Driving-induced many-body localization: Supplemental Material

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A. Relaxation of initial product states at low frequencies

In addition to the increase of the level statistics parameter with system size, the lack of localization at low driving frequencies is also manifested in the relaxation of initial product states of particle occupations. When these states are driven at $\omega = 3.5J$, their generalized imbalance decays to a value which decreases with system size (Fig. S1), as in the undriven case. Note that the decay as a function of time is slower when compared to the decay in the undriven case. Presumably, this is due to the proximity to the MBL transition [1] for this value of the driving frequency.

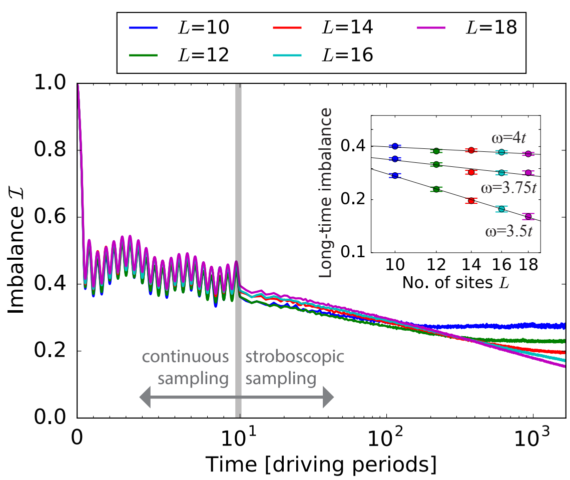

Figure S1. Imbalance as a function of time for $J_{eff} = 0$ $[A/\omega = (A/\omega)^*]$ at a low driving frequency $\omega = 3.5J$. The imbalance at long times decays with system size, indicating that the system is in the delocalized phase as expected from level statistics analysis. This is in contrast to the localized case for $\omega = 5J$ (Fig. 3 in the main text), where the long-time imbalance does not depend on system size. Inset: imbalance at long times (averaged over $1.4 \times 10^3 T < t < 1.5 \times 10^3 T$) as a function of system size for different driving frequencies. Error bars indicate one standard deviation over disorder realizations. The values for the slopes are -0.91 ± 0.04, -0.33 ± 0.12, -0.15 ± 0.06 for $\omega = 3.5J$, $3.75J$, $4J$ respectively. The diminishing slope as the driving frequency approaches $\omega = 4J$ indicates slowing down of the dynamics, due to the proximity to the transition into the MBL phase.

As the driving frequency approaches the speculated critical frequency $\omega_c \approx 4J$, the remaining imbalance at long times declines much slower with system size (Fig. S1 inset). The quick flattening of the slope in the inset of Fig S1, for a small change of driving frequency, provides an independent corroboration for the value of the critical frequency.

B. Phase boundaries between the ergodic and MBL phases

The phase boundaries in Fig. 1 were obtained by finite-size scaling of the quasi-energy level statistics $\langle r \rangle$. Examples of such data are given in Figs. 2, 4, S2. Namely, a consistent increase of $\langle r \rangle$ with system size is interpreted as an indication for the ergodic phase, while a consistent decrease of $\langle r \rangle$ with system size is interpreted as an indication for the MBL phase. Error bars indicate parameter ranges where the trend in level statistics with increasing system size is not statistically significant (according to the error bar for $\langle r \rangle$, as shown for example in Figs. 2, 4, S2).

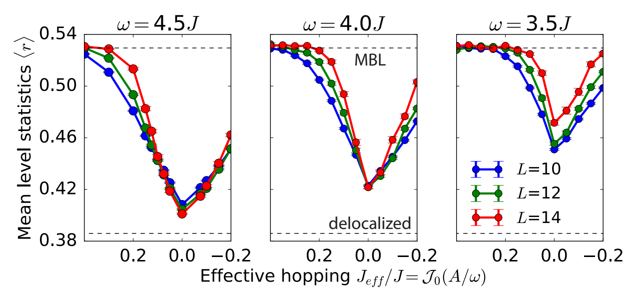

Figure S2. Quasi-energy level statistics as a function of driving amplitude at a few driving frequencies near ω_c. When the driving frequency is lowered, the range of driving amplitudes which induce many body localization narrows around $(A/\omega)^*$ corresponding to the first root of $\mathcal{J}_0$, for which $J_{eff} = 0$. At $\omega = 4J$, the level statistics parameter at $(A/\omega)^*$ changes very slowly with system size, indicating proximity to the critical frequency; at a lower frequency $\omega = 3.5J$, the level statistics parameter tends to the delocalized value at any driving amplitude $A/\omega > 0$ up to the first minimum of $\mathcal{J}_0$.

C. Level statistics as a function of driving amplitude at low frequencies

At $\omega = 5J$ we found driving-induced many-body localization in a range of driving amplitudes around $(A/\omega)^*$, corresponding to the first root of $\mathcal{J}_0$, and for which

$J_{eff} = 0$ (Fig. 2 in main text). We expect this range to narrow for lower driving frequencies, and to vanish altogether for $\omega < \omega_c \approx 4J$; at these frequencies, the drive fails to induce localization even at $J_{eff} = 0$ (Fig. 4). Indeed, when we perform finite-size scaling of the level statistics as a function of driving amplitude at frequencies lower than $5J$, we find that the range of localization-inducing driving amplitudes continuously narrows, and all but shrinks to a point $J_{eff} = 0$ at $\omega = 4J$ (Fig. S2 left, middle). Below this frequency, the level statistics parameter increases with system size for any driving amplitude $A/\omega > 0$ up to the first minimum of $\mathcal{J}_0$ (Fig. S2 right), indicating that the drive fails to induce localization for this range of driving amplitudes.

D. Driving-induced MBL beyond the first minimum of $\mathcal{J}_0$

So far, we have analyzed a range of driving amplitudes $0 \le A/\omega \le (A/\omega)_{\min}$, where $(A/\omega)_{\min}$ is the first minimum of the Bessel function (bright green in Fig. S3b). We performed additional simulations which suggest a qualitatively similar phase diagram at higher driving amplitudes.

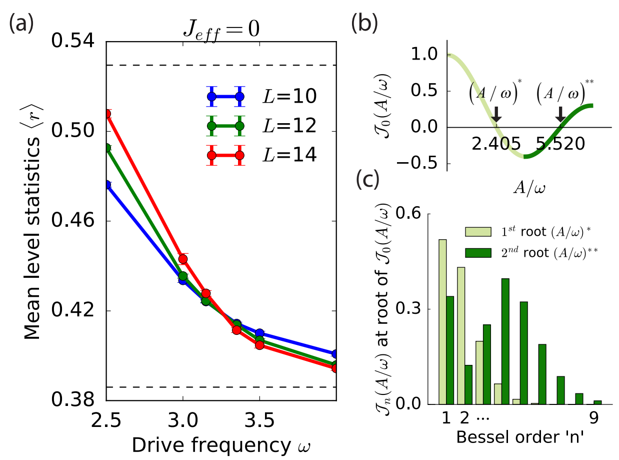

Figure S3. Driving-induced MBL at a driving amplitude corresponding to $(A/\omega)^{**}$, the second root of $\mathcal{J}_0(A/\omega)$. (a) Finite-size scaling of quasi-energy level statistics as a function of driving frequency. The critical frequency at the second root ($\omega_c \approx 3.25J$) is smaller than the one found at the first root, $\omega_c \approx 4J$. (b) Plot of the zeroth Bessel function $\mathcal{J}_0$: bright green indicates the amplitude range considered in the main text, additional simulations performed at the dark green amplitude range show qualitatively similar results. (c) Values of the various Bessel functions $\mathcal{J}_n$ at the first root $(A/\omega)^*$ (light green) vs. second root $(A/\omega)^{**}$ (dark green) of $\mathcal{J}_0(A/\omega)$, indicating the values of the non-zero Fourier modes of the Hamiltonian when the time-averaged Hamiltonian has no hopping term, $J_{eff} = 0$. The values of the negative orders are related by $\mathcal{J}_{-n}(A/\omega) = (-1)^n \mathcal{J}_n(A/\omega)$.

Specifically, we obtained the finite-size scaling of the quasi-energy level statistics for driving amplitudes corresponding to the interval between the first minimum of $\mathcal{J}_0$ to its next maximum (dark green in Fig. S3b). The result of this analysis also shows localization in a range of driving amplitudes above a critical driving frequency. To determine the critical driving frequency for values of A/ω corresponding to $(A/\omega)^{**}$, the second root of $\mathcal{J}_0$, we varied the driving frequency while tuning the driving amplitude such that A/ω remains fixed. The results, shown in Fig S3a, indicate that the critical driving frequency for inducing MBL at $(A/\omega)^{**}$ is $\omega_c \approx 3.25J$, which is lower compared the one found at the first root ($\omega_c \approx 4J$).

This decrease in the critical driving frequency can be understood by comparing the Fourier spectrum of the Hamiltonian at the first two roots of the zeroth Bessel function (Fig. S3c). While the first Fourier mode of the Hamiltonian is the most dominant when A/ω is tuned to the first root of $\mathcal{J}_0$, at the second root of $\mathcal{J}_0$ the bulk of its Fourier spectrum shifts to the higher harmonics. Intuitively, driving at a larger amplitude therefore has a similar effect to increasing the driving frequency.

E. Inducing MBL with a square-wave electric field

Our analysis so far focused on an AC electric field oscillating in time as $E(t) = A\cos(\omega t)$. However, the effective hopping in H_{eff} is suppressed also for other functional forms for periodic time dependence of the AC electric field. We tested the possibility to induce an MBL phase with a square-wave AC electric field:

$$E(t) = \begin{cases} A & 0 \le t < T/2 \\ -A & T/2 \le t < T \end{cases} \tag{S1}$$

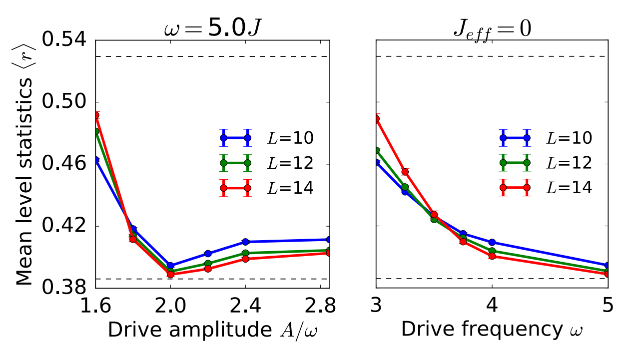

Figure S4. Finite-size scaling of quasi-energy level statistics $\langle r \rangle$ for the square-wave electric field (S1). Left: as a function of rescaled driving amplitude A/ω at a fixed driving frequency $\omega = 5J$. Right: as a function of driving frequency for $A = 2\omega$, corresponding to $J_{eff} = 0$ for the square-wave drive [see Eq. (S2)] .

Such a field can be used for exact dynamical localization in models with hopping beyond nearest-neighbor [2, 3]. In our model, performing the Peierls substitution and time-averaging the acquired phase leads to the effective hopping amplitude:

$$J_{eff}/J = e^{-i\frac{\pi}{2}\frac{A}{\omega}} \cdot \mathrm{sinc}\left(\frac{\pi A}{2\omega}\right) \tag{S2}$$

Again, finite-size scaling of quasi-energy level statistics shows localization in a range of driving amplitudes around $J_{eff} = 0$ (here obtained at $A/\omega = 2$) below a critical driving frequency $\omega \approx 3.5J$ (Fig. S4).

F. Driving-induced MBL at a lower filling fraction

Our numerical simulations so far focused on the case of half filling. This filling fraction was chosen to maximize the width of the many-body spectrum for a given number of sites, thus minimizing finite-size effects.

We expect qualitatively similar results at different particle fillings. The critical frequency might slightly decrease though, since the interactions become effectively weaker away from half filling (this is apparent when the filling is decreased below 1/2, but is also true when it is increased due to particle-hole symmetry). In the parameter range we use for our simulations, weaker interactions imply stronger localization with a shorter localization length; therefore, the local spectrum becomes narrower and the critical frequency should correspondingly decrease.

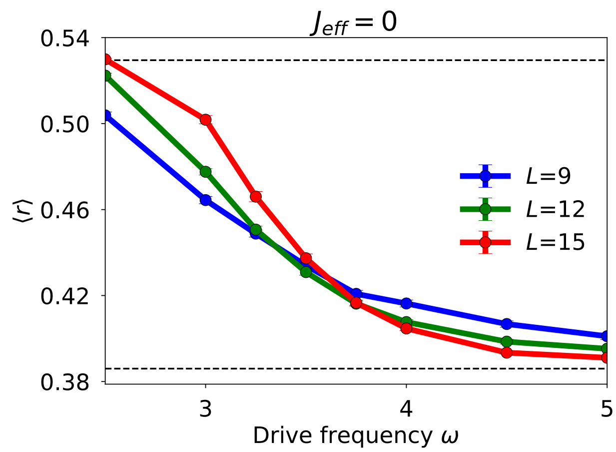

Figure S5. Finite-size scaling of quasi-energy level statistics $\langle r \rangle$ at filling fraction 1/3. We tune the driving frequency ω while fixing the rescaled driving amplitude A/ω at the first root of the zeroth Bessel function ($J_{eff} = 0$). We find a critical frequency $\omega \approx 3.75$ for inducing the MBL phase, which is slightly lower than the critical frequency found at half filling ($\omega \approx 4$).

To test this, we repeat the procedure of figure 4 at a different filling fraction 1/3 (Fig. S5). Namely, we fix the rescaled driving amplitude A/ω at the first root of the zeroth Bessel function ($J_{eff} = 0$) and perform finite-size scaling of the quasi-energy level statistics as a function of the driving frequency ω. Indeed, we find that the MBL phase is induced above a critical frequency $\omega \approx 3.75$, which is slightly smaller than the critical frequency $\omega \approx 4$ found at half filling.

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