HIGH ANGULAR MOMENTUM HALO GAS: A FEEDBACK AND CODE-INDEPENDENT PREDICTION OF LCDM

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ABSTRACT

We investigate angular momentum acquisition in Milky Way sized galaxies by comparing five high resolution zoom–in simulations, each implementing identical cosmological initial conditions, but utilizing different hydrodynamic codes: Enzo, Art, RAMSES, Arepo, and Gizmo–PSPH. Each code implements a distinct set of feedback and star formation prescriptions. We find that while many galaxy and halo properties vary between the different codes (and feedback prescriptions), there is qualitative agreement on the process of angular momentum acquisition in the galaxy’s halo. In all simulations, cold filamentary gas accretion to the halo results in ∼ 4 times more specific angular momentum in cold halo gas (λcold ≈ 0.15) than in the dark matter halo. At z > 1, this infall frequently results in the formation of transient cold flow disks—large co–rotating gaseous structures in the halo of the galaxy that are fueled, aligned, and kinematically connected to filamentary gas infall along the cosmic web. Due to the qualitative agreement among disparate simulations, we conclude that the buildup of high angular momentum halo gas and the transitory formation of cold flow disks are robust predictions of LCDM galaxy formation. A growing body of observational evidence suggests that this process is borne out in the real universe.

Subject headings: galaxies:formation—halos—evolution — methods:numerical—hydrodynamic—simulation

1. INTRODUCTION

In the standard Lambda Cold Dark Matter (LCDM) picture of galaxy formation, gas accreting onto a growing dark matter halo shock-heats to the virial temperature of the halo, giving the gas time to virialize and eventually cool out of the hot gaseous halo and sink onto the central galaxy [Rees & Ostriker 1977; Silk 1977; White & Rees 1978; White & Frenk 1991; Maller & Bullock 2004]. Under this picture of galaxy growth, it is expected that the resulting angular momentum distribution of galaxies should mimic the spin of their dark matter, resulting in rotationally supported galaxy disks (and presumably hot gaseous halos as well) that are proportional to the spin of the dark matter halo [Fall & Efstathiou 1980; Mo et al. 1998], which has been well studied in dissipationless N-body simulations (e.g. Bullock et al. 2001; Vitvitska et al. 2002; Maller et al. 2002; Avila-Reese et al. 2005; D’Onghia & Navarro 2007; Bett et al. 2010; Muñoz-Cuartas et al. 2011; Ishiyama et al. 2013; Trowland et al. 2013; Kim et al. 2015). However, recent advances in hydrodynamic simulations and galaxy formation theory have increasingly emphasized the importance of “cold flows”—gas accretion onto galaxy halos via filamentary streams with cooling times shorter than the compression time for establishing a stable shock13, either when the halo is below a critical mass threshold, or even for massive halos at sufficiently high redshift (e.g. Binney 1977; Keres et al. 2005; Dekel & Birnboim 2006; Ocvirk et al. 2008; Brooks et al. 2009; Dekel et al. 2009; Faucher-Giguère & Kereš 2011; Faucher-Giguère et al. 2011; Stewart et al. 2011a; van de Voort et al. 2011; Hobbs et al. 2013; van de Voort et al. 2015). In the cold flow paradigm, gas that is accreted in the cold mode tends to have specific angular momentum considerably higher than the dark matter [Chen et al. 2003; Sharma & Steinmetz 2005; Keres et al. 2009; Keres & Hernquist 2009; Agertz et al. 2009; Brook et al. 2011a; Stewart et al. 2011b; Kimm et al. 2011], inconsistent with the previous picture of galaxy angular momentum buildup. The resulting angular momentum of the stellar disk may be rather different than that of the accreted gas because of feedback effects [Maller & Dekel 2002; Brook et al. 2011b].

13 Some recent moving–mesh simulations have called into question whether these cold streams deliver unshocked gas to the galaxy without heating in the inner regions of the halo (e.g. Torrey et al. 2012; Nelson et al. 2015, 2015, 2016). As our focus in this work is on gas accretion into the halo, not the eventual transition from the halo to the galaxy, this distinction should have minimal impact on topics discussed here.
As a result of this changing paradigm for cosmological gas accretion and galaxy growth, a new scenario of angular momentum acquisition in galaxies and galaxy halos seems to be emerging. In this picture (Stewart et al. 2011b; Kimm et al. 2011; Pichon et al. 2011; Codis et al. 2012; Danovich et al. 2012; Stewart et al. 2013; Codis et al. 2015; Danovich et al. 2015; Prieto et al. 2015; Tillson et al. 2015) the particularly high angular momentum of cold flow gas is related to its coherent, filamentary origin, coupled with the specific geometry of the cosmic web in the environment of a given galaxy. These filamentary cold flows deliver significant angular momentum to galaxy halos, with the cold gas orbiting for \(\sim 1 - 2\) dynamical times before spiraling into the central galaxy. At any given time, galaxy disks typically have lower spin than halo gas owing to the fact that the specific angular momentum of infalling material increases with time. Halo gas is “younger” and this correlates with higher spin.

Importantly, this scenario is predictive. The high-spin halo gas is often (but not always) coherent in its spin direction. It often forms a thick planar structure of high angular momentum cool gas that co-rotates with the central disk. Throughout this work, we will refer to this transient structure as a “cold flow disk,” though we note that this gas is not rotationally supported—it typically spirals in within \(\sim 2\) dynamical times. Nor are these structures especially thin or perfectly aligned with the orientation of the the galactic disk. The “disk” here refers to how the accreting cold-mode gas shows observational signs of coherent rotation along a preferred plane.

Encouragingly, an increasing number of observations have begun to demonstrate the abundance of high angular momentum material in galaxy halos, qualitatively consistent with this emerging theoretical picture. In the local universe, some of these observations include detection of high angular momentum extended HI disks and XUV disks (Oosterloo et al. 2007; Christlein & Zaritsky 2008; Sancisi et al. 2008; Lemanis et al. 2011; Holwerda et al. 2012), as well as low metallicity high angular momentum gas (presumably from fresh accretion) in polar ring galaxies (Spavone et al. 2010). There is even indication that local extended HI disks may be environmentally dependent on the galaxy’s filamentary environment (Courtois et al. 2015). At moderate redshift (\(z \sim 0.5-1.5\)) numerous absorption line studies of the circumgalactic medium of galaxies have begun to emphasize the bi-modal properties of absorbers, where absorption along the galaxy’s major axis tends to show high angular momentum (co–rotating) infall, and absorption along the galaxy’s minor axis shows observational signatures of outflow (Kacprzak et al. 2010, 2012a, b; Bouché et al. 2012, 2013; Crighton et al. 2013; Nielsen et al. 2015; Diamond-Stanic et al. 2015; Bouché et al. 2016). At higher redshift (\(z \sim 2-3\)) kinematic studies of Ly\(\alpha\) “blobs” have observed large scale rotation consistent with high angular momentum cold gas accretion (Martin et al. 2014; Prescott et al. 2015). There are also recent detections of massive protogalactic gaseous disks kinematically linked to gas inflow along a cosmic filament, strikingly similar to the theoretical “cold flow disk” structure (Martin et al. 2015; 2016).

In this context, it is important that we ascertain how robust the predictions of these cosmological simulations are—a difficult task considering that many properties of simulated galaxies depend sensitively on the implementation of uncertain subgrid physics models such as gas cooling, star formation, radiation pressure, and supernova feedback (e.g., Thacker & Couchman 2000; Kay et al. 2002; Scannapieco et al. 2012; Gnedin et al. 2011; Piontek & Steinmetz 2011; Martíz et al. 2012; Agertz et al. 2013; Vogelsberger et al. 2013; Vogelsberger et al. 2014; Marasco et al. 2015; Genel et al. 2015; Ceverino et al. 2014; Agertz & Kravtsov 2015). In addition, even with identical subgrid implementations, there are inherent numerical advantages and disadvantages between different hydrodynamic code implementations—for example, Lagrangian smooth particle hydrodynamic (SPH) versus Eulerian grid codes—that result in artificial differences between galaxies simulated with different codes (e.g., Morris 1996; Agertz et al. 2007; Wadsley et al. 2008; Cullen & Delm2 2010; Hahn et al. 2010; Springel 2010; Hopkins 2014; Richardson et al. 2016).

In order to test the validity of the emerging cold flow picture of angular momentum acquisition we must ascertain the dependency of these predictions on the use of different numerical techniques and a variety of cutting-edge subgrid physics models. In this paper, we run five hydrodynamic zoom-in simulations of a Milky Way sized galaxy, each with identical cosmological initial conditions but with different codes: Enzo, Ramses, Art, Arepo, and Gizmo-SPH, each implemented with current subgrid physics models. In order to ensure uniform analysis for different hydrodynamic codes, we utilize the analysis software yt (which allows a single analysis routine to be run on different code architectures; Turk et al. 2011) to explore the angular momentum content of halo gas and whether or not the expected “cold flow disk” prediction is robust across these disparate platforms. We introduce the simulations in §2, present our main results from the comparison of §2, present our main results from the comparison of §3–5, finding that the same qualitative picture of high angular momentum halo gas and frequent formation of the cold flow disk phenomenon is present in all simulations, a seemingly natural consequence of filamentary gas accretion in LCDM. We discuss the implications of these results and the growing observational evidence of their existence in §6 and summarize and conclude in §7.

2. THE SIMULATIONS

2.1. Overview

The simulations used in this paper are all part of the Scylla Multi–Code Comparison Project. This project resimulates a Milky Way halo mass zoom–in simulation (originally performed by Ryan Joung with Enzo in Joung et al. 2012) using other cosmological hydrodynamic codes. The codes are all run with their current subgrid models in order to compare state of the art simulations across codes. Thus the project is much like the Aquila code comparison (Scannapieco et al. 2012) but with higher resolution. Our resolution is similar to the Agora code comparison project (Kim et al. 2014), but that project is seeking to use uniform physics while we

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14 Though, inevitably, there are bound to be recent improvements to some of the subgrid models during the time it took to run the simulations, analyze and publish the results.
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HM96 — Haardt & Madau (1996), FG09 — Faucher-Giguère et al. (2009),
† Subsequent versions of Arepo have switched to a different time integration (Pakmor et al. 2016) using Heun’s method
‡ For Lagrangian codes, high resolution gas particle mass is $3.3 \times 10^4 M_\odot$
* See text for detailed descriptions of feedback models.

are running each code as it has been used for other science papers. The codes used here are Enzo, Art, Ramses, Arepo and Gizmo-PSPH. For all runs the cosmology, dark matter particle mass and box size are identical—the box is 25 Mpc/h across with a much smaller region simulated at high resolution, using dark matter particles of mass $1.75 \times 10^5 M_\odot$. All adaptive mesh refinement codes reach the same maximum refinement of $95 \ h^{-1}$ comoving pc, which is identical to the force resolution of the Lagrangian codes, with the exception of gas particles in Gizmo-PSPH, which uses an adaptive gravitational softening with a minimum value of $14 \ h^{-1}$ comoving pc. A flat cosmology consistent with WMAP5 (Komatsu et al. 2009) is used throughout, with $\Omega_m = 0.279$, $\Omega_{\Lambda} = 0.721$, $\Omega_b = 0.046$, $h = 0.70$, $\sigma_8 = 0.82$ and $n_s = 0.96$.

The initial conditions for the original Enzo run (Joung et al. 2012) were generated with the code Graphic (Bertschinger 2011) with a starting redshift of $z = 99$. The same code with the same seed was used to generate the initial conditions for the Ramses run. For all other runs the dark matter particles from the Enzo run were used to determine the initial conditions. That is, the dark matter particles were set identical to those in the Enzo run and baryons were added based on the dark matter distribution (no separate transfer function). We expect these differences to be negligible by the redshift where galaxies are forming.

Based on the cosmological model specified above, all Lagrangian codes set the gas mass resolution ($m_{\text{gas}} = 3.3 \times 10^4 M_\odot$) relative to the dark matter particle mass ($m_{\text{DM}} = 1.75 \times 10^5 M_\odot$). Table 1 outlines many of the pertinent details for each code, including star formation (SF) density thresholds and efficiency parameters, epoch of reionization, UV background model, and the type of stellar feedback model adopted. Below, we describe the gas cooling and feedback physics of each individual run in more detail and include references to recently published science papers that utilize similar subgrid physics models as those implemented here.

For all analysis that follows, we make the distinction between “cold” and “hot” gas by a temperature cutoff of 250,000 K (commonly used as the distinction between “cold-mode” and “hot-mode” gas accretion, e.g., Keres et al. 2005, 2009; Stewart et al. 2011b, 2013). For the sake of uniform comparison between codes we choose a fiducial value of 300 comoving kpc as an approximation of the virial radius at each epoch, due to the fact that halo–finding algorithms produce slightly different virial radii at the same epoch for different simulations (varying within the range of $\sim 280–320$ comoving kpc for $z < 3$). In order to guarantee uniform analysis for the varied code architectures and file formats, all analysis presented here has been performed utilizing the yt\(^{16}\) analysis software (Turk et al. 2011; Turk & Smith 2011; Turk 2013), an open source project which has been developed and is continually being maintained and improved by the astrophysical community for the intended purpose of supporting cross–code compatible hydrodynamic analysis routines.

2.2. Enzo

The Enzo (Bryan et al. 2014) run serves as the basis for the Scylla simulation suite, and was performed in 2010 by Ryan Joung and discussed in Joung et al. (2012); Fernández et al. (2012); and Putman et al. (2012). Enzo uses an Adaptive Mesh Refinement (AMR) grid to solve the equations of hydrodynamics, with this particular run using a version of Enzo before the uniform release of Enzo 2.0. Enzo uses an FFT in the root grid gravity

\(^{15}\) http://web.mit.edu/edbert/ \(^{16}\) http://yt-project.org
solvotherm and a 3rd order piecewise parabolic method hydrodynamics solver. Feedback is thermal as described in Cen et al. (2005). The simulation includes metallicity-dependent cooling to a temperature of 10 K (Dalgarno & McCray 1972), neutral hydrogen shielding from UV radiation, and diffuse photoelectric heating (Abbott 1982; Joung et al. 2009).

2.3. Art

The Art (Kravtsov et al. 1997; Kravtsov 2003) run uses an AMR grid to solve the equations of hydrodynamics. Art uses a multilevel particle mesh gravity solver and a 2nd order Godunov method hydrodynamics solver. Our run uses the star formation and feedback models described in Ceverino et al. (2014) and includes thermal feedback from supernovae explosions and stellar winds (Ceverino & Klypin 2009; Ceverino et al. 2010) as well as radiative feedback (model HadPreLSR in Ceverino et al. 2014). This model of radiative feedback includes radiation pressure from ionizing and infrared photons, photoheating, and photoionization from massive stars. Other recent papers using similar physics include Zolotov et al. (2015); Snyder et al. (2015a); Ceverino et al. (2015); Goerdt & Ceverino (2015); Mandelker et al. (2015); Vecchella et al. (2016a); Ceverino et al. (2016); and Tomassetti et al. (2016).

2.4. Ramses

The Ramses (Tezgier 2002) run (Ramses version 3.0) uses an AMR grid to solve the equations of hydrodynamics. Ramses uses a particle mesh gravity solver and a 2nd order MUSCL scheme hydrodynamics solver. The gas cooling is based on a metallicity-dependent cooling, including metal line cooling down to a temperature floor of 100 K. A stiffening of the interstellar medium (ISM) equation of state (chosen as a power law with $\gamma = 4/3$) was used to prevent gas with densities higher than the 1 atom/cm$^{-3}$ threshold to cool further than 100 K and artificially fragment. Feedback includes energy from stellar winds and supernovae (deposited in kinetic form) following Dubois & Teyssier (2008), where the proper distributions of type II supernova lifetimes are based on Leitherer et al. (1999) and Leitherer et al. (2010), such that energy from type II supernovae is injected continuously between 2 and 50 Myr. Feedback from type Ia supernovae are also included, following Greggio & Renzini (1983) to compute the SN frequency. This run has essentially the same physics as in Dubois et al. (2014); Welker et al. (2014); Codis et al. (2015); and Chisari et al. (2015) with the exception that we have not included any AGN physics here.

2.5. Arepo

The Arepo (Springel 2010) run employs a quasi-Lagrangian finite volume method for solving the hydrodynamic equations of motion (Vogelsberger et al. 2013). The version of Arepo used here employs a tree multipole expansion gravity solver and a 2nd order Godunov method hydrodynamics solver with a MUSCL scheme, however subsequent versions of Arepo have switched to a different time integration in the hydrodynamics solver (Pakmor et al. 2016) using Heuns method. Radiative gas cooling includes both primordial cooling (Katz et al. 1996) as well as line cooling from heavy elements (Wiersma et al. 2009a; Vogelsberger et al. 2013). Pressurization of the ISM, star formation, and associated feedback is handled using the Springel & Hernquist (2003) subgrid model. Time delayed stellar mass return and metal enrichment is carried out (Wiersma et al. 2009b; Vogelsberger et al. 2013), and kinetic star formation driven winds are employed with a wind velocity scaled to the local dark matter velocity dispersion. Winds are launched carrying 40% of the local ISM metallicity to prevent over ejecting metal mass from the dense ISM (Zahid et al. 2014).

This run includes a physics implementation that is similar to that used in the Illustris simulation (Vogelsberger et al. 2014; Genel et al. 2014) with the notable difference that no AGN physics is included here. Other recent work that contains similar physics include Tor-

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**Figure 1.** Hydrogen number density and temperature comparison at $z = 3$. All panels show the circumgalactic region, with panel widths of 300 physical kpc. Physical size scales are given in the left-most panels, and circles denote a radius of 300 comoving kpc (75 physical kpc), which is roughly the virial radius of the halo. **Top:** Projected gas density, showing the gas accretion onto the galaxy via cosmic filaments. Detailed morphology of the resulting galaxy varies between simulation codes, but the same filamentary accretion structure is apparent. **Bottom:** Projected density–weighted gas temperature. Note that temperature of gas in the CGM is highly dependent on the specific feedback and code implementation.
2.6. Gizmo-PSPH

The Gizmo-PSPH (Hopkins 2015) run uses a tree multipole expansion for the gravity solver and the pressure−energy formulation of smoothed particle hydrodynamics (PSPH; Hopkins 2013) together with a number of additional improvements to artificial viscosity, timestepping, and higher−order kernels, to solve the equations of hydrodynamics. Radiative gas cooling includes both primordial cooling (Katz et al. 1996) as well as cooling from 11 separately tracked metal species (Wiersma et al. 2009a). Gas follows an ionized + atomic + molecular cooling curve from $T = 10^{-3}$ to $10^{10}$ K.

Star formation and feedback uses the Feedback In Realistic Environments (FIRE) prescriptions from Hopkins et al. (2014), which explicitly follow the mass, metal, momentum, and energy deposition by radiation pressure, photo−ionization and photo−electric heating, stellar winds, and SNe (Types II and Ia), with all rates tabulated from the stellar population model STARBURST99 (Leitherer et al. 1999) assuming a Kroupa (2001) IMF. They do not include AGN feedback. Unlike the other codes here, which assume stars form with a relatively low efficiency per free−fall time in all gas above some relatively large density threshold $\sim 10^{-3}$ cm$^{-3}$, the FIRE models restrict star formation only to gas which is locally self−gravitating (following Hopkins et al. 2013), self−shielding and molecular (following Krumholz & Gnedin 2011), Jeans−unstable, and exceeds a higher density $n > 5$ cm$^{-3}$, but within this highly restricted gas assumes the star formation occurs on a free−fall time. The FIRE code and methods are identical in this paper and in all published papers including e.g. Onorbe et al. (2015), Chan et al. (2015), Ma et al. (2015), Faucher-Giguere et al. (2015), Wheeler et al. (2015) and others.

3. BASIC HALO PROPERTIES

3.1. Large Scale Structure and Mass Growth

We begin with a visual inspection of the region around the galaxy for each simulation. Figure 1 shows the gas density (number density of H; top) and density-weighted temperature (bottom) projections at $z = 3$ through a cube of width 300 physical kpc ($\sim 4 R_{\text{vir}}$ at this redshift). The top panels of Figure 1 show qualitative agreement between the simulations on the general geometry and structure of the forming disk galaxy at this redshift, as well as its placement in a large−scale cosmic filament that is continually delivering an inflow of cold gas into the virial radius of the galaxy. However, the detailed structure of the galaxy—and even that of the filament into which the galaxy is embedded—does appear to vary significantly between simulations. For example, the width of the cosmic filament, the size and structure of the galactic disk, and the peak density of infalling satellite galaxies all vary on a noticeable level.

Perhaps more striking is the temperature differences among simulations shown in the bottom panels of Figure 1. All simulations demonstrate the presence of a significant gaseous halo around the galaxy, as well as streams of filamentary gas that penetrate the halo and deposit cold gas in the inner halo, near the galactic region. However, the extent that feedback has enriched the CGM and IGM, the density structure of the gaseous halo, the temperature distribution of hot gas, and the precise structure of the cold flows as they interact with the gaseous halo of the galaxy vary significantly among simulations.

To illustrate some of the similarities and differences among the simulations, Figure 2 shows the mass growth of the halo as a function of time—including the total mass (black), total gas mass within the virial radius (cyan), total stellar mass within the virial radius (yellow) and the total hot ($T > 250,000$ K) gas mass within the virial radius (magenta). Note that the total virial mass and the total gas mass are both quite robust among simulations—despite very different feedback implementations. However, the total stellar mass and the total mass in hot gas varies significantly among some simulations. (Alternatively, the total mass in cold gas also differs significantly among simulations, however for the sake of clarity cold gas masses are not shown in the figure).

We note that the Gizmo−PSPH code shows a significantly lower stellar masses than any of the other codes used here, likely as a result of strong feedback implementations. Interestingly, the galaxy stellar mass for Gizmo−PSPH is much closer to observational expectations—for example, abundance matching suggests that a $10^{12} M_\odot$ halo at $z = 1$ should host a central galaxy with a stellar mass of $\sim 2 \times 10^{10} M_\odot$ (Behroozi et al. 2010). As our goal in this work is to focus on similarities between codes, with emphasis on the galaxy halos and not the galaxies themselves, we defer a more detailed discussion of the numerous differences between the simulations and their implications for galaxy formation as a topic for future study.

3.2. Angular Momentum

One fundamental result of the recent emerging picture of angular momentum acquisition in galaxies is that gas in the halos of galaxies tends to have specific angular momentum $\sim 3−5$ times higher than the dark matter (Stewart et al. 2011b, Kimm et al. 2011, Stewart et al. 2013, Danovich et al. 2015). We revisit these previous findings by comparing the spin parameter, $\lambda$, of both the cold halo gas and the dark matter in the halo for all our simulations. We adopt the spin parameter from Bullock et al. (2001): $\lambda_\star \equiv j_x/\sqrt{2VR}$, where $j_\star$ is the spin parameter of a given component, based on that component’s specific angular momentum, $j_x$, and $V$ and $R$ are typically defined by the virial velocity and virial radius of the halo, respectively. However, in order to make a uniform comparison between simulations, we approximate the halo virial radius by $R = 300$ comoving kpc, where $V$ is then defined by the circular velocity at this radius, $\sqrt{GM/R}$. Figure 3 shows the spin parameter for each saved output of each simulation between $z = 3$ to $z = 1$, where we only include material inside the virial radius but outside of the central region, $R > 0.1R_{\text{vir}}$, in
Figure 2. Masses enclosed within 300 comoving kpc (roughly the virial radius) as a function of time. Note that the total virial mass and total gas masses are similar. However, the total stellar mass and the total hot gas in the halo (or total cold gas—not shown for the sake of clarity) may vary significantly depending on which code and feedback implementations are used. Our calculations (after all, were are interested in the halo, not the galaxy itself). While the simulations vary in the precise value (and direction—not shown) of the angular momentum of their gaseous halos, we find several important qualitative agreements across all the simulations.

- Both cold halo gas (and hot halo gas—not shown in the figure, for clarity) consistently have more specific angular momentum than the dark matter component.

- While simulations agree that dark matter halo spin parameters are typically $\lambda_{DM} \approx 0.04$, the average cold halo gas spin parameter across our simulations is $\lambda_{\text{cold}} \approx 0.15$.

- In agreement with previous work, in all simulations the cold halo gas contain $\approx 4$ times the specific angular momentum of the dark matter halo (though with considerable variation), while the hot gas typically has $\approx 2$ times the specific angular momentum of the dark matter.

These findings confirm previous results: the angular momentum of galaxy halos varies significantly among components; the dark matter invariably measures a cumulative combination of past accretion, resulting in the lowest specific angular momentum; the hot gaseous halo is typically built and maintained both by non-filamentary “hot-mode” gas accretion, as well as feedback and outflows (which are sensitive to subgrid physics models); and the cold halo gas traces filamentary “cold-mode” accretion and has the highest specific angular momentum. Thus, while our simulations agree with previous $N$-body simulations for a dark matter halo spin parameter of $\lambda_{DM} \approx 0.04$, one should expect to observe typical cold halo gas with significantly higher angular momentum, with spin parameters closer to $\lambda_{\text{cold}} \approx 0.15$.

In a previous study of four cosmological zoom–in simulations (all using the same hydrodynamic code) Stewart et al. [2013] found no significant trend between cold gas spin parameter and cosmological time (or redshift). Therefore, while Figure 3 arguably shows a trend of increasing cold gas spin parameter from $z = 3 - 1$, this is likely a consequence of this particular halo’s unique merger and accretion history, and not general result of galaxy formation in LCDM.

4. LARGE SCALE FILAMENTARY INFLOW

In order to place this high angular momentum cold halo gas in the proper cosmological context, Figure 4 shows the large–scale environment around the simulated galaxy—where, for purposes of this work, we define the halo environment by box widths of 2400 comoving kpc (300 physical kpc at $z = 3$). The top panels again show the gas density (H number density), similar to Figure 1 but zoomed out by a factor of two and viewed along an orthogonal orientation. The bottom panels show the line of sight velocity of all cold gas above a minimum density threshold in hydrogen (all forms) of $n_{H} > 3 \times 10^{-4} \text{ cm}^{-3}$, which was chosen to select only gas sufficiently dense to be embedded in filamentary (or dark matter halo) structures on these large scale environments.

Because the galaxy is the most massive halo in its environment (i.e. not a member of a group or cluster) the cosmic filaments in its environment are strongly af-
Figure 4. Large scale environment at $z = 3$ (along an orthogonal line of sight from Figure 1). In all panels, circles denote 300 comoving kpc (roughly the virial radius of the halo) and the size scale in physical kpc is indicated in the left–most panels, with panel widths of 600 physical kpc. Top: Gas density projections showing the structure of the cosmic web near the galaxy. Bot: Line of sight velocity of dense cold gas with a density in hydrogen of $n_H > 3 \times 10^{-4} \text{ cm}^{-3}$ in an identical orientation and scale as the top panels. There is a clear line of sight velocity signature—the top left filament is blueshifted while bottom filaments are redshifted—indicating the motion of the cosmic web as it flows onto the massive galaxy halo. The particularly chaotic structure of the Gizmo–PSPH simulation is due to a violent merger–induced outflow at this epoch; see §5.1.

Figure 5. Large scale environments at $z = 2$ (top), $z = 1.5$ (middle) and $z = 1$ (bottom) for various subsets of the simulation runs. As with figure 4, the panels show the line of sight velocities of dense gas with hydrogen density of $n_H > 3 \times 10^{-4} \text{ cm}^{-3}$. Each box has a width of 2400 comoving kpc across (800, 960, 1200 physical kpc at $z = 2, 1.5, 1$, respectively). Circles denote a 300 comoving kpc radius (roughly the virial radius of the halo) and the size scale in physical kpc is indicated in the left–most panels of each row. Again note the clear signature of large scale filamentary gas flowing onto the galaxy.

Figure 5 show line of sight velocity maps along an orthogonal orientation for dense cold gas at $z = 2$ (top), $z = 1.5$ (middle) and $z = 1$ (bottom) for various subsets of the simulation runs (as labeled). While the basic filamentary nature of the gaseous inflows becomes less apparent at decreasing redshift (when the filaments are less dense), we can still note the same qualitative behavior of inflowing gas. On environmental scales, filamentary inflow results in the same clear line of sight velocity signature as before; across all simulations, gas flowing into the virial radius from the top of the panels is redshifted, while gas flowing in from the bottom is blueshifted. The only notable exception is the Ramses code at $z = 1$, which is likely the result of the lack of self–shielding from the UV background leaving very little cold gas above our minimum density threshold, so almost no cold dense inflow redshifted.

This result is perhaps not surprising, as any three–dimensional filamentary structure where matter flows along the cosmic web towards a central overdensity (and is viewed along an arbitrary axis) is unlikely to show multiple filaments all flowing perpendicular to the line of sight. Thus one should naively expect strong line of sight velocities to be apparent when viewing large–scale filamentary gas flows. While this may not be a surprising result, however, it is important to keep these large scale gas flows in mind for future discussion of the origin of cold flow disks. We will see in §5 that these large–scale filamentary flows have a direct impact on the behavior of the cold gas within the virial radius of the halo.

Note that the line of sight velocity structure of the filament flowing in from the right of the galaxy shows considerably more variation between the simulations. This occurs because this filament does happen to be roughly perpendicular to the line of sight. Thus the velocities along the right–most filament are more sensitive to the peculiar velocities of galaxies, gas streams, and outflows, which vary more strongly between simulations than the gross large–scale flows towards the central halo.

The line of sight velocity of dense cold gas at $z = 2$ (top), $z = 1.5$ (middle) and $z = 1$ (bottom) for various subsets of the simulation runs (as labeled). While the basic filamentary nature of the gaseous inflows becomes less apparent at decreasing redshift (when the filaments are less dense), we can still note the same qualitative behavior of inflowing gas. On environmental scales, filamentary inflow results in the same clear line of sight velocity signature as before; across all simulations, gas flowing into the virial radius from the top of the panels is redshifted, while gas flowing in from the bottom is blueshifted. The only notable exception is the Ramses code at $z = 1$, which is likely the result of the lack of self–shielding from the UV background leaving very little cold gas above our minimum density threshold, so almost no cold dense inflow redshifted.
as shown in previous work (Stewart et al. 2011b, 2013; Danovich et al. 2015), a natural consequence of high angular momentum inflow of cold filamentary gas is the existence of cold flow disks—extended, flattened rotating structures of high-angular momentum material, aligned and fueled by filamentary inflow. We now investigate the robustness of these previous results using our suite of simulations. Figure 6 is analogous to Figure 4, except that the panels now focus on material within the virial radius (panel widths of 150 physical kpc at $z = 3$). The bottom panels again show line of sight velocity maps of cold dense gas in the halo, except that we have increased the minimum density threshold by a factor of 10 when compared to Figure 4, to a hydrogen density of $n_H > 3 \times 10^{-3} \text{ cm}^{-3}$ (this should correspond to a minimum hydrogen column density of $N_{HI} \gtrsim 10^{17} \text{ cm}^{-2}$, Altay et al. 2011; Schaye 2001).

The exact morphology of gas in the halo of the galaxy varies considerably between the simulations, not surprisingly, given the vastly different feedback mechanisms implemented in each simulation—some of which drive explosive spherical outflows that violently shred the ISM and CGM of the galaxy (e.g., Gizmo-PSPH) and some of which instead drive high-velocity bi-conical outflows out of the plane of the galaxy (e.g., Enzo). However, we also note that some of the morphological differences are also influenced by the precise timing of galaxy mergers. For example the Gizmo-PSPH simulation is in the

We select gas based on temperature and density rather than HI content or species column density because we want to avoid any differences in ionization fractions among simulations when making our comparison. The qualitative trend that there is always orbiting “cold flow disk” gas in the halo does not depend on the details of this selection criterion, though quantitative measures (e.g., the apparent covering fraction of this gas) will of course depend on these details—a topic we plan to revisit in future work.

is still visible in the figure. While the detailed structure of the inflowing gas again varies among simulations, it seems apparent that filamentary gas accretion along a three-dimensional cosmic web onto an overdense region (at this mass scale from $1 < z < 3$) tends to produce the same qualitative picture across all the simulations, regardless of subgrid physics.
Figure 8. Time lapse of a post–merger violent outflow event in Gizmo–PSPH at $z \sim 3$, proceeding from left–to–right. The overlaid circles denote 300 ckpc, roughly the halo virial radius and the overlaid ellipse in each panel roughly corresponds to the “cold flow disk” region, to aid the eye in comparison between images. Top: density map of the gas in the halo. Bottom: line of sight velocity of cold dense gas in the halo (identical analysis to Figure 6). The cold flow disk apparent in the left panel by the coherent rotation from $z = 3.2$–3.0, but once the outflow event is over, fresh high angular momentum inflow along the cosmic web begins to establish a new cold flow disk by $z = 2.9$, demonstrating the robustness of the cold flow disk phenomenon.

midst of a violent outflow at this epoch, due to a recent major merger, which partially explains the chaotic structure shown in Figure 6 (We will demonstrate in §5.1 that the Gizmo–PSPH simulation shows the presence of a cold flow disk immediately before and after this merger–driven outflow event). While the same general merger and accretion history takes place for each simulation, the exact timing of these mergers at a given epoch may vary, and any coherent velocity structure for cold gas in the galaxy’s halo is typically destroyed during a sufficiently strong outflow event.

Despite these varied differences in morphology, the bottom panels of Figure 6 show a remarkably similar qualitative picture. As was the case with the large scale environment, the cold gas entering the virial radius from the upper–left filament shows a dramatic blueshift in each simulation, while the cold gas entering from the bottom filaments show dramatic redshifts. (As before, the line of sight velocity of the material in the upper–right quadrant of these panels is less uniform, as it probes a gas accreting along a filament that is roughly perpendicular to the line of sight.) The qualitative result in each case is a massive clumpy and chaotic disk–like structure of co–planar high angular momentum cold gas accretion kinematically linked to inflow from the cosmic web—i.e., what has been previously referred to as a “cold flow disk.” To aid the eye in comparing the images, an ellipse has been overlaid on each image roughly corresponding to this cold flow disk region, inside of which coherent rotation of inflowing cold gas is apparent). Though the precise extent, orientation, clumpiness, size, and structure of these disks vary between simulations, each code produces a qualitatively similar picture, where there is a clear rotational velocity structure within the virial radius of the halo that is kinematically linked to that of the large–scale filamentary environment of Figure 4, with the exception of the Gizmo–PSPH simulation at this epoch; see §5.1.

We repeat a similar line of sight analysis along an orthogonal orientation within the halo virial radius for $z = 2$ (top), $z = 1.5$ (middle) and $z = 1$ (bottom) for various subsets of the simulation runs in Figure 7. Again, the precise structure of these disks varies between simulations, but most of the simulations produce qualitatively similar pictures; there is a clear rotational velocity structure within the virial radius of the halo that is kinematically linked to the large–scale filamentary inflow shown in Figure 5. As with Figure 5, the only exception appears to be Ramses at $z = 1$, which has evacuated most of its halo of cold dense gas altogether, likely as a result of the lack of UV self–shielding implementation in the simulation. These qualitatively similar results, across a broad range in hydrodynamic code types and subgrid physics models of galaxy formation, seem to suggest that the formation of cold flow disks at $z > 1$ is a natural consequence of high angular momentum filamentary inflow along the cosmic web, and represents a robust prediction of cosmological gas accretion in LCDM.

5.1. The rapid destruction and reformation of a cold flow disk at $z = 3$

In the discussion of Figures 4 and 6, we noted that the velocity structure of the galaxy in the Gizmo–PSPH simulation is not nearly as clean and orderly as the other simulations at $z = 3$, and therefore does not seem to host a clear cold flow disk structure. While we thought it important to show all galaxies at precisely the same epoch, we note that in Gizmo–PSPH, the galaxy happens to be in the midst of a post–merger starburst, accompanied with a violent outflow event at this epoch, due to this code’s strong feedback physics. Halo–halo mergers, of course, tend to occur at broadly similar times in all codes, but differences in galaxy masses and halo baryonic mass distributions mean that the galaxy–galaxy mergers can and do occur at significantly different times, and with different mass ratios and corresponding consequences for star formation, at the halo center (see e.g. Stewart et al.).

19 As another example of different merger timing, note the location and structure of the infalling galaxy embedded in the filament to the upper–left of the main galaxy in each panel.
in the improved P–SPH implementation; this is supported by early results from the FIRE–2 simulations, which use a different, mesh–free Godunov–type finite volume method to solve the hydrodynamics (P. Hopkins, private communication).

In Figure 5 we show the structure of this galaxy immediately before and after this violent merger event. The time sequence begins in the left panel at $z = 3.3$, where inflowing cold gas demonstrates coherent rotation in the galaxy halo, including a large cold flow disk of accreting gas. (To aid the eye in comparing the images, some of which are quite chaotic during the outflow, an ellipse has been overlaid on each image roughly corresponding to this initial cold flow disk region—identical to those in Figure 6.) At this epoch, the recent influx of of fresh gas onto the central regions of the galaxy results in a spike in star formation, and consequently a violent spherical outflow event from $z = 3.2–3.0$ that effectively destroys the ISM of the galaxy (leaving a deficit of gas in the center of the galaxy, as seen at $z = 3.1$) and disrupting the inflowing filamentary gas in the CGM of the halo. However, the filamentary gas continues to flow into the halo, and this inflow continues to contribute substantial angular momentum. As a result, a new cold flow disk begins to form almost immediately after the outflow event has subsided, again showing orderly rotation of fresh gas inflow along a very similar orientation to the original cold flow disk (right–most panel). We argue that the bursty nature (Muratov et al. 2015) of the subgrid physics as implemented in GIZMO–PSPH coupled with this demonstration of the near–immediate regrowth of a cold flow disk after a massive outflow event only reinforces the robust nature of cold flow disk formation in LCDM.

6. DISCUSSION

Our results indicate that the existence of the cold flow disk phenomenon is expected to be a robust prediction of LCDM that is not sensitive to variations in current subgrid physics models or hydrodynamic astrophysical code implementations. This finding has significant implications for the observational verification of the cold flow scenario (as discussed in the Introduction). While the cold flow paradigm of gas accretion and galaxy formation has been increasingly emphasized in galaxy formation theory, it is quite difficult to observationally verify this mode of gas accretion onto galaxies. Extended, high–angular momentum, co–rotating gaseous structures are a testable observational signature of the cold flow model of galaxy formation.

Encouragingly, there is a growing body of observational evidence that seems to indicate that co–rotating halo gas of this kind is present around real galaxies. For example, kinematic studies of some Ly–α nebulae suggest rotational velocities and inflow rates consistent with those expected for cold flow disks (Martin et al. 2014; Prescott et al. 2015). Similarly, absorption line studies are beginning to emphasize the bimodal distribution of absorption detections, where detections along the galaxy’s minor axis tend to show absorption properties consistent with outflowing gas, while detections roughly along the galaxy’s major axis demonstrate properties (such as co-rotational inflow) that are consistent with cold flow disks (Kacprzak et al. 2010, 2012a,b; Bouché et al. 2012, 2013; Crigton et al. 2013; Nielsen et al. 2015; Bouché et al. 2016).

In perhaps the most direct confirmation of the existence of cold flow disks, Martin et al. (2015) performed a spectroscopic analysis on the cosmic filament (illuminated by two nearby QSOs) first detected by Cantalupo et al. (2014) at $z \sim 2$, and found that a substantial fraction of what was initially thought to be part of the illuminated filament was in fact a massive rotating gaseous structure. This extremely extended gaseous disk (extending to $\sim R_{\text{vir}}/2$, corresponding to a width of 125 physical kpc) showed smooth rotation kinematics, with one side of the disk kinematically linked to the inflow velocity of the nearby cosmic filament. This very closely resembles what we have presented here for cold flow disks, though we note that the particular system observed by Martin et al. (2015) was estimated to be a much more massive halo than what we have simulated here ($M_{\text{vir}} \sim 10^{13}M_{\odot}$) and it therefore had a correspondingly more massive cold flow disk than shown here, as might be expected for a larger, more massive halo. A similar cold flow protodisk, again fed by a cosmic filament that was first detected in Ly–α emission, was also reported in Martin et al. (2016), suggesting that cold flow disks may be common phenomena for massive galaxies at high redshift.

While not seen in our particular simulations, we also speculate that polar ring galaxies—which have previously been suggested as evidence of cold flow gas accretion onto galaxies (Maccio et al. 2006; Brook et al. 2008; Spavone et al. 2010)—may be a result of a similar phenomenon. Such galaxies could reasonably occur when strong central torques (e.g., from a major galaxy merger) result in a near perpendicular misalignment between the angular momentum of the central galaxy and that of the inflowing cold mode gas.

We note that the cold flow disks in our simulations are significantly more massive and extended (relative to the halo virial radius) at high redshift, when cosmic filaments are more narrowly defined and contain higher density gas flows. However, Figure 3 and previous work (e.g., Stewart et al. 2013) both demonstrate that accreting cold gas continues to have high angular momentum, even at later times where the rotational signature of a cold flow disk is less clear. We speculate that it may be possible that this high angular momentum accretion helps to explain observations of extended XUV disks (e.g., Thilker et al. 2005, 2007; Lemonias et al. 2011; Holwerda et al. 2012), local extended H1 disks (e.g., García–Ruiz et al. 2012; Oosterloo et al. 2007; Walter et al. 2008; Christlein & Zaritsky 2008; Sancisi et al. 2008; Wang et al. 2013; Huang et al. 2014; Courtois et al. 2015) and co–rotating cold halo gas around local Milky Way analogs (e.g., Diamond–Stanic et al. 2015).

Indeed, these growing observations of high angular momentum material in the outskirts of galaxy halos would be quite difficult to explain if one were to assume the canonical picture of galaxy formation whereby baryons in galaxy halos share the same distribution of angular momentum as the dark matter. In contrast, the cold flow paradigm naturally predicts that halo gas (and particularly the cold halo gas) preferentially constitutes re-
cent gas accretion from the cosmic web, with $\sim 3$–5 times the angular momentum of the dark matter, naturally explaining the kinds of high angular momentum phenomena being observed. We caution, however, that we have not focused on the gaseous halos in our simulations at $z < 1$ here, and leave a more detailed comparison between simulations and low–$z$ observations as a topic of further study.

7. CONCLUSION

We have simulated the evolution of a Milky Way sized galaxy from identical cosmological initial conditions with a variety of simulation codes: Enzo, Art, Ramses, Arepo, and Gizmo–PSPH. Each code has used subgrid physics models drawn from scientific literature common to each simulation type, and we have compared the simulations in an attempt to draw robust conclusions about galaxy formation in LCDM (focusing on $z > 1$) that are not sensitive to uncertain aspects of galaxy formation simulations. To ensure uniform analysis among the various code types, we have used the hydrodynamic analysis software yt, which enables the same analysis routine to be performed on each code.

While we found many aspects of the simulated galaxies that did vary substantially among the simulations (e.g. morphology, stellar mass, hot halo temperature and density, to be discussed further in future work), we found the following qualitative features common to all simulations, regardless of which subgrid physics model or hydrodynamic code was used:

1. Gas in the galaxy halo has substantially higher specific angular momentum than the dark matter in the halo, with mean values of $j_{\text{cold}} \simeq 4j_{\text{DM}}$ and $j_{\text{hot}} \simeq 2j_{\text{DM}}$ (though with considerable scatter), leading to a typical cold halo gas spin parameter of $\lambda_{\text{cold}} \simeq 0.15$.

2. The large scale filamentary structure is qualitatively similar in all simulations (with minor variations, for example regarding lower mass streams of secondary importance to the galaxy’s growth). The three–dimensional geometry of these filaments, which are all flowing towards the central galaxy (the highest overdensity in its environment) results in a strong line of sight velocity structure. FILaments flowing onto the galaxy from opposite directions (along an arbitrary line of sight) tend to show alternating blueshifted and redshifted velocities relative to the galaxy as they flow towards the galaxy center.

3. As the filamentary gas accretion enters the virial radius, the large scale velocity structure of the accreting filaments often leads to the creation of cold flow disks—massive gaseous disks of high angular momentum cold gas that is transitioning from the cosmic web, though the halo, to eventually accrete onto the galactic disk. This process is continuous and dynamic; even after violent outflow events disrupt the CGM, inflowing gas quickly regrows the cold flow disk soon after the outflow has subsided. The maximum line of sight velocity expected for these cold flow disks is $\sim 250$ km/s (corresponding to roughly 1.5 times the virial velocity of the halo).

In this work, we have limited our analysis to the growth of a single Milky Way size halo at $z > 1$ using a variety of different hydrodynamic codes and feedback physics implementations. It is difficult to draw general conclusions about galaxy formation from the simulation of a single halo, however, a number of theoretical works have previously established the high angular momentum nature of filamentary gas accretion, using various hydrodynamic codes, larger cosmological volumes, and/or analysis of multiple zoom–in simulations. For example: Pi–chon et al. (2011) analyzed $\sim 15,000$ halos at $z > 1.5$ from a (lower resolution) cosmological–scale simulation using the Ramses code; in a companion work, Kimm et al. (2011) also included $\sim 900$ intermediate resolution halos and 2 high resolution zoom–in simulations to $z = 0$ using Ramses; Stewart et al. (2011b, 2013) analyzed 4 zoom–in simulations to $z = 0$ using the SPH code Gasoline; and Danovich et al. (2015) analyzed 29 zoom–in simulations at $z > 1.5$ using the Art code. The results presented here demonstrate that the high angular momentum nature of cold gas accretion in LCDM is not likely to change (in the qualitative sense) among a broad range of different physics implementations and hydrodynamic codes, suggesting that high angular momentum halo gas (sometimes taking the form of cold flow disks) appears to be a robust expectation of LCDM.

We note that the prevalence of cold flow disks is yet to be fully understood, and thus cannot be determined from the single high resolution simulation presented here. The buildup of a cold flow disk is dependent on the geometry and kinematics of the cosmic web in the galaxy’s environment, so we speculate that there are likely to be significant environment effects even at fixed halo mass. For example, Milky Way sized galaxies near the outskirts of galaxy clusters would not dominate the gravitational potential of the cosmic web in their large-scale environment, so we may not expect to find the same clear velocity signature of filamentary inflow (i.e., Figure 4) for such systems.

While all simulations presented here produced cold flow disks of a qualitatively similar nature at some point during the growth of the halo, we also note that there are considerable variations in their quantitative nature (morphology, rotational velocity, size, temperature, density, etc.). The exploration of the prevalence of cold flow disks in simulations, for different environments and halo masses, would be a useful topic of further study, especially in light of recent observations of a cold flow disks at $z \sim 2$ (Martin et al. 2015, 2016) that are strikingly similar to the qualitative results presented here.

Computations described in this work were performed using the publicly-available Enzo code (http://enzo-project.org), and the publicly-available yt toolkit (http://yt-project.org/), both of which are the products of collaborative efforts of many independent scientists from numerous institutions around the world. Their commitment to open science has helped make this work possible. KRS would especially like to thank Matt Turk and Nathan Goldbaum for their support with yt. KRS also thanks Volker Springel, Lars Hernquist and Paul
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