NEW CASCADED LATTICE STRUCTURES FOR FIR FILTERS HAVING EXTREMELY LOW COEFFICIENT SENSITIVITY

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ABSTRACT

It is shown that any arbitrary FIR transfer function can be implemented in the form of a passive structure, resulting in low coefficient sensitivity. The building blocks are planar-rotation operators, and internal signal nodes are automatically L_2-scaled. Denormalized versions of the building blocks are also shown. Design examples are included.

I. INTRODUCTION

The design of low-sensitivity structures for finite impulse response (FIR) filters has received some attention in the past [1]–[4]. It is generally recognized that the direct-form structure displays poor stopband sensitivity, whereas the cascade-form has poor passband sensitivity, particularly for large orders [1]. Certain other types of structures have also been known in the past [3]–[4].

In the case of infinite impulse response filters (IIR), there exist several systematic procedures for the synthesis of structures with low passband-sensitivty. It is also known that many of these structures share a common basic property known as structural passivity [6]. The role of structural passivity in forcing a low passband-sensitivity is also known [6],[8]. In the case of FIR filters, structurally passive implementations are known only for the restricted case of Type 1 linear-phase filters [9]. A natural question that arises, therefore, is whether it is possible to synthesize passive FIR structures, for *arbitrary* FIR transfer functions. The answer is in the affirmative, and we show in this paper how such structures can be obtained. Numerical design examples are also included in order to demonstrate the low-sensitivity behavior of the new passive FIR structures.

II. PASSIVE FIR STRUCTURES

Let P_{N-1} be an FIR transfer function of order $N-1$, given by

$$P_{N-1}(z) = \sum_{n=0}^{N-1} p_{N-1,n} z^{-n} \qquad (1)$$

where $p_{N-1,n}$ are real-valued. Assume, without loss of generality, that

$$\left|P_{N-1}(e^{j\omega})\right| \leq 1, \qquad \text{for all } \omega\,. \qquad (2)$$

In other words, $P_{N-1}(z)$ is FIR BR [9], where the acronym BR stands for *bounded real*. The phase response of $P_{N-1}(z)$ may or may not be linear; no assumption is made in this regard. We wish to find an implementation of (1) with N parameters $k_0, k_1, \ldots, k_{N-1}$ such that, as long as the parameters are bounded by

$$|k_m| \leq 1, \qquad 0 \leq m \leq N-1 \qquad (3)$$

the boundedness of (2) automatically holds. Such implementations are called structurally bounded (or passive) and have low passband-sensitivity as explained in [6].

It is clear that $P_{N-1}(z)$ is completely characterized by the N parameters $p_{N-1,0}, p_{N-1,1}, \ldots, p_{N-1,N-1}$. A direct-form implementation of (1) has multiplier values precisely equal to $p_{N-1,n}$. In view of (2), it is clear that the coefficients $p_{N-1,n}$ are bounded as

$$\left|p_{N-1,n}\right| \leq 1, \qquad 0 \leq n \leq N-1\,. \qquad (4)$$

However, for an arbitrary FIR function, if (4) holds, it does not necessarily imply (2). In other words, the direct-form implementation is not structurally bounded. Given an arbitrary FIR BR $P_{N-1}(z)$ (*i.e.*, given the set of N coefficients $p_{N-1,n}$), our goal is to find an equivalent set of parameters k_m such that

1. The set $k_0, k_1, \ldots, k_{N-1}$ completely characterizes $P_{N-1}(z)$.
2. Inequality (3) holds.
3. Conversely, and most important, every set of N k_m-parameters satisfying (3) necessarily implies (2). As a result, even if k_m are quantized after their computation, as long as they satisfy (3), the transfer function remains BR. Thus, $\{k_m\}$ is a passive characterization.

Once we describe how to find such a set of parameters, a passive digital filter structure is immediately placed in evidence, and has low passband-sensitivity. The synthesis procedure, described below, explains how the above characterization can be achieved.

III. THE SYNTHESIS PROCEDURE

Given an FIR BR transfer function $P_{N-1}(z)$, let us define a complementary transfer function $Q_{N-1}(z)$ such that

$$\left|P_{N-1}(e^{j\omega})\right|^2 + \left|Q_{N-1}(e^{j\omega})\right|^2 = 1 \qquad (5)$$

for all ω. Since $P_{N-1}(z)$ satisfies (2), we can always find $Q_{N-1}(z)$ with real coefficients satisfying (5). The vector $\mathbf{G}_{N-1}(z) \triangleq \left[P_{N-1}(z) \;\; Q_{N-1}(z)\right]^t$ is said to be a lossless FIR vector, or simply an FIR allpass function, in view of (5). We now show how to synthesize a structure of the form in Figure 1, to realize $P_{N-1}(z)$ and $Q_{N-1}(z)$. In this structure, the rectangular building blocks (characterized by the real parameters k_m) are memoryless, *i.e.*, have no delay elements. The structure will be such that, as long as the parameters are bounded as in (3), the equality (5) holds, hence (2) holds. The basic step in the synthesis procedure is the following order reduction problem:

The Order Reduction Problem

Let $P_{m+1}(z)$ and $Q_{m+1}(z)$ be two FIR BR functions

$$P_{m+1}(z) = \sum_{n=0}^{m+1} p_{m+1,n} z^{-n}\,, \qquad (6a)$$

$$Q_{m+1}(z) = \sum_{n=0}^{m+1} q_{m+1,n} z^{-n} \qquad (6b)$$

such that

$$\left|P_{m+1}(e^{j\omega})\right|^2 + \left|Q_{m+1}(e^{j\omega})\right|^2 = 1 \qquad (7)$$

for all ω. We wish to construct these functions in terms of two lower-order functions $P_m(z)$ and $Q_m(z)$

$$P_m(z) = \sum_{n=0}^{m} p_{m,n} z^{-n},\; Q_m(z) = \sum_{n=0}^{m} q_{m,n} z^{-n} \qquad (8)$$

as shown in Figure 2, such that

$$\left|P_m(e^{j\omega})\right|^2 + \left|Q_m(e^{j\omega})\right|^2 = 1\,. \qquad (9)$$

If this process is repeated starting from $m+1 = N-1$, we eventually obtain the structure of Figure 1, with $P_0^2 + Q_0^2 = 1$. In order to accomplish this, first note that given two arbitrary polynomials $P_{m+1}(z)$ and $Q_{m+1}(z)$ as in (6), it is easy to obtain $P_m(z)$ as in (8) by defining

$$P_m(z) = -q_{m+1,m+1} P_{m+1}(z) + p_{m+1,m+1} Q_{m+1}(z) \qquad (10)$$

because the highest power in (10) simply cancels. Next, since $P_{m+1}(z)$ and $Q_{m+1}(z)$ are constrained by (7), they satisfy the following additional property:

$$p_{m+1,0} p_{m+1,m+1} + q_{m+1,0} q_{m+1,m+1} = 0 \qquad (11)$$

which can be verified by equating the highest power of $e^{j\omega}$ in both sides of (7). Accordingly, the polynomial

$$p_{m+1,m+1} P_{m+1}(z) + q_{m+1,m+1} Q_{m+1}(z) \qquad (12)$$

has the form $z^{-1}Q_m(z)$ where $Q_m(z)$ is in the form (8). We can obtain normalized versions of (10),(12) by redefining

$$P_m(z) = k_{m+1} P_{m+1}(z) + \hat{k}_{m+1} Q_{m+1}(z) \qquad (13)$$

$$z^{-1}Q_m(z) = -\hat{k}_{m+1} P_{m+1}(z) + k_{m+1} Q_{m+1}(z) \qquad (14)$$

where

$$k_{m+1} = \frac{-q_{m+1,m+1}}{\sqrt{p_{m+1,m+1}^2 + q_{m+1,m+1}^2}}\,, \qquad (15a)$$

$$\hat{k}_{m+1} = \frac{p_{m+1,m+1}}{\sqrt{p_{m+1,m+1}^2 + q_{m+1,m+1}^2}}\,. \qquad (15b)$$

Clearly, $|k_{m+1}| \leq 1$. In matrix form, the inverse of (13),(14) is given by

$$\begin{bmatrix} P_{m+1}(z) \\ Q_{m+1}(z) \end{bmatrix} = \begin{bmatrix} k_{m+1} & -\hat{k}_{m+1} \\ \hat{k}_{m+1} & k_{m+1} \end{bmatrix} \begin{bmatrix} P_m(z) \\ z^{-1}Q_m(z) \end{bmatrix}\,. \qquad (16)$$

Figure 3 shows the interconnection pattern represented by (16).

Since the 2×2 matrix in (16) is orthogonal, equation (9) holds, as long as (7) holds. As a result, the

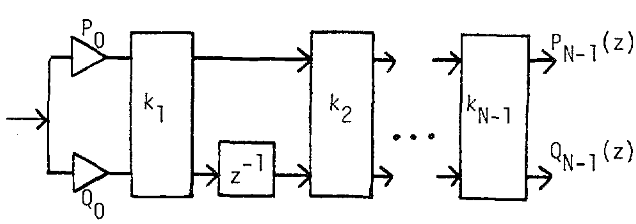

Fig. 1. The overall structural appearance.

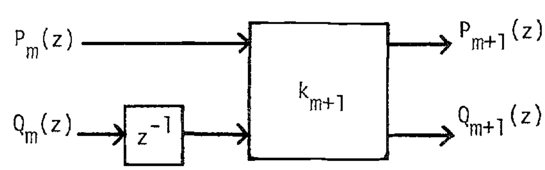

Fig. 2. The order-reduction process.

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above order reduction process can be repeated, completing the synthesis.

Each building block in Figure 1 is lossless in view of the orthogonality of its transfer matrix

$$\mathbf{k}_{m+1} = \begin{bmatrix} k_{m+1} & -\hat{k}_{m+1} \\ \hat{k}_{m+1} & k_{m+1} \end{bmatrix} \quad (17)$$

and hence the structure can be implemented as a sequence of planar-rotation operations. In summary, we have obtained a passive implementation of an arbitrary FIR BR function, as a cascade of lossless building blocks.

Complexity of the New Implementation

First notice that, if we assume $\hat{k}_{m+1}$ to be nonnegative and adjust, if necessary, the sign of k_{m+1} accordingly, this does not affect the overall transfer functions $P_{N-1}(z)$ and $Q_{N-1}(z)$, except for a possible change of sign. Thus we can always take $\hat{k}_{m+1} = +\sqrt{1-k_{m+1}^2}$. Accordingly, there are only N independent parameters in the structure of Figure 1.

In view of the presence of 4 multipliers per section, the new structures have more multipliers than the direct form. This overhead can be justified by the observation that all internal nodes of interest in Figure 1 are scaled in an L_2 sense, which is often an advantage. Notice, in addition, that we obtain two transfer functions simultaneously out of the structure of Figure 1. Moreover, in practice each building block can be implemented by using cordic forms [7] rather than by using 4 multipliers explicitly. Finally, notice that the following operation

$$\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \ \cos\theta = k_{m+1} \quad (18)$$

is equivalent to a single complex multiplication, $c+jd = e^{j\theta}\cdot(a+jb)$ and hence Figure 1 can be implemented as a sequence of N complex multiplications. Thus, if complex-arithmetic is readily available, this seems to be the most convenient implementation scheme.

A denormalized version of the structure of Figure 1 can be obtained simply by employing the building block of Figure 4 instead of Figure 3. Notice that $P_{N-1}(z)$ and $Q_{N-1}(z)$ are unaffected, except for a common scale factor. A total of about $2N$ multipliers is then required, *i.e.*, N multipliers per transfer function, which is the same as for direct-form implementations of (nonlinear-phase) FIR transfer functions.

Generality of the Passive Structure of Figure 1

Notice that any arbitrary FIR function $P_{N-1}(z)$ (scaled such that (2) holds) can be implemented as in Figure 1, with all k_m satisfying (3). The location of zeros and the phase response of $P_{N-1}(z)$ are unrestricted. Thus $P_{N-1}(z)$ could be linear-phase of any Type [10], or minimum-phase, or mixed-phase. In particular, if $P_{N-1}(z)$ happens to have linear-phase, then it can be shown that $k_m^2 = k_{N-1-m}^2$. Accordingly, in spite of quantization of parameters, linearity of phase can be maintained.

In this context, recall that the FIR lattice structures arising out of the theory of linear prediction [11] have an overall appearance as in Figure 1 (except that the building blocks are different), but can be used to realize only minimum and maximum phase transfer functions when k_m are restricted as in (3).

A Comment on Quantization

In practice, the physical multipliers k_m and $\hat{k}_m$ cannot be simultaneously quantized while satisfying $k_m^2 + \hat{k}_m^2 = 1$. Accordingly, under quantized conditions, the structure of Figure 1 is not *lossless*, *i.e.*, (5) does not hold; but we can quantize k_m and $\hat{k}_m$ such that equality in (5) is replaced with "$\leq$." Clearly, this is sufficient to ensure passivity and hence low passband-sensitivity.

A design example:

A linear-phase FIR function $P_{N-1}(z)$ of order $N-1 = 60$ was synthesized in the above manner. Figure 5 shows the complementary responses of $P_{N-1}(z)$ and $Q_{N-1}(z)$. An implementation as in Figure 1, with multipliers quantized to 3 bits, has response $|P_{N-1}(e^{j\omega})|$ as in Figure 6. The passband details of the 3 bit implementation are as in Figure 7, where a direct-form response is also shown for comparison. The excellent passband-

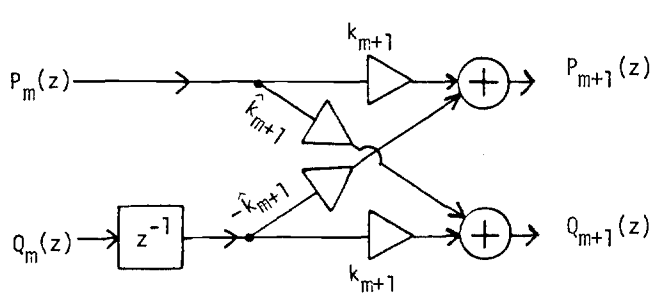

Fig. 3. Structural pattern for Eqn. (16)

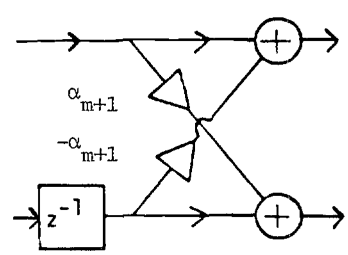

Fig. 4. A de-normalized section. Here $\alpha_{m+1} = \hat{k}_{m+1}/k_{m+1}$

10B. 1. 3

sensitivity of the new passive structures is clearly in evidence.

CONCLUDING REMARKS

A procedure for synthesizing passive structures for arbitrary FIR transfer functions has been presented and demonstrated. We feel that the results can be extended to the case of FIR filter banks with M filters satisfying an extended version of (5). Finally, we believe that the number of multipliers in a de-normalized version can be further reduced to a theoretical minimum. These and related issues are currently being studied.

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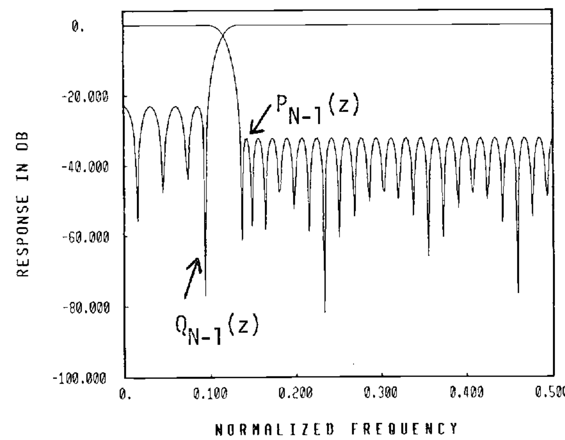

Fig. 5. Simulated lattice response.

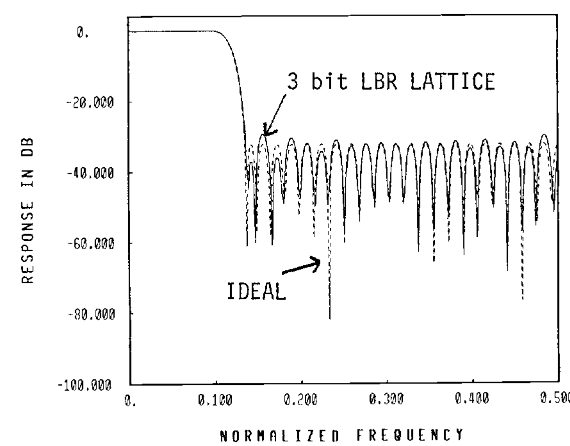

Fig. 6. LBR lattice response, 3-bits.

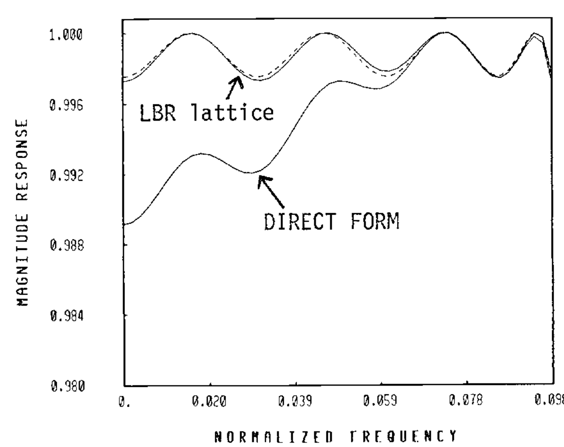

Fig. 7. Passband details for 3-bits. (broken line is for ideal response)

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