most likely modestly misaligned with the orbital angular momentum, with a misalignment angle γ ranging between 25° and 80°.

Because of significant precession, the spin-orbit misalignments which LIGO directly measures and reports, corresponding to GW frequencies of 20 Hz, in principle must be evolved backwards in time to identify the spin orientations when the BHs first formed [28, 29]. Although this process turns out to be crucial to extract astrophysical information from full GW data, its details are not important for this study where we only focus on loose constraints on the measured spin direction (i.e. $25° \lesssim \gamma \lesssim 80°$). Moreover, in the simple assumption adopted here where additional alignment processes (such as tidal interactions) are neglected, previous work showed there is no net tendency to align or anti-align the BH spins [10]. This is a crucial point which will be specifically addressed in future work.

Formation and misalignment of GW151226 from isolated evolution– GW151226 could have formed from the evolution of a pair of isolated massive stars "in the field" [1]. Concrete formation scenarios for this event can be easily extracted from exhaustive simulations of binary evolution over cosmic time [2] (the evolutionary scenarios described here are drawn from the publicly available "Synthetic Universe"[1]). As a representative example, GW151226 could have formed from a pair of $53M_\odot$ and $25M_\odot$ stars, initially in a relatively close and modestly elliptical orbit with semimajor axis $R = 4000R_\odot$; as the stars evolve and the more massive star transfers and loses mass, the binary evolves to a $22M_\odot$ helium star and a $26M_\odot$ companion in a modestly tighter and circularized orbit of $900R_\odot$; the primary then undergoes a SN explosion, losing a small amount of mass to form a $19.7M_\odot$ BH. The kick following this first explosion tilts the orbital plane, changing relative alignment between the orbital plane and the BH's spin direction – presumed to be parallel to the pre-explosion orbital angular momentum. Subsequent phases of stellar interaction – notably, when the BH spirals through the envelope of the secondary star, stripping it and leaving behind a helium core – cause the binary to progress to a much tighter circular orbit of a few $R_\odot$ prior to the second SN. Because the common-evelope phase typically shrinks the orbital separation of a factor $\gtrsim 100$, the orbital velocity $v = \sqrt{GM/R}$ (where M is the binary's total mass) at the second SN event is typically an order of magnitude larger than the velocity prior to the first. Since the effect of the kick onto the binary only depends on the ratio v_k/v (see below), this second SN has a minimal impact on the misalignment of the orbital angular momentum [10]. If SN kicks are indeed responsible for the observed misaligned primary BH in GW151226, it is likely this formed during the first SN. Moreover, the first-born BH accretes too little matter to appreciably change its angular momentum direction, even during the common-envelope phase [33, 34].

Spin-orbit misalignment from natal kicks– The orbital-plane tilt angle introduced by the first SN kick can be calculated using simple Newtonian kinematics [10, 11]. For simplicity, here we only study the typical case in which strong binary interactions have circularized the pre-SN orbit [35]. We likewise assume for simplicity the initially most massive object undergoes the first SN explosion. If $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ is the relative orbital separation, $\mathbf{v} = d\mathbf{r}/dt$ is the orbital velocity and $\mathbf{v}_k$ is the imparted kick velocity, then the orbital angular momentum per unit reduced mass changes from $\mathbf{L}/\mu = \mathbf{r}\times\mathbf{v}_k$ to $\mathbf{L}_f/\mu_f = \mathbf{r}\times(\mathbf{v}+\mathbf{v}_k)$, where $\mu_f \neq \mu$ because of mass loss during the explosion. The orbital plane tilt γ reads

$$\cos\gamma = \hat{\mathbf{L}}\cdot\hat{\mathbf{L}}_f = \frac{(\mathbf{v}+\mathbf{v}_k)\cdot\hat{\mathbf{v}}}{\sqrt{(\mathbf{v}+\mathbf{v}_k\cdot\hat{\mathbf{v}})^2+(\mathbf{v}_k\cdot\hat{\mathbf{L}})^2}}\,. \qquad (1)$$

Assuming the spin of the collapsing star $\mathbf{S}$ was aligned to the orbital angular momentum before the explosion (i.e. $\hat{\mathbf{S}} = \hat{\mathbf{L}}$), γ also equals the spin misalignment angle of the newly-formed BH. If the kick imparted by the explosion is sufficiently large, the post-SN eccentricity exceeds unity and the binary does not remain bound. If $\beta = M_f/M$ denotes the fraction of total mass retained by the binary after the explosion, disruption occurs if $F(\mathbf{v}_k) < 0$ where

$$F(\mathbf{v}_k) = 2\beta - 1 - \frac{|\mathbf{v}_k|^2}{v^2} - 2\frac{\mathbf{v}_k\cdot\mathbf{v}}{v^2}\,. \qquad (2)$$

Finally, the cumulative distribution of the misalignment angle γ between pre- and post-SN angular momenta can be expressed as

$$P(\gamma<\gamma_*) = \frac{\int d\mathbf{v}_k\, p(\mathbf{v}_k)\Theta[\gamma_* - \gamma(\mathbf{v}_k)]\Theta[F(\mathbf{v}_k)]}{\int d\mathbf{v}_k\, p(\mathbf{v}_k)\Theta[F(\mathbf{v}_k)]} \qquad (3)$$

where $\Theta(x)$ is the Heavyside step function and $p(\mathbf{v}_k)$ is the kick velocity probability distribution. For simplicity, in the following we assume $p(\mathbf{v}_k)$ is an isotropic Maxwellian distribution characterized by a single 1D width σ (corresponding a mean square velocity $\langle v_k^2\rangle = 3\sigma^3$), as found for neutron stars [12]. Motivated by the

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formation scenario illustrated above, we assume modest mass loss in SN explosions, adopting $\beta = 0.98$ as a representative example of the narrow range of β found in typical population-synthesis studies $(0.95 - 1)$; we stress this choice does not significantly influence our results.

Because the dimensionless quantities γ and F depend on natal kicks only through the ratio $\mathbf{v}_k/v$, the probability $P(\gamma < \gamma_*)$ depends on σ only through the dimensionless ratio σ/v. In the limit of large σ/v, the distribution of misalignments among surviving binaries approaches a nearly uniform distribution, i.e. $P(\gamma < \gamma_*) \simeq \gamma_*/\pi$. The left panel of Figure 1 shows the misalignment distribution pertinent to GW151226 (i.e. $25° < \gamma < 80°$), as a function of the unknown dimensionless kick magnitude σ/v; for comparison, horizontal lines show the range of misalignments implied by the LIGO observations. On the right, we show the probability of a kick misalignment that is both consistent with these limits *and* does not unbind the orbit. Only modest SN kicks of $\sigma \gtrsim 0.5v$ allow a wide range of spin-orbit misalignments consistent with GW151226.

To convert from a relative to an absolute velocity scale, we adopt a distribution of progenitor masses and separations consistent with GW151226 and with observations of massive stars [2, 36]. We assume the binary is circular; the primary mass is drawn from a power-law distribution $p(m_1) \propto m_1^{-2.35}$ between $30M_\odot$ and $100M_\odot$, m_2 is drawn from a uniform distribution between $20M_\odot$ and m_1, and the orbital period P_{orb} is drawn from a distribution $p(P_{orb}) \propto (\log P_{orb}/\mathrm{day})^{-0.5}$, with limits set by twice the radius of the stars of interest ($R = 40R_\odot$) and by the maximum radius of one of the two stars' giant phase ($R = 3\times10^3R_\odot$). We then compute the ensemble-averaged cumulative probability distribution

$$\langle P(\gamma < \gamma_*)\rangle = \int P(\gamma < \gamma_*|m_1, m_2, P_{\rm orb}, \sigma)p(m_1)p(m_2) \times p(P_{\rm orb})dm_1dm_2dP_{\rm orb} \quad (4)$$

For simplicity, we neglect mass transfer before the first SN and assume all binaries which survive the first SN kick are equally likely to form a binary BH similar to GW151226. To the extent it holds, our calculations can be applied to generic binary BHs formed from isolated evolution, not just GW151226.

Figure 2 shows the distribution of kick misalignments as a function of σ. As expected given the characteristic velocity of bound orbits of massive stars, a kick of at least $\sqrt{\langle v_k^2\rangle} \simeq 45$ (62) km/s must be imparted to the first-born BH to obtain the misalignment of GW151226 in 5% (10%) of the realizations. If BH kicks are as large as those imparted to neutron stars ($\sigma \simeq 265$ km/s [12]), up to $\sim 39\%$ of our realizations are found consistent with the observed spin misalignment.

Distinguishing from alternative models– Coalescing binary BHs could form in dense interacting environments, where the spin and orbital angular momentum directions will be randomized, i.e. $P(\gamma < \gamma_*) = \cos\gamma_*/2$. The right panel of Figure 1 also shows a horizontal line corresponding to the probability that a randomly-oriented binary will lie within the region observed for GW151226. Field binaries with $0.5 \lesssim \sigma/v \lesssim 1.5$ have a higher probability to produce misalignment consistent with GW151226 than binaries formed through dynamical interactions. As pointed out in [30], modest SN kicks cannot produce an isotropic spin distribution. As as σ increases, the misalignment distribution becomes uniform in γ, below the randomly-oriented result (which predict a distribution uniform in $\cos\gamma$). However, strong SN kicks $\sigma \gg 2v$ on BHs both disrupt most field binaries and eject BHs from globular clusters, dramatically reducing the rate and creating difficulties for any stellar-evolution-based formation scenarios. While SN kicks can more easily explain the observed spin-orbit misalignments for the particular case of GW151226, observations of the spin misalignment *distribution* from many future events will be crucial to support or rule out different formation scenarios.

The Kullback-Leibler (KL) divergence $D_{KL} = \int dx p(x) \ln[p(x)/q(x)]$ provides a measure of the difference between two distributions $p(x), q(x)$, and hence the number of detections needed before we can distinguish between models (i.e., $N \simeq 1/D_{KL}$) [37]. We can calculate the KL divergence between the isotropic spin misalignment distribution and the distributions implied by any σ/v shown in Figure 1 or any σ shown in Figure 2. Even loosely accounting for measurement error (e.g., using the width of the distribution of GW151226 as an estimate of the relative misalignment accuracy), we find $\mathcal{O}(10)$ events similar to GW151226 are needed to distinguish between an isotropic distribution and a distribution misaligned by natal BH kicks, in agreement with other estimates [26, 38]

Discussion– LIGO should detect several hundred more binary BHs over the next five years [1, 39]. These observations will support or rule out whether binaries are born with spin strictly aligned with their orbital angular momentum or obtained significant misalignment from natal kicks. They will also provide strong constraints on the strength of such kicks.

Relatively low-mass binaries like GW151226 provide the simplest, cleanest laboratory to study the impact of SN kicks. First and foremost, the explosions that form them are not expected to result from direct collapse [2], so some residual linear momentum will be imparted to the ejected material and the BHs. Second, low-mass, unequal-mass-ratio binaries like GW151226 accumulate many precession cycles prior to merger in LIGO's sensitive band [40]. Third, this regime of precessing inspiral is relatively well-modeled theoretically [28, 29, 40–43]; and accessible with current parameter-estimation techniques [23–25]. LIGO has therefore the best chance to make precise measurements about misalignment for low-mass binaries, where the merger phase is relatively unimportant. By contrast, for more massive BHs like GW150914, fewer cycles are available in the LIGO band and the merger phase becomes crucial [44, 45]. Phenomenological mod-

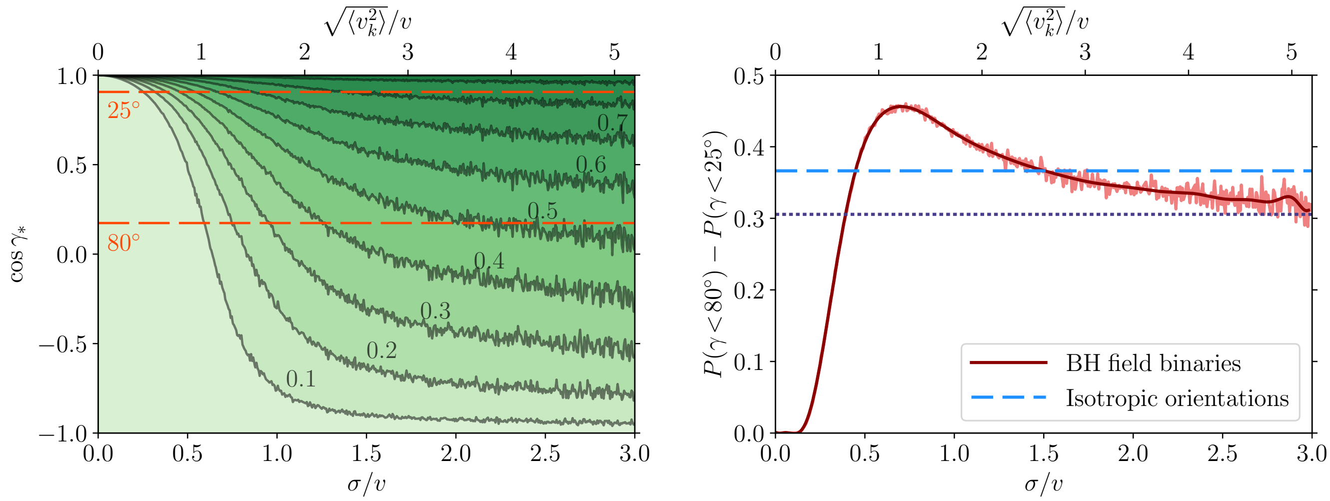

FIG. 1. **Comparing kick-induced misalignments with GW151226.** *Left:* Contour plot of the cumulative probability distribution $P(\gamma < \gamma_*)$ of the spin misalignment γ produced by the first SN kick in a binary similar to the progenitor of GW151226. The binary kick is assumed to be drawn from a Maxwellian distribution characterized by σ, which enters our predictions only through its ratio with the binary orbital velocity v. For a sense of scale, horizontal dashed lines are drawn at $\gamma = 25°$ and $\gamma = 80°$ as found for GW151226 [1]. *Right*: Fraction of surviving binaries with spin misalignment consistent with GW151226 as a function of the dimensionless kick magnitude σ/v. The lighter pink line shows $P(\gamma < 80°) - P(\gamma < 25°)$ from our Monte Carlo runs, while the darker red curve shows a polynomial fit. For context, the horizontal dashed line shows $(\cos 25° - \cos 80°)/2$, as expected from random spin-orbit alignment, while the horizontal dotted line corresponds to $(80° - 25°)/180°$, as expected in the limit of large σ. As SN natal kicks increase in magnitude, the fraction with misalignment consistent with GW151226 first increases substantially, as most surviving binaries have been modestly kicked relative to their orbital speed.

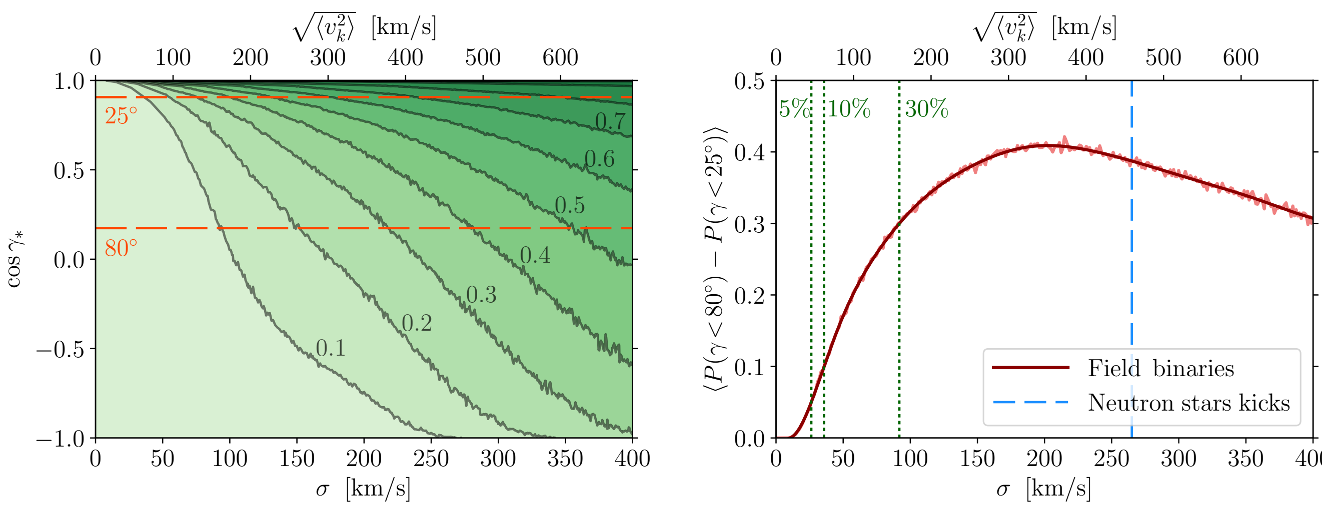

FIG. 2. **Kick velocities consistent with GW151226 misalignment**: *Left*: Cumulative distribution $\langle P(\gamma < \gamma_*)\rangle$ averaged over masses and separations as a function of misalignment angle $\cos\gamma$ and physical kick strength σ. The top axis shows the correspondent 3D root-mean-square velocity $\sqrt{\langle v_k^2\rangle} = \sqrt{3}\sigma$. *Right*: Difference $\langle P(\gamma < 80°) - P(\gamma < 25°)\rangle$ versus σ and $\sqrt{\langle v_k^2\rangle}$, illustrating how the expected fraction of binaries with misalignments consistent with GW15226 changes with the characteristic natal kick magnitude. The lighter pink line shows results of our Monte Carlo runs, while the darker red curve correspond to a polynomial fit. Vertical green dotted lines are drawn at $\sigma \simeq 26, 36$ and 92 km/s, corresponding to probabilities of 5%, 10% and 30%; the dashed blue line at $\sigma = 265$ km/s marks the typical kick magnitude imparted to neutron stars [12].

els that approximate full solutions of Einstein's equations are known to omit important physics, which can in turn lead to biases when these models are applied to parameter estimation [46]. Robust spin-orbit misalignment measurements for heavy BHs will require improved waveform modeling and/or more extensive use of numerical relativity data [44, 47].

The natal kicks required to explain the misalignment of GW151226 are in excess of the fallback-suppressed kicks adopted by default in current binary evolution models [2, 16, 30] (though note models M4, M5, and M6 in [2]). Notably, these natal kicks are consistent with the observed recoil velocity of BH X-ray binaries in our own galaxy [13–15] (c.f. [16] for an extensive discussion on these measurements). For isolated binary evolution models, a modest increase in SN kicks diminishes the expected event rate – more binary BHs are disrupted by the first SN – but otherwise produces predictions for the population of merging binary BHs that are consistent with existing observations [2, 48]. The impact of recent physically-motivated prescriptions that relate kick magnitude and ejected mass [18, 49] has yet to be fully explored with large-scale population-synthesis studies

Large natal kicks $\sqrt{\langle v_k^2 \rangle} \gtrsim 50$ km/s that must be imparted to BHs of mass $\gtrsim 15 M_\odot$ at formation could be a significant challenge for SN physics. For example, one of the leading models used to explain the kicks imparted to neutron stars invokes gravitational attraction by the newly-formed compact object of some of the material ejected asymmetrically during the explosion (the so-called "gravitational tug-boat mechanism" [18, 19]). While this requires significant and quite asymmetric mass ejection, many of the formation scenarios explored for GW151226 assume very modest mass loss ($\beta \sim 0.98$), with most of the material falling back on to (and slowing down) a proto-neutron star core that later collapses to a BH (see, e.g., [50, 51]).

Our analysis assumes SN kick provide the principal mechanism for binary spin-orbit misalignment in field binaries. Alternatively, binaries could be born with primordial spin-orbit misalignment, or gain comparable misalignment early in their life via either interactions with by a tertiary companion [52] or core-envelope interactions [53]. If such misalignment can persist or grow during the long lifetime and many interactions necessary to form a coalescing BH, then LIGO observations might be an indicator of primordial spin misalignment processes.

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