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CANDIDATE ENTRY AND POLITICAL POLARIZATION: AN ANTI-MEDIAN VOTER THEOREM

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Abstract

We study a citizen candidate entry model with private information about ideal points. We fully characterize the unique symmetric equilibrium of the entry game, and show that only relatively "extreme" citizen types enter the electoral competition as candidates, whereas more "moderate" types never enter. It generally leads to substantial political polarization, even when the electorate is not polarized and citizens understand that they vote for more extreme candidates. Our results are robust with respect to changes in the implementation of a default policy if no citizen runs for office. We show that polarization increases in the costs of entry and the degree of risk aversion, and decreases in the benefits from holding office. Finally, we provide a simple limiting characterization of the unique symmetric equilibrium when the number of citizens goes to infinity. In the limit, only the very most extreme citizens, with ideal points at the boundary of the policy space, become candidates.

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1 Introduction

We present a theoretical argument that polarization of political candidates relative to the distribution of preferences in the underlying citizenry may be an unavoidable consequence of having open elections—that is, elections where any citizen is eligible to run for office—if there is asymmetric information between candidates and voters about the policy intentions of the candidates they are voting for. Political scientists have known for decades that, despite a barrage of information from the media, many voters are poorly informed about the true preferences of candidates at the time they are running for office, or where they sit on an ideological scale (Campbell et al. 1960, Palfrey and Poole 1987, and others). Asymmetry of information would seem to be, if anything, a greater problem the more open is the election. The question we then ask in this paper, for open-entry winner-take-all elections where ideal points are privately known and impossible to credibly reveal, what is the ideological distribution of the entering candidates, and how does the equilibrium outcomes depend on the underlying distribution of ideal points? The answer is an anti-median voter theorem. In large elections only the very most extreme citizens will compete for office. The result does not depend on the distribution of voter preferences, and the outcomes correspond to the unique symmetric equilibrium of the entry game.

The stark result of the model suggests that one should expect the distribution of preferences of political elites to be more polarized than the distribution of voter preferences. On the empirical side, there is some evidence suggesting this to be the case in western democracies with relatively open entry. For example, several political scientists have argued that the currently high polarization of political elites in the United States, especially elected officials (Poole and Rosenthal 1984, McCarty et al. 2006) is not matched by, let alone a result of, a high level of polarization of policy preferences in the underlying citizenry (Fiorina and Abrams 2008; Fiorina et al. 2006, DiMaggio et al. 1996). One possible explanation for this is that more extreme members of a polity may have a greater incentive to run for office than more moderate members. The difficulty in answering this question theoretically is that the incentives for entry into politics are determined endogenously, as they depend crucially on the entry strategies of other potential candidates. Thus, one needs to analyze the equilibrium of an entry game. Here we explore a model with asymmetric information, free entry of candidates, and limited ability for voters to control politicians once they are in office to identify the extent to which the combination of these factors may explain, as an equilibrium phenomenon, relatively extreme preferences of elected politicians and hence political outcomes.

Our analysis follows a similar approach as the citizen candidate models of entry in simple majority elections with complete information pioneered by Besley and Coate (1997) and Osborne and Slivinski (1996). However, we reach much different conclusions. These models, and ours, depart from standard spatial models of electoral competition (Downs 1957; Hotelling 1929) by introducing endogenous entry of candidates, or parties, when these have policy preferences of their own.[1] As a consequence, the benefits candidates

[1] The citizen candidate models have their roots in the earlier work on strategic entry, models related to Duvergers' law, and models with policy motivated candidates. See, for example, Feddersen (1992), Feddersen et al. (1990), Fey (1997, 2007), Osborne (1993), Palfrey (1984, 1989), and Wittman (1983).

enjoy from winning the elections do not only include direct personal benefits from holding office, but also the fact that by winning they can implement their preferred public policy outcomes. The baseline model with complete information about citizen ideal points proceeds as follows: A community is electing a new leader to implement a policy decision. In the first stage, each citizen can enter the electoral competition as a candidate, at some commonly known costs, and make a policy promise for the event of being elected (if nobody enters, a default policy is implemented). In the second stage, simple majority elections take place in which each citizen prefers the candidate whose ideal point is closest to her own ideal point, that is, yields her the highest utility. In the third stage, the newly elected leader implements her policy preferences as the new policy. If there are no barriers such as political primaries or nominating conventions, and no incumbents or political reputations or future elections, then incomplete information of voters about candidates would be a natural and potentially important component of the model. Any entrant willing to pay the cost of entry can do so, and the potential entrants are all identical except for their ideal point. The incorporation of the private (incomplete) information about citizens' and candidates' preferences is our main point of departure from the seminal citizen candidate models, and most others that have followed, which assume that the policy preferences of all citizens and hence all candidates are common knowledge. We use the same notion of entry equilibrium: entry strategies must be such that all citizens' entry decisions are optimal given the entry strategies of other citizens.

The two seminal citizen candidate models differ to some extent. For example, Osborne and Slivinski (1996) assume a continuum of citizens (i.e., potential candidates) and that each votes sincerely for her most preferred candidate. By contrast, Besley and Coate (1997) assume a finite number of citizens and strategic voting (i.e., a Nash equilibrium in undominated strategies for the elections). The models have some differences in their implications about candidates and outcomes, but have two important results in common. For most environments, they identify multiple equilibria with different numbers of candidates, which support both median and non-median policy decisions. Moreover, they show how the equilibrium set depends on the cost of entry, benefits from holding office, and the exact location of voter ideal points.

Here, we develop a citizen candidate model with a finite (possibly very large) number of citizens whose ideal points are private information and iid draws from a continuous distribution on the policy space. As in the seminal models, we look at an equilibrium in multiple stages that satisfy sequential rationality of *all* voters and candidates. Because we have private information, we look at perfect Bayesian equilibrium. We prove that a unique symmetric equilibrium exists, and provide a full characterization of it.[2] Because our model does not allow any coordination among the voters or candidates, and all citizens are drawn independently from the same distribution of types and have the same payoff functions, symmetric equilibria seem appropriate to focus on. This equilibrium in the entry stage of the game is characterized by a pair of cutpoint policies that determine the entry decisions, one on the left side of the ideological spectrum and the other on

[2] The uniqueness result is both strong and quite surprising in view of the dramatic multiple equilibrium problem in citizen candidate models with complete information. See, for example, Dhillon and Lockwood (2002), Eguia (2007), Roemer (2003), and the references they cite.

the right side. It has the property that citizens with "moderate" preferences (between the two cutpoints) never enter; only citizens with ideal points more extreme than one of the two cutpoints enters. A leftist citizen enters if and only if her ideal point is to the left of (or equal to) the left cutpoint, while a rightist citizen enters if and only if her ideal point is to the right of (or equal to) the right cutpoint. This unique cutpoint equilibrium implies a unique probability distribution of the number of candidates, and we derive the following comparative statics results about how this distribution changes with the underlying parameters of the model: if (*i*) the costs of entry increase, (*ii*) the benefits from holding office decrease, or (*iii*) the degree of risk aversion increases, then fewer candidates enter, in the sense of first order stochastic dominance, and they are more extreme on average. Finally, we derive the expected number of candidates for a very large citizenry and show that in the limit only the very most extreme possible citizens enter the electoral competition. Thus, in both small and large electorates, the distribution of ideal points of candidates will necessarily be more polarized—by any measure one might use—than the distribution of ideal points in the population they are representing. Moreover, this "political polarization" effect is greater the larger is the electorate. Thus in very large electorates, outcomes will typically coincide with the most extreme policies in the policy space, rather than the median ideal point—hence the term "anti" median voter theorem.

The model extends the analysis of citizen candidate models in another important direction. So far, citizen candidate models have utilized a fixed exogenous default policy, for example the status quo policy, in the event that nobody runs for office.[3] In contrast, we employ a default policy the effect of which is endogenously determined in equilibrium. Specifically, our default policy randomly selects one citizen as the new leader to implement a policy decision. For example, consider a community without any reasonable candidate. Under pressure to make important policy decisions, a citizen who has substantial policymaking experience but did not campaign is convinced to act as the interim leader, irrespective of her political leaning. On the face of it, this sounds like just another exogenous specification of the default decision. However, in our model with private information, the equilibrium cutpoints affect the distribution of ideal points that are sampled in the event of no entry. Thus, our random default policy has the advantage that it determines the stochastic policy decision endogenously, as part of the equilibrium.

At the end of the paper, we also show that the result can be extended to other variations on the model, in natural ways. For example, we consider a variety of other default specifications and show that our polarization results are robust. The equilibrium cutpoints change in minor ways, but the qualitative results are unchanged. In addition to various exogenous specifications, we also consider an alternative endogenous default policy with multiple entry rounds. That is, if nobody runs for office, another entry round follows and this continues until eventually at least one candidate enters. While we do not analytically characterize the solution, we can show that it will also lead to political polarization. In fact, the first round entry cutpoints in the multi-round model are more extreme than in the one-round model.

We also consider relaxing the informational asymmetry by assuming that some infor-

[3]Some specification of a no-entry outcome is needed for the game to be well defined.

mation about the candidates' ideal points can be identified by voters. An important piece of information that citizens could base their vote on is a candidate's political leaning "left" or "right", without full detailed knowledge of their exact stands on all issues. We investigate how such partially private information, that is *directional* information about the candidates' ideal points, would affect the results of our model with private information. To incorporate directional information in our model, we introduce common knowledge about whether each candidate is a "leftist" or "rightist" on the policy space. We show that this kind of realistic additional information does not affect our results at all when the citizens' ideal points are symmetrically distributed around the median.

Several papers have begun to explore the effects of uncertainty on citizen candidate equilibria, in several different ways. For example, Eguia (2007) allows for uncertain turnout and shows how this can reduce somewhat the set of equilibria in the model of Besley and Coate (1997). Moreover, Fey (2007) uses the Poisson game approach to study entry when there is an uncertain number of citizens. Brusco and Roy (forthcoming) add aggregate uncertainty, allowing for shifts in the distribution of ideal points. Casamatta and Sand-Zantman (2005) study a model with private information and three types of citizens, and analyze the asymmetric equilibria of the resulting coordination game. Finally, although not a citizen candidate model, Osborne et al. (2000) study a model where extreme types participate in costly meetings and the moderate policy outcome is the result of a bargaining process of extremists interests in both directions left and right. In their model, those who enter the bargaining are extreme types because their benefits are high relative to the meeting costs, but no uncertainty about their ideal points is needed to produce this result, and more moderate types abstain because the bargaining outcome is moderate. In the framework of a citizen candidate model with public information about the candidates' ideal points, moderates close to the median ideal point are a threat in the sense that they can win the elections outright if they enter. By allowing for private information, these moderate types are kept out not only because of their lower expected utility from implementing the own preferred policy, but also because their chances of winning the elections is equal to that of all other types, including extremists.

2 General model

A community of $n \geq 2$ citizens is electing a new leader to implement a policy decision. The policy space is represented by the $[-1, 1]$ interval of the real line. Each citizen $i = 1, ..., n$ has preferences over policies, which are represented by a concave utility function $-\left|\frac{1}{2}(x_i - \gamma)\right|^{\alpha}$, $\alpha \geq 1$, that is decreasing in the Euclidean distance between the policy decision, $\gamma \in [-1, 1] \subset \mathbb{R}$, and her ideal point (or, type), $x_i \in [-1, 1] \subset \mathbb{R}$. This is a special case of power utility functions, and one can think of α as a measure of the citizens' risk aversion, where risk aversion is strictly increasing in α.[4] Examples are the commonly-used

[4]Essentially all the results extend to general concave single-peaked utility functions, with few restrictions. This is explained in the proof of Lemma 1, which is central to all the results and is is done for general utility functions. We use power utility functions becuase it is a convenient parametrization for comparative statics and computing examples later in the paper.

limit case of risk neutrality, $\alpha = 1$, and the quadratic specification of risk aversion, $\alpha = 2$.

An individual's ideal point is private information, therefore, only citizen i knows x_i. Moreover, the ideal points are distributed according to a cumulative probability distribution function F, and we assume that $F(x)$, $x \in [-1, 1] \subset \mathbb{R}$, is common knowledge. We make the following additional assumptions about $F(.)$:

A1 : $F(-1) = 0$;
A2 : $F(1) = 1$;
A3 : $F(.)$ is continuous, strictly increasing, and twice differentiable on $[-1, 1]$, where $f(.)$ is the density function of $F(.)$.

There are four decision making stages. In the first stage (*Entry*), all citizens decide simultaneously and independently on whether to run for office, $e_i = 1$, and bear the entry costs $c \geq 0$, or not run, $e_i = 0$, and bear no costs. The number of citizen candidates is denoted by $m \equiv \sum_{i=1}^{n} e_i$. In the second stage (*Policy promises*), each candidate publicly announces a non-binding policy promise. In the third stage (*Voting*), each citizen i makes a costless decision on whether to vote for one of the candidates, possibly for herself, or to abstain. The new leader is determined by simple majority rule (with random tie breaking) and announced publicly.[5] In the final stage (*Policy decision*), the leader implements a policy γ. Then, each citizen i's total payoff is given by

$$\pi_i (x_i, \gamma, e_i, w_i) = - \left| \tfrac{1}{2} (x_i - \gamma) \right|^{\alpha} - c e_i + b w_i, \tag{1}$$

where $w_i = 1$ if i is elected as the new leader, in which case she receives private benefits from holding office, $b \geq 0$. If i is not the new leader, then $w_i = 0$. We assume citizens maximize their expected own payoffs. Note that we assumed all citizen have the same entry costs, c, leadership benefits, b, and degree of risk aversion, α.

3 Political equilibrium

A "political equilibrium" is a perfect Bayesian equilibrium of the citizen candidate model with private information described above. In the characterization of equilibrium, all the action is in the *Entry* stage. Here, we briefly discuss the final three decision making stages, and analysis of the entry stage is carried out in the next section. In the *Policy decision* stage, the newly elected leader's only credible policy decision is to implement her own ideal point, $\gamma^* = x_i$ (see Alesina 1988), a strictly dominant strategy in that stage. In the *Policy promises* stage, policy announcements are cheap talk since each candidate has an incentive to misrepresent her ideal point to increase her chances of winning the elections. In the *Voting* stage, all non-candidate citizens are indifferent over all candidates since the candidates' ideal points are private information and policy promises are cheap talk. Hence we simply assume that each non-candidate either votes for each candidate with equal probability of $\frac{1}{m}$, or abstains. For each candidate, on the other hand, it is a

[5] If the outcome of the entry stage is $m = 0$, one citizen i is randomly selected as the new leader with equal probability of $\frac{1}{n}$, and the game proceeds straight to the policy decision stage. Default policies are discussed in more detail later.

weakly dominant strategy to vote for herself. This is because in case of becoming the new leader, implementing her own ideal point yields her no loss in payoff, compared to a strict loss with probability one if another candidate is elected (note that the probability of any other candidate having the same ideal point as herself is equal to zero). Given these vote decisions, each candidate has an equal chance of winning the election.[6]

4 Symmetric entry equilibrium in cutpoint strategies

In this section, we analyze symmetric entry equilibria. We prove two main results. First, symmetric entry equilibria are *always* in cutpoint strategies. A cutpoint strategy is characterized by two critical ideal points, $(\check{x}_l, \check{x}_r)$ with $-1 \leq \check{x}_l \leq \check{x}_r \leq 1$, such that a citizen enters if and only if $x_i \leq \check{x}_l$ or $x_i \geq \check{x}_r$. This is true because the best response strategy of a citizen to any symmetric strategy of the other citizens are always cutpoint strategies (Section 4.1), even if the other citizens are not using cutpoint strategies. Second, we show that there is always a unique symmetric equilibrium and fully characterize it. These results hold for any continuous cumulative probability distribution, $F(x)$, of ideal points $x \in [-1, 1]$ satisfying A1-A3 (Section 4.2), and for any concave single peaked utility function of citizens, and a stochastic default policy, d, in which one citizen is randomly selected if no candidate emerges in the entry stage. The rest of the section explores the comparative statics results for these entry equilibria (Section 4.3), examines the limiting case of large communities (Section 4.4), and we illustrate these results with several examples (Section 4.5).

4.1 Cutpoint strategies and best response condition

Consider citizen i. Suppose all citizens $j \neq i$ are using an entry strategy defined by two cutpoints:

$$\check{e}_j = \begin{cases} 0 & if \quad x_j \in (\check{x}_l, \check{x}_r) \\ 1 & if \quad x_j \in [-1, \check{x}_l] \cup [\check{x}_r, 1], \end{cases} \tag{2}$$

where $(\check{x}_l, \check{x}_r)$ is some pair of ideal points with $-1 \leq \check{x}_l \leq \check{x}_r \leq 1$ and the subscripts denote their relative locations *l*eft and *r*ight, respectively. In words, the cutpoint strategy $\check{e}$ determines that a citizen with ideal point equal to or more "extreme" than $\check{x}_l$ or $\check{x}_r$ runs for office, and citizens with ideal points more "moderate" than $\check{x}_l$ and $\check{x}_r$ do not run.

Recall that if neither citizen i nor any other citizen runs for office ($m = 0$), a stochastic default policy, d, takes effect, which randomly selects one of the n citizens as the new leader with equal probability of $\frac{1}{n}$ for each. In this event, it follows from Bayesian updating that $x_j \in (\check{x}_l, \check{x}_r), \forall j \neq i$.

To derive the equilibrium pair of cutpoint policies, or *equilibrium cutpoints*, $(\check{x}_l^*, \check{x}_r^*)$, we must compare a citizen i's expected payoffs as both a candidate and a non-candidate,

[6] Note that voting equilibria exist in which some candidates have strictly larger probabilities of being elected than others. By assumption, we rule out the possibility of any kind of coordination prior or after entry decisions are made. Hence, ex ante, each candidate has an equal probability of becoming the new leader.

given the equilibrium decisions in subsequent stages (see Section 3). Then, $(\check{x}_l^*, \check{x}_r^*)$ is an equilibrium if and only if $\check{e}_i(\check{x}_l^*, \check{x}_r^*)$ is a best response for citizen i when $\check{e}_j(\check{x}_l^*, \check{x}_r^*)$ is the entry strategy of all $j \neq i$.

Citizen i's expected payoff for *entering,* $\check{e}_i = 1$, can be written as[7]

$$E[\pi_i \mid x_i, \check{e}_i = 1] = (1-p)^{n-1}\, b$$
$$+ \sum_{m=2}^{n} \binom{n-1}{m-1} p^{m-1}(1-p)^{n-m}\left[\frac{b}{m} - \frac{m-1}{m}\; E[\left|\tfrac{1}{2}(x_i - \gamma)\right|^{\alpha} \mid \gamma \notin (\check{x}_l, \check{x}_r)]\right] - c, \quad (3)$$

where p denotes the probability that a randomly selected $j \neq i$ enters, if each j is using strategy $(\check{x}_l, \check{x}_r)$. So, $p \equiv p_l + p_r$, with $p_l \equiv \Pr(x_j \leq \check{x}_l) = F(\check{x}_l)$ and $p_r \equiv \Pr(x_j \geq \check{x}_r) = 1 - F(\check{x}_r)$ for our $F(x)$, $x \in [-1, 1] \subset \mathbb{R}$.

Citizen i's expected payoff loss from the policy outcome if some $j \neq i$ is elected (a term inside the summation) equals:

$$E\left[\left|\tfrac{1}{2}(x_i - \gamma)\right|^{\alpha} \mid \gamma \notin (\check{x}_l, \check{x}_r)\right]$$
$$= \frac{p_l}{p}\frac{\int_{-1}^{\check{x}_l} f(x)\left|\frac{1}{2}(x_i - x)\right|^{\alpha} dx}{p_l} + \frac{p_r}{p}\frac{\int_{\check{x}_r}^{1} f(x)\left|\frac{1}{2}(x_i - x)\right|^{\alpha} dx}{p_r} \quad (4)$$
$$= \frac{\int_{-1}^{\check{x}_l} f(x)\left|\frac{1}{2}(x_i - x)\right|^{\alpha} dx + \int_{\check{x}_r}^{1} f(x)\left|\frac{1}{2}(x_i - x)\right|^{\alpha} dx}{p} \qquad \textit{for } \check{x}_l \neq -1 \wedge \check{x}_r \neq 1,$$

which accounts for the possibility that the policy outcome will be in the left or right direction, γ_l or γ_r, with probability p_l/p and p_r/p, respectively. The first term in expression (3) gives the case where i receives b since she is the only candidate, which occurs with probability $(1-p)^{n-1}$. The second term gives the cases where $m - 1 \geq 1$ candidates enter in addition to herself, which occurs with probability $\binom{n-1}{m-1}p^{m-1}(1-p)^{n-m}$ and yields her expected benefits from holding office of $\frac{b}{m}$. The summation accounts for all possible $m = 2, ..., n$. Moreover, i will not be elected with probability $(m-1)/m$ and her expected loss in payoffs for this event is $E\left[\left|\frac{1}{2}(x_i - \gamma)\right|^{\alpha} \mid \gamma \notin (\check{x}_l, \check{x}_r)\right]$, given in expression (4). Finally, i bears the entry costs, c, independent of how many other candidates enter, which gives the third term in expression (3).

By contrast, citizen i's expected payoff for *not entering*, $\check{e}_i = 0$, is

$$E[\pi_i \mid x_i, \check{e}_i = 0] = (1-p)^{n-1}\left[\frac{b}{n} - \frac{n-1}{n}E[\left|\tfrac{1}{2}(x_i - d)\right|^{\alpha} \mid d \in (\check{x}_l, \check{x}_r)]\right]$$
$$- \sum_{m=2}^{n} \binom{n-1}{m-1} p^{m-1}(1-p)^{n-m}\; E\left[\left|\tfrac{1}{2}(x_i - \gamma)\right|^{\alpha} \mid \gamma \notin (\check{x}_l, \check{x}_r)\right]. \quad (5)$$

[7] Note that since i is entering, the default policy will not take force.

The first term corresponds to the event where, like herself, no other citizen enters, which occurs with probability $(1-p)^{n-1}$. In this case the stochastic default policy, d, takes effect. Then, citizen i's expected benefits from holding office if being randomly selected as the new leader is b/n (we assume that d does not invoke any entry costs in this event),[8] and with probability $(n-1)/n$ she will not be selected which yields her an expected payoff loss equal to:

$$E[\left|\tfrac{1}{2}(x_i - d)\right|^\alpha \mid d \in (\check{x}_l, \check{x}_r)] = \frac{\int_{\check{x}_l}^{\check{x}_r} f(x)\left|\frac{1}{2}(x_i - x)\right|^\alpha dx}{1-p} \quad \textit{for } \check{x}_l \neq \check{x}_r. \tag{6}$$

Observe that if $\check{x}_l = \check{x}_r$, the default policy is irrelevant because all citizens enter. The remaining terms in expression (5) correspond to the events where $m-1 \geq 1$ other citizens choose to enter.

Finally, it is readily verified that relating expressions (3) and (5) and rearranging yields the *best response* entry strategy for a citizen with ideal point x_i, if all other citizens are using cutpoint strategy $\check{e}$, which is to enter if and only if[9]

$$\begin{aligned}
&(1-p)^{n-1}\left(\tfrac{n-1}{n}\right)\left[b + E[\left|\tfrac{1}{2}(x_i - d)\right|^\alpha \mid d \in (\check{x}_l, \check{x}_r)]\right] \\
&\qquad + \sum_{m=2}^{n}\binom{n-1}{m-1} p^{m-1}(1-p)^{n-m}\frac{1}{m}\left[b + E[\left|\tfrac{1}{2}(x_i - \gamma)\right|^\alpha \mid \gamma \notin (\check{x}_l, \check{x}_r)]\right] \geq c,
\end{aligned} \tag{7}$$

where the left-hand and right-hand sides (henceforth LHS and RHS) give citizen i's expected net-benefits and costs from running for office, respectively.

The key observation, however, concerns the properties of $LHS(7)$. In particular, because $\alpha \geq 1$, it is a U-shaped (convex) function in x_i with a unique minimum strictly between -1 and 1. In fact, the U-shape is not restricted to the case where all other citizens $j \neq i$ are playing a cutpoint entry strategy $\check{e}_j$. Rather, this cutpoint strategy is the unique equilibrium outcome of citizen i best responding to any symmetric (type-dependent) entry strategy of all other citizens $j \neq i$ (and it would even be straightforward to extend this result to any type-dependent asymmetric strategies played by all others, following a similar proof that we use in this paper). For an arbitrary (possibly mixed) entry strategy, $\sigma(x) : [-1, 1] \to [0, 1]$, played by all $j \neq i$, where $\sigma(x)$ denotes the probability of entering for a citizen with ideal point x, the left-hand side of the best response condition (7) can be written more generally as:

$$\begin{aligned}
&Q_{ne}(n,q)\int_{-1}^{1} f_{ne}(x|\sigma)\left|\tfrac{1}{2}(x_i - x)\right|^\alpha dx \\
&\qquad + Q_e(n,q)\int_{-1}^{1} f_e(x|\sigma)\left|\tfrac{1}{2}(x_i - x)\right|^\alpha dx + Q_b(n,q)b \;\geq\; c,
\end{aligned} \tag{8}$$

[8] The assumption that a randomly selected leader does not bear any entry costs (e.g., there are no campaign costs) simplifies our analysis, but is innocent regarding our equilibrium results derived in the following (sub)sections.

[9] Without loss of generality, we assume that indifferent citizen types choose to enter.

where $Q_{ne}(n,q) \equiv (1-q)^{n-1}\left(\frac{n-1}{n}\right)$ corresponds to the case where no $j \neq i$ enters, $Q_e(n,q) \equiv \sum_{m=2}^{n}\binom{n-1}{m-1}q^{m-1}(1-q)^{n-m}\frac{1}{m}$ corresponds to the case where at least one $j \neq i$ enters, and $Q_b(n,q) \equiv (1-q)^{n-1}\left(\frac{n-1}{n}\right) + \sum_{m=2}^{n}\binom{n-1}{m-1}q^{m-1}(1-q)^{n-m}\frac{1}{m}$ corresponds to the case where i enters and wins, and the probability of each $j \neq i$ entering is given by $q \equiv \int_{-1}^{1}\sigma(x)f(x)dx$. The conditional distribution of types in the 'ne' and 'e' events are given by f_{ne} and f_e, respectively, where, assuming $q \in (0,1)$:[10]

$$f_{ne}(x|\sigma) = \frac{[1-\sigma(x)]f(x)}{1-q} \tag{9}$$

and

$$f_e(x|\sigma) = \frac{\sigma(x)f(x)}{q}. \tag{10}$$

The following lemma implies the cutpoint property of best replies:

Lemma 1 *For any symmetric entry strategy, $\sigma(x) : [-1,1] \to [0,1]$, played by all $j \neq i$, where $\sigma(x)$ denotes the probability of entering for a citizen with ideal point x, the left-hand side of the best response condition (8) is a U-shaped function in $x_i \in [-1,1]$ with a unique minimum at $x_{\min}$ and two relative maxima at $x_i = -1$ and $x_i = 1$.*

Proof. See Appendix 7.1. ■

4.2 Equilibrium characterization

In this subsection, we characterize all symmetric equilibria. We begin by looking at a special case where the logic is especially transparent, the case of *symmetric* distributions with $f(x) = f(-x), \forall x \in [0,1]$. Thereafter, we extend our analysis to all *asymmetric* distributions, and the proofs (see appendix) are all done for the general case. In particular, we show that there always exists a unique equilibrium in cutpoint strategies that uses equilibrium cutpoints, $(\check{x}_l^*, \check{x}_r^*)$ with $\check{x}_l^* \leq x_{\min} \leq \check{x}_r^*$. For the symmetric case, $x_{\min} = 0$ always and $\check{x}_l^* = -\check{x}_r^*$. Our next result characterizes the symmetric equilibrium in cutpoints for the case of symmetric distributions.

Proposition 1 (Entry equilibria with symmetric cutpoints) *For any continuous cumulative probability distribution, $F(x)$, of ideal points, $x \in [-1,1] \subset \mathbb{R}$, with symmetric density $f(x) = f(-x)$, $\forall x \in [0,1]$, and for a stochastic default policy, d, that randomly selects one citizen as the new leader if nobody runs for office, the political equilibrium is characterized by a unique pair of cutpoints $(-\check{x}^*, \check{x}^*)$, with $\check{x}^* \in [0,1]$, where each citizen i with a more extreme ideal point in the left or right direction (i.e., $x_i \leq -\check{x}^*$ or $x_i \geq \check{x}^*$, respectively) enters the electoral competition as a candidate, $\check{e}_i^* = 1$, and each citizen i with a more moderate ideal point, $-\check{x}^* < x_i < \check{x}^*$, does not enter, $\check{e}_i^* = 0$. The equilibrium cutpoint $\check{x}^*$ is characterized as follows:*

[10] The boundary cases of $q = 0$ and $q = 1$ simply eliminate one of the terms in condition (8).

(*i*) *If* $c \leq \underline{c} \equiv \frac{1}{n}\left[b + \int_{-1}^{1} f(x)\left|\frac{x}{2}\right|^{\alpha} dx\right]$, *then* $\check{x}^* = 0$ *and* $\check{e}_i^* = 1, \forall i$ *("everybody enters", or* $m = n$*);*

(*ii*) *If* $c \geq \overline{c} \equiv \frac{n-1}{n}\left[b + \int_{-1}^{1} f(x)\left|\frac{1}{2}(1-x)\right|^{\alpha} dx\right]$, *then* $\check{x}^* = 1$ *and* $\check{e}_i^* = 0, \forall i$ *("nobody enters", or* $m = 0$*);*

(*iii*) *If* $\underline{c} < c < \overline{c}$, *then* $\check{x}^* \in (0,1)$ *is the unique solution to:*

$$(1-p)^{n-1}\left(\tfrac{n-1}{n}\right)\left[b + \frac{\int_{-\check{x}^*}^{\check{x}^*} f(x)\left|\frac{1}{2}(\check{x}^* - x)\right|^{\alpha} dx}{1-p}\right] + \sum_{m=2}^{n}\binom{n-1}{m-1} p^{m-1}(1-p)^{n-m}$$
$$\times \frac{1}{m}\left[b + \frac{\int_{-1}^{-\check{x}^*} f(x)\left|\frac{1}{2}(\check{x}^* - x)\right|^{\alpha} dx + \int_{\check{x}^*}^{1} f(x)\left|\frac{1}{2}(\check{x}^* - x)\right|^{\alpha} dx}{p}\right] = c. \qquad (11)$$

where $p \equiv 2F(-\check{x}^*)$.

Proof. This proposition is a special case of Proposition 2, which we prove in Appendix 7.2. ■

Next, we generalize the symmetric case of Proposition 1 to any well-behaved continuous cumulative probability distribution, $F(x)$, of ideal points $x \in [-1,1] \subset \mathbb{R}$ (see Section 2). This gives:

Proposition 2 (Entry equilibria): *For any continuous cumulative probability distribution, $F(x)$, of ideal points $x \in [-1,1] \subset \mathbb{R}$ with density $f(x)$, and for a stochastic default policy, d, that randomly selects one citizen as the new leader if nobody runs for office, the political equilibrium is characterized by a unique pair of cutpoints, $(\check{x}_l^*, \check{x}_r^*)$, with $\check{x}_l^* \leq \check{x}_{min} \leq \check{x}_r^*$, where each citizen i with a more extreme ideal point in the left or right direction (i.e., $x_i \leq \check{x}_l^*$ or $\check{x}_r^* \leq x_i$,) enters the electoral competition as a candidate, $\check{e}_i^* = 1$, and each citizen i with a more moderate ideal point, $\check{x}_l^* < x_i < \check{x}_r^*$, does not enter, $\check{e}_i^* = 0$. Four different kinds of entry equilibria can arise: (i) "everybody enters", (ii) "nobody enters", (iii) some citizens with more extreme ideal points in only one direction are expected to enter, and (iv) some citizens with more extreme ideal points in both directions are expected to enter.*

Proof. See Appendix 7.2. ■

While the proofs are tedious, the intuition behind the results of Propositions 1 and 2 can be explained as follows. If the costs of entry are very small relative to the expected net-benefits (i.e., from holding office and avoiding a payoff loss due to the distance between the preferred and implemented policy), everybody has an incentive to become a candidate. On the other hand, if the entry costs are sufficiently high relative to the expected net-benefits, nobody wants to run for office.[11] If the distribution is symmetric, then $LHS\,(7)$

[11] Both the universal entry equilibrium and the zero entry equilibrium are in fact cutpoint equilibria, corresponding to cutpoints $\check{x}_{\min} = \check{x}_l^* = \check{x}_r^*$ and ($\check{x}_l^* = -1$, $\check{x}_r^* = 1$), respectively. We show in the proof of Propositon 2 that it always holds that $\check{x}_l^* < \check{x}_{\min} < \check{x}_r^*$ if $p < 1$.

is symmetric around 0 so if net-benefits and costs are in the intermediate range, then the best response condition (7) must hold as equality (i.e., $LHS\,(7) = c$) for exactly two citizen types $\check{x}_l^*$ and $\check{x}_r^*$ with $\check{x}_r^* = -\check{x}_l^*$. When ideal points are asymmetrically distributed, differences in expected net-benefits in both directions must be balanced out by asymmetric cutpoints, $-\check{x}_l^* \neq \check{x}_r^*$. This either results in a pair of interior cutpoints, $-1 < \check{x}_l^* < \check{x}_{\min} < \check{x}_r^* < 1$ or possibly a fourth kind of equilibrium where (7) holds with equality for only one ideal point in $(-1, 1)$ and either $LHS\,(7) < c$ when $x_i = -1$ or $LHS\,(7) < c$ when $x_i = 1$. In the former case, the left equilibrium cutpoint is $\check{x}_l^* = -1$ and there is only entry by candidates on the right; in the latter case, the right equilibrium cutpoint is $\check{x}_r^* = 1$ and there is entry only by candidates on the left. These are citizen candidates in the opposite direction of where the probability densities amass (i.e., the cutpoints are pulled towards the bulk of density), because these tend to have higher expected net-benefits from entering as they have higher expected losses from the distance between the preferred and implemented policy. In the following subsections, we use Propositions 1 and 2 to derive comparative statics results and characterize limit results for large communities.

4.3 Comparative statics

In this subsection, we derive comparative statics results for the unique equilibrium characterized in Propositions 1 and 2. To be precise, we analyze the effects of changes in c, b, and α on the equilibrium cutpoints, $(\check{x}_l^*, \check{x}_r^*)$, where $\check{x}_\delta^*(\check{x}_{-\delta}^*, n, c, b, \alpha)$ for $\delta = l, r$ and $\delta \neq -\delta$, for the region of the parameter space where the solution is interior for at least one cutpoint, that is, where $\check{x}_l^* \in (-1, \check{x}_{\min}]$ and/or $\check{x}_r^* \in [\check{x}_{\min}, 1)$ (see Proposition 2 (*iii*) and (*iv*)). Thus, we are excluding cases (*i*) and (*ii*), where there is either universal entry or no entry, respectively.[12]

Proposition 3 (Comparative statics) *An increase in the costs of entry, c, or the degree of risk aversion, α, yields more extreme interior equilibrium cutpoints, $(\check{x}_l^*, \check{x}_r^*)$—i.e., $\check{x}_l^*$ strictly decreases or $\check{x}_r^*$ strictly increases, or both—while an increase in the benefits from holding office, b, yields more moderate cutpoints. A decrease in $\check{x}_l^*$ (increase in $\check{x}_r^*$) implies that candidates and policy outcomes in the left (right) direction get more extreme, on average. It also implies a decrease in the expected number of candidates, $E[m] = np$, when c or α increases or b decreases. Finally, if n gets very large, $\check{x}_l^*$ approaches minus one and $\check{x}_r^*$ approaches one, that is,* $\lim_{n\to\infty} \check{x}_l^*(\check{x}_r^*, n) = -1$ *and* $\lim_{n\to\infty} \check{x}_r^*(\check{x}_l^*, n) = 1$.

Proof. See Appendix 7.3. ■

The intuition that an increase in the degree of risk aversion, α, yields more extreme equilibrium cutpoints is the following. In any given equilibrium $(\check{x}_l^*, \check{x}_r^*)$ with at least one interior cutpoint, each citizen i compares the entry costs, c, with the expected net-benefits from the lottery given by $LHS\,(7)\,|_{(\check{x}_l^*, \check{x}_r^*)}$, which is a function of $p(\check{x}_l^*, \check{x}_r^*)$. If the

[12] This is done for convenience and is essentially without loss of generality. For any $c > 0$, case (*i*) does not apply for sufficiently large n (see our analysis in the proof of Proposition 3). Case (*ii*) is satisfied for all n unless c is very large.

degree of risk aversion increases, this simply means that for each citizen the utility of the lottery must decrease relative to the utility of the secure option of saving c. Therefore, $LHS\,(7)\mid_{(\check{x}_l^*,\check{x}_r^*)}$ decreases for all citizens. This, in turn, means that in addition to the citizen types $x_i \in (\check{x}_l^*, \check{x}_r^*)$ now also (at least) citizen types $\check{x}_l^*$ and $\check{x}_r^*$ have no longer an incentive to enter, so that the new equilibrium cutpoints are more extreme than $(\check{x}_l^*, \check{x}_r^*)$ (i.e., if both cutpoints are interior, while if only one cutpoint is interior, this cutpoint gets more extreme and the boundary cutpoint remains).

Except for the limiting result, Proposition 3 does not give comparative statics results for the effect of changes in n on the equilibrium expected number of candidates, $E[m]$. The reason is that if c is small enough then this comparative static can go either way. Specifically, there may be more or fewer candidates if n increases, because there are two effects on entry that result from increasing the community size from n to $n+1$. First, there is the direct effect that the number of *potential* citizen candidates has increased by 1. If the equilibrium cutpoint were to remain unchanged, this effect works to increase the expected number of candidates. The second effect is the indirect equilibrium effect. Because the expected number of candidates would increase if the cutpoint remains unchanged, the probability of winning if one enters is lower, so the incentive to enter is reduced. This effect works in the opposite direction of more extreme cutpoints and a lower expected number of entrants. The total effect is generally going to be ambiguous.

Finally, Proposition 3 shows that $\lim_{n\to\infty} \check{x}_l^*(\check{x}_r^*, n) = -1$ and $\lim_{n\to\infty} \check{x}_r^*(\check{x}_l^*, n) = 1$. That is, in very large communities, only the most extreme citizens throw their hat in the ring. Of course, this does not imply there is zero entry! We can use this result to derive the limiting distribution of the number of candidates, which is fully characterized next.

4.4 Large communities

Here, we use the result of Proposition 3 that $\lim_{n\to\infty} \check{x}_l^*(\check{x}_r^*, n) = -1$ and $\lim_{n\to\infty} \check{x}_r^*(\check{x}_l^*, n) = 1$ to examine the limiting distribution of the number of candidates. This gives:

Proposition 4 (Large communities) *If the number of citizens, n, gets very large and if both equilibrium cutpoints are interior (see Proposition 2 (iv)), then the expected number of candidates, $\tau \equiv \lim_{n\to\infty} E[m] = \lim_{n\to\infty} np$, with $\tau = \tau_l + \tau_r$, $\tau_l \equiv \lim_{n\to\infty} np_l$, and $\tau_r \equiv \lim_{n\to\infty} np_r$, is given by the two conditions*

$$\tau_l = -\tau_r + (\tau_l + \tau_r)e^{-(\tau_l+\tau_r)}\frac{v_l - \frac{1}{2}}{c} + \left[1 - e^{-(\tau_l+\tau_r)}\right]\frac{b + \frac{1}{2}}{c} \tag{12}$$

and

$$\tau_r = -\tau_l + (\tau_l + \tau_r)e^{-(\tau_l+\tau_r)}\frac{v_r - \frac{1}{2}}{c} + \left[1 - e^{-(\tau_l+\tau_r)}\right]\frac{b + \frac{1}{2}}{c}, \tag{13}$$

where $v_l \equiv \int_{-1}^{1} f(x)\left|\frac{1}{2}(-1-x)\right|^{\alpha} dx$ and $v_r \equiv \int_{-1}^{1} f(x)\left|\frac{1}{2}(1-x)\right|^{\alpha} dx$. If only one equilibrium cutpoint, $\check{x}_\delta$, is interior and $|\check{x}_{-\delta}| = 1$, with $\delta = l, r$ and $\delta \neq -\delta$ (see Proposition 2 (iii)), then $\tau_{-\delta} = 0$ and the expected number of candidates $\tau_\delta \equiv \lim_{n\to\infty} np_\delta$ is determined *by the single condition*

$$\tau_\delta = \tau_\delta e^{-\tau_\delta} \frac{v_\delta - \frac{1}{2}}{c} + \left[1 - e^{-\tau_\delta}\right] \frac{b + \frac{1}{2}}{c}. \tag{14}$$

Proof. See Appendix 7.4. ■

The result follows from a Poisson approximation argument. Note that if ideal points are symmetrically distributed, we know from Proposition 1 that $p_l = p_r$ (since $-\check{x}^* = \check{x}^*$), and thus, both conditions (12) and (13) collapse to the single condition $\tau = \tau e^{-\tau} \frac{v - \frac{1}{2}}{c} + (1 - e^{-\tau}) \frac{b + \frac{1}{2}}{c}$, which has a unique solution in τ, with $\tau_l = \tau_r = \frac{\tau}{2}$ and $v = v_l = v_r$.

4.5 Examples

In this subsection, we give specific parametric examples of entry equilibria and comparative statics using $F(x) = \frac{1}{8}(x+1)^2 + \frac{1}{4}(x+1)$, $x \in [-1,1] \subset \mathbb{R}$, with density $f(x) = \frac{1}{4}(x+1) + \frac{1}{4}$ and $f'(x) = \frac{1}{4}x$. Thus, we use asymmetrically distributed ideal points for which the density is linearly increasing in x. The examples illustrate graphically the key equilibrium properties of our model.

A. Equilibrium net-benefits and variations in the costs of entry, c

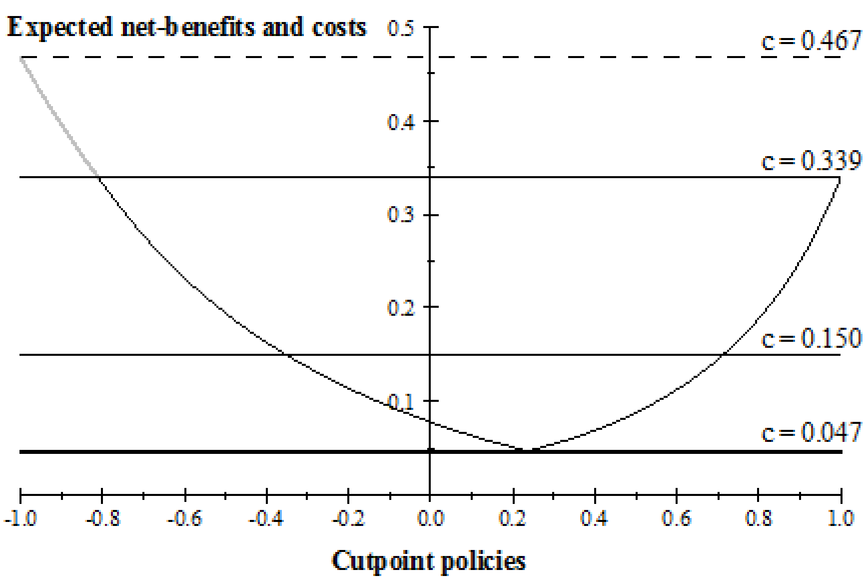

Figure 1: Entry equilibria in asymmetric pairs of cutpoint policies and variations in the entry costs, c, for $n = 5$, $b = 0$, and $\alpha = 1$.

To show our comparative statics results for changes in the costs of entry, we use $n = 5$, $b = 0$, and $\alpha = 1$ and vary the costs between $c = 0.047$, 0.150, 0.339, and 0.467. Figure 1 gives the feasible cutpoints $\check{x}_l \in [-1, \check{x}_{\min}]$ and $\check{x}_r \in [\check{x}_{\min}, 1]$ on the horizontal axis and the expected net-benefits and costs of entry on the vertical axis. Expected net-benefits are represented by the U-shaped curve and the various costs by horizontal lines (see $LHS\,(30)$ and $RHS\,(30)$ in the proof of Proposition 2).

The equilibrium pairs of cutpoints, $[\check{x}_l^*(c),\ \check{x}_r^*(c)]$, for the various costs are determined by the intersection of the expected net-benefits curve and the respective cost lines. A pair gets more extreme if c increases (i.e., $\check{x}_l^*$ decreases in c, $\check{x}_r^*$ increases in c, or both). Specifically, we have $\check{x}_l^*(c = 0.047) = \check{x}_r^*(c = 0.047) = 0.236$; $\check{x}_l^*(c = 0.150) = -0.354$ and $\check{x}_r^*(c = 0.150) = 0.714$; $\check{x}_l^*(c = 0.339) = -0.811$ and $\check{x}_r^*(c = 0.339) = 1$; and $\check{x}_l^*(c = 0.467) = -1$ and $\check{x}_r^*(c = 0.467) = 1$. Note that $\check{x}_{\min} = 0.236$, $\underline{c} = \frac{1}{5}\left[0 + 0.235\right] = 0.047$, $\overline{c} = \overline{c}_r(\overline{x}_l = 0.811, \check{x}_r = 1) = 0.339$ and $\overline{\overline{c}} = \overline{c}_l = \frac{5-1}{5}\left[0 + 0.58\overline{3}\right] = 0.467$. Moreover, note that everybody enters if $c \leq \underline{c}$, some citizen types in both directions of $\check{x}_{\min}$ are expected to enter if $\underline{c} < c \leq \overline{c}$, some citizen types only in the left direction of $\check{x}_{\min}$ are expected to enter if $\overline{c}_r < c < \overline{\overline{c}}$, and nobody enters if $\overline{\overline{c}} \leq c$ (see the proof of Proposition 2). Finally, as the equilibrium cutpoints get more extreme if c increases, expected entry decreases ($E[m(c)] = 5$, 2.091, 0.259, and 0 for our ascending c) and expected policy outcomes get more extreme ($[E[\gamma_l^*(c)], E[\gamma_r^*(c)]] = [-0.303, 0.637]$, $[-0.651, 0.859]$, $[-0.903, 1]$, and $[-1, 1]$).[13]

B. Variations in the benefits from holding office, b

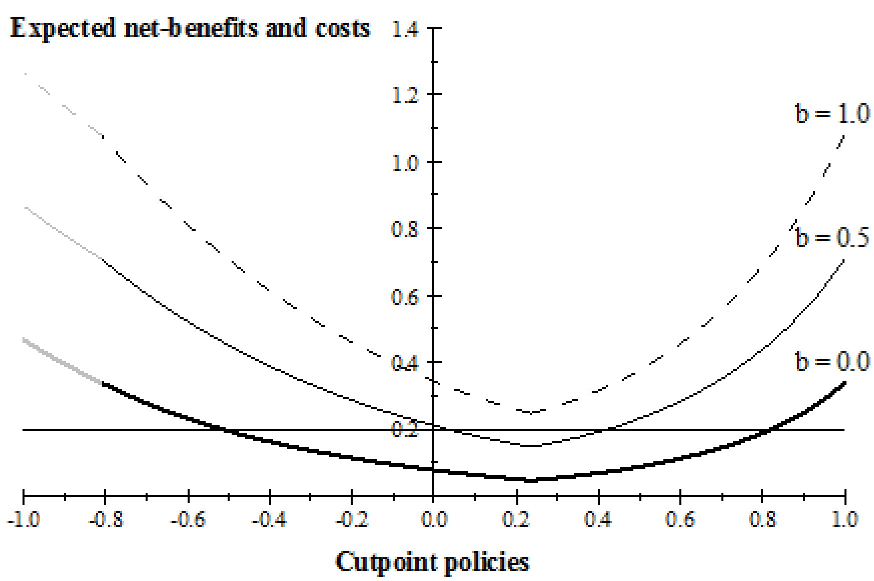

Figure 2: Entry equilibria in asymmetric pairs of cutpoint policies and variations in the benefits from holding office, b, for $n = 5$, $c = 0.2$, and $\alpha = 1$.

Next, we look at the effects of changes in the benefits from holding office. The example uses $n = 5$, $c = 0.2$, and $\alpha = 1$ and varies the benefits between $b = 0, 0.5$, and 1. Figure 2 shows that the equilibrium cutpoints, $[\check{x}_l^*(b),\ \check{x}_r^*(b)]$, are getting less extreme (more entry) as b increases. Specifically, $[\check{x}_l^*(0) = -0.517,\ \check{x}_r^*(0) = 0.822]$, $[\check{x}_l^*(0.5) = 0.033,\ \check{x}_r^*(0.5) = 0.422]$

[13] Expected entry is given by $E[m(\check{x}_l, \check{x}_r)] = np = n\left[1 - F(\check{x}_r) + F(\check{x}_l)\right]$, and expected policy outcomes in both directions by $E[\gamma_l(\check{x}_l, \check{x}_r)] = \frac{\int_{-1}^{\check{x}_l} f(x)x dx}{\int_{-1}^{\check{x}_l} f(x)dx}$ and $E[\gamma_r(\check{x}_l, \check{x}_r)] = \frac{\int_{\check{x}_r}^{1} f(x)x dx}{\int_{\check{x}_r}^{1} f(x)dx}$.

(see the intersections of the respective net-benefits curves and the cost line), and $\check{x}_l^*(1) = \check{x}_r^*(1) = \check{x}_{\min} = 0.236$ (since the net-benefits curve lies above the cost line). Finally, the increase in the benefits from holding office raises expected entry ($E[m(b)] = 1.397$, 3.917, and 5 for our ascending b) and expected policy outcomes become more moderate ($[E[\gamma_l^*(b)], E[\gamma_r^*(b)]] = [-0.743, 0.912]$, $[-0.425, 0.721]$, and $[-0.303, 0.637]$).

C. Variations in the degree of risk aversion, α

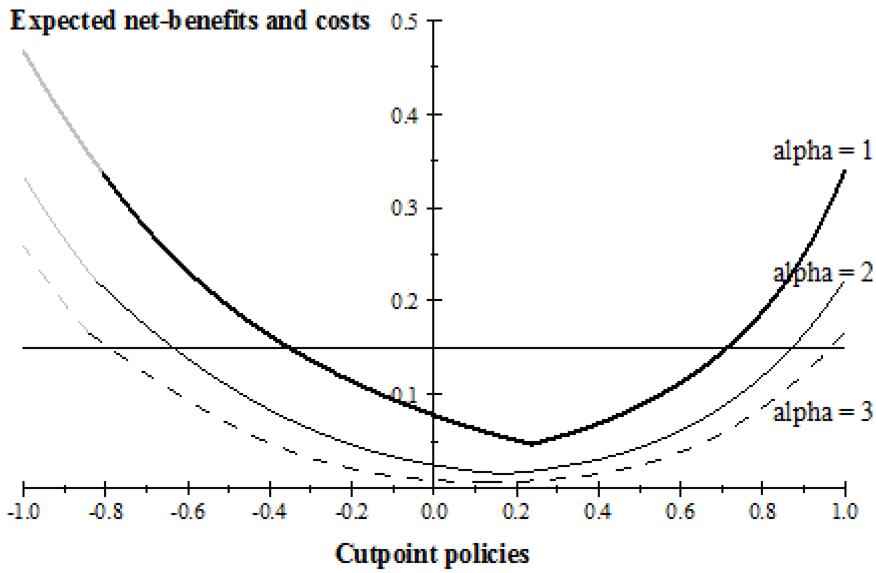

Figure 3: Entry equilibria in asymmetric pairs of cutpoint policies and variations in the degree of risk aversion, α, for $n = 5$, $b = 0$, and $c = 0.15$.

This example examines the comparative statics for changes in the degree of risk aversion. It uses $n = 5$, $b = 0$, and $c = 0.15$ and varies the degree of risk aversion between $\alpha = 1$ (tent preferences), 2, and 3. Figure 3 shows that the equilibrium cutpoints, $[\check{x}_l^*(\alpha), \check{x}_r^*(\alpha)]$, are getting more extreme as α increases. The intersections of the respective net-benefits curves and the cost line yield $[\check{x}_l^*(1) = -0.354, \check{x}_r^*(1) = 0.714]$, $[\check{x}_l^*(2) = -0.636, \check{x}_r^*(2) = 0.871]$, and $[\check{x}_l^*(3) = -0.791, \check{x}_r^*(3) = 0.966]$. The increase in the degree of risk aversion decreases expected entry ($E[m(\alpha)] = 2.091$, 1.013, and 0.415 for our ascending α) and expected policy outcomes become more extreme ($[E[\gamma_l^*(\alpha)], E[\gamma_r^*(\alpha)]] = [-0.651, 0.859]$, $[-0.808, 0.936]$, and $[-0.892, 0.983]$).

5 Extensions

5.1 Variations in the default policy

In the following, we discuss variations in the default policy that takes effect if no citizen runs for office. In the model of Osborne and Slivinski (1996) where the types of all citizens and candidates are complete information, the default policy is an infinite loss, $-\infty$. Applying this drastic measure to our model with incomplete (private) information would result in a unique equilibrium of universal entry, regardless of the specific citizen types and their distribution. To see this, note that in any equilibrium without universal entry we have $p < 1$, and thus, the expected utility from not entering is $-\infty$ and the expected utility from entering is bounded below by $-c - 1$. In other words, there is no feasible policy that gives any citizen a utility of a magnitude that can match an infinite loss. Therefore, an equilibrium would only exists if the conditions of universal entry are satisfied (see Proposition 2 (i)), which seems to be a very restrictive political scenario.

An obvious alternative to modeling the no-entry outcome would be to have a fixed default policy, $\overline{d} \in \mathbb{R}$, such as the status quo policy (e.g., Besley and Coate 1997). Applying $\overline{d} \in [-1, 1]$ to our model yields only slightly different conditions compared to our stochastic default policy, d, where one citizen is randomly selected as the new leader if nobody enters. To see this, let us first introduce $\overline{d}$ to our best response entry condition (7), which is a function of the cutpoints $(\check{x}_l, \check{x}_r)$:

$$\begin{aligned}(1-p)^{n-1}\left(\tfrac{n-1}{n}\right)\left[b+\left|\tfrac{1}{2}\left(x_i-\overline{d}\right)\right|^{\alpha}\right] \\ +\sum_{m=2}^{n}\binom{n-1}{m-1}p^{m-1}(1-p)^{n-m}\frac{1}{m}\left[b+E[\left|\tfrac{1}{2}(x_i-\gamma)\right|^{\alpha}\mid\gamma\notin(\check{x}_l,\check{x}_r)]\right]\geq c.\end{aligned} \tag{15}$$

This looks very similar to condition (7) and it is straightforward to establish the U-shaped property of the left-hand side (cf. Lemma 1 and footnote 18 to the proof of Proposition 2), which is key to characterizing a unique symmetric equilibrium in cutpoints. When comparing the expected loss from our endogenous stochastic default policy and fixed default policy (shown in the first terms of conditions (7) and (15)), some citizens with $E[\left|\frac{1}{2}(x_i - d)\right|^{\alpha} \mid d \in (\check{x}_l, \check{x}_r)] > (<) \left|\frac{1}{2}\left(x_i - \overline{d}\right)\right|^{\alpha}$ would enter (abstain) with d but not with $\overline{d}$. The effect of moving the fixed default policy along the real line of the policy space is that expected entry decreases in the direction where $\overline{d}$ moves and increases in the opposite direction. In other words, both equilibrium cutpoints $\check{x}_l^*$ and $\check{x}_r^*$ move in the same direction as $\overline{d}$. As such it has a qualitatively similar effect as shifting the distribution of ideal points to the left or right.

A slightly more general formulation of the status quo would be to allow randomization over exogenous status quo points. Again, this is different from what we do in the paper, but the characterization of a unique symmetric cutpoint equilibrium still goes through because a similar result to Lemma 1 (U-shaped net-benefits curve) can be shown for arbitrary exogenous stochastic defaults. Also, note that for very large n for cases (iii) and (iv) of Proposition 2, using an exogenous stochastic status quo given by F will be approximately the same as our endogenous model of randomly selecting a leader from the population, because for large n the equilibrium cutpoints converge to -1 and 1.

The alternative default policies discussed above have the advantage of being simple, but they are exogenous and the policy decision is made by constitutional fiat rather than reflecting the preferences of the citizens. By contrast, in our stochastic default policy the distribution of potential leaders from the citizenry results endogenously from the rational calculus of the players, and indeed is part of the equilibrium.[14]

[14] Note that in our stochastic default policy the selected leader receives benefits from holding office but

A third possibility is an alternative way to endogenize the no-entry outcome by allowing for multiple rounds in the entry stage: if nobody runs for office in round 1, another round follows and this continues until eventually there is at least one candidate. Importantly, after each unsuccessful entry round the citizens can Bayesian update that nobody's type is more extreme than the equilibrium cutpoints in that round. Then, the entry decision in round 2 will be derived just like the one in round 1, except that the earlier probability distribution of ideal points will be truncated, and so forth. As a consequence, the equilibrium cutpoints get more moderate with every additional entry round. A caveat of the process of multiple entry rounds is that it may continue without end (e.g., if the costs of entry are larger than the payoff from holding office and the difference between the largest and smallest actual types). To avoid this problem, one could for example assume a maximum number of possible entry rounds after which our stochastic default policy is invoked, or assume benefits from holding office that are sufficiently large relative to the costs of entry such that a $\check{x}_{min}$-type would prefer to enter, if she believes nobody else would ever enter.

The equilibrium conditions for the cutpoints would be more complicated with multiple entry rounds, compared to our stochastic default policy. Specifically, one cannot solve the model "forward" in a straightforward recursive fashion by using initial cutpoints derived from the equilibrium conditions of Proposition 2. This is because the cutpoints in round 1 depend on beliefs about the cutpoints in round 2, and so forth. However, one can characterize equilibrium conditions in a recursive fashion by using the monotonicity established in Lemma 1 (i.e., the U-shape best response condition), which will continue to hold. Specifically, a citizen whose type corresponds to an equilibrium cutpoint in round 1 is indifferent between entering in round 1 and postponing her entry decision to round 2, if nobody runs for office. However, the expected payoffs in round 2 are determined by the equilibrium cutpoints in round 2, which in turn are a function of the cutpoints in round 3, and so forth. In other words, the multiple entry rounds create a nested system of expected continuation payoffs in entry round t, $E\left[CV_t\left(x_i, (\check{x}_l, \check{x}_r)_{\forall t'>t}, n, c, b, \alpha\right)\right]$, which depends on citizen i's type, all future cutpoints $(\check{x}_l, \check{x}_r)_{\forall t'>t}$, and all exogenous parameters. To get an intuition how this default policy can affect the equilibrium cutpoints, we compare the best response entry condition (7) with the following one for entry round 1:

$$\begin{aligned}(1-p)^{n-1}&\left[b - E\left[CV_{t=1}\left(x_i, (\check{x}_l, \check{x}_r)_{\forall t>1}, n, c, b, \alpha\right)\right]\right] \qquad (16)\\ &+\sum_{m=2}^{n}\binom{n-1}{m-1}p^{m-1}(1-p)^{n-m}\frac{1}{m}\left[b+E[\left|\tfrac{1}{2}(x_i-\gamma)\right|^{\alpha} \mid \gamma \notin (\check{x}_l, \check{x}_r)]\right] \geq c,\end{aligned}$$

where convexity of the left-hand side is guaranteed because $E\left[CV_{t=1}(.)\right]$ is convex (cf. Lemma 1).[15] Observe that conditions (7) and (16) differ only with respect to their first terms. After simple rearrangements, directly comparing both terms yields: $LHS\,(7) >$

does not bear any entry costs. This simplifies the equilibrium analysis and none of our propositions would change if such costs would be introduced. To see this, consider costs $\widetilde{c}$, smaller or larger than c, for the randomly selected leader. Compared to the best response entry condition (7), a term $-(1-p)^{n-1}\widetilde{c}/n$ would be added to the left-hand side, with no essential consequence for our propositions.

[15]This best response entry condition is derived using citizen i's expected payoff for *entering* from

$LHS\,(16) \Rightarrow$

$$\left(\tfrac{n-1}{n}\right) E[\left|\tfrac{1}{2}\,(x_i - d)\right|^{\alpha} \mid d \in (\check{x}_l, \check{x}_r)] > \frac{b}{n} - E\left[CV_{t=1}\left(x_i, (\check{x}_l, \check{x}_r)_{\forall t>1}, n, c, b, \alpha\right)\right]. \qquad (17)$$

First, note that the left-hand side is strictly positive. Moreover, following the cutpoint strategy $\check{e}$, on the right-hand side extreme citizens have higher expected payoffs than moderate citizens from the b-part of $E\left[CV_{t=1}(.)\right]$ $(> b/n)$ and also from the $(\check{x}_l, \check{x}_r)_{\forall t>1}$-part of $E\left[CV_{t=1}(.)\right]$ (i.e., the expected net-benefits minus the expected costs if $b = 0$; cf. Lemma 1). Therefore, the right-hand side is negative for some extreme citizens. Then, because $LHS\,(7) > LHS\,(16)$ for some extreme citizens, the equilibrium cutpoints in round 1 of multiple entry rounds are more extreme than with our stochastic default policy.

Of course, there are other ways of modeling multiple entry rounds. Here, our aim was only to provide some insights of how this default policy can affect the equilibrium cutpoints.

5.2 Directional information about candidates' political leanings

Next, we relax our assumption of private information about each candidate's type by allowing for partially private information, that is, *directional* information about a candidate's political leaning (whereas all other assumptions made in Section 4 continue to hold). Specifically, after entry decisions are made, the electorate learns whether a candidate's ideal point lies to the "left" or "right" of $\check{x}_{min}$ (see Proposition 2 and its proof), but not her exact ideal point. Hence, policy promises remain cheap talk within the set of "left" and "right" types, respectively.

Here, we focus on symmetric probability distributions of ideal points. This case is simple, because the equilibrium results are not different between our citizen candidate model with private and partially private information. The reason is that, due to symmetry, when making an entry decision the probability of any candidate being elected is the same $(1/m)$ under both assumptions. To see the intuition, consider the following simple example. Suppose citizen i enters from the left, there are exactly two other candidates, and exactly fifty percent of the citizens are "leftist" and "rightist", respectively. Moreover, if there are more candidates with the same political leaning, each receives the same number of votes from citizens with that leaning. This gives three possible events: with probability 1/4 both other candidates are "rightists" and i wins; with probability 1/4 both other candidates are also "leftists" and i wins with probability 1/3; and with probability 1/2 one of the others is a "leftist" and the other a "rightist" and i loses. Thus, overall, i wins with probability $1/4 + 1/12 = 1/3$. It is straightforward to derive the general case

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expression (3) and her expected payoff for *not entering*, that is:

$$(1-p)^{n-1}\, E\left[CV_{t=1}(.)\right] - \sum_{m=2}^{n} \binom{n-1}{m-1} p^{m-1}\,(1-p)^{n-m}\;\; E\left[\left|\tfrac{1}{2}\,(x_i - \gamma)\right|^{\alpha} \mid \gamma \notin (\check{x}_l, \check{x}_r)\right].$$

Using similar rearrangements as for condition (7), both expected payoffs yield condition (16).

for symmetrically distributed types (i.e., each other candidate is equally likely to have either political leaning), that is, for any m the probability that i will win is equal to $1/m$. Note that for the case with asymmetrically distributed types, the effect of directional information is not innocuous and deriving the equilibrium cutpoints is more tedious. This is because a candidate's probability of being elected now depends on whether she is a "leftist" or "rightist".

There are interesting extensions to the citizen candidate model with directional information. For example, consider a two-party version of the model with one "left" and one "right" party, which have nominating conventions. One could define the parties in terms of the cutpoints as follows: label any candidate entering to the left of $\check{x}_l$ (right of $\check{x}_r$) a contender for the nomination of party L (R). Then one could introduce a party nomination stage where, for example, each party leader: (i) is selected randomly from all contenders of this party; (ii) is the one closest to the party median (defined as the median of the distribution of *potential* party contenders, i.e., the distribution truncated by the respective cutpoint); or (iii) the most moderate or most extreme contender. We leave to future research the analysis of these more complex entry stages.

5.3 Asymmetric entry equilibria in type-independent strategies

Up to this point, we have assumed that there are no coordination possibilities or focal points that would allow citizens to arrive at an asymmetric equilibrium. Thus we have analyzed symmetric entry equilibria in cutpoints. Like most entry games there can be asymmetric equilibria. In the following, we examine equilibria where the number of candidates, m, is commonly known and citizen types are private information but equilibrium entry strategies are type-independent (while all other assumptions made in Section 4 continue to hold).[16] Proposition 2 (i) and (ii) characterize universal entry and universal abstention, respectively. Both equilibria are also boundary cases of type-independent equilibria where exactly m citizens enter and exactly $n-m$ citizens do not enter (regardless of x_i). We call such a solution an "m-equilibrium". Obviously, if the parameters of the model allow for a (type-independent) m-equilibrium to exist, then there are potentially more of them ($\frac{n!}{m!(n-m)!}$, to be precise). These equilibria are characterized by two conditions. To keep notation simple, we only study symmetric probability distributions of ideal points (it is straightforward to extend the conditions to asymmetric distributions). The first condition states that *all types* are weakly better off entering than not entering if exactly $m-1$ other citizens enter for sure. For symmetric distributions, this is true if and only if the 0-type is weakly better off entering than not entering. The second condition is that *all types* are strictly better off not entering than entering if exactly m other citizens enter for sure. For symmetric distributions, this is true if and only if the 1-type is strictly better off not entering than entering. Thus, for $1 < m < n$, these two conditions are:

$$\frac{1}{m}\left[b+\int_{-1}^{1} f(x)\left|\tfrac{x}{2}\right|^{\alpha} dx\right] \geq c \tag{18}$$

[16] In constrast, Besley and Coate (1997) and Osborne and Slivinski (1996) study entry equilibria where m is commonly known and types are public information and relevant.

and

$$\frac{1}{m+1}\left[b+\int_{-1}^{1} f(x)\left|\tfrac{1}{2}(1-x)\right|^{\alpha} dx\right] < c, \tag{19}$$

Putting them together, we have an m-equilibrium for $1 < m < n$ if and only if:

$$\frac{1}{m+1}\left[b+\int_{-1}^{1} f(x)\left|\tfrac{1}{2}(1-x)\right|^{\alpha} dx\right] < c \leq \frac{1}{m}\left[b+\int_{-1}^{1} f(x)\left|\tfrac{x}{2}\right|^{\alpha} dx\right]. \tag{20}$$

For $m = 1$, the first of the two conditions is slightly different because if nobody enters, a citizen "wins" with probability $1/n$. Therefore, the condition that a citizen is weakly better off entering than not entering if nobody else is entering is given by:

$$\frac{n-1}{n}\left[b+\int_{-1}^{1} f(x)\left|\tfrac{x}{2}\right|^{\alpha} dx\right] \geq c. \tag{21}$$

Thus we have a 1-equilibrium if and only if:

$$\frac{1}{2}\left[b+\int_{-1}^{1} f(x)\left|\tfrac{1}{2}(1-x)\right|^{\alpha} dx\right] < c \leq \frac{n-1}{n}\left[b+\int_{-1}^{1} f(x)\left|\tfrac{x}{2}\right|^{\alpha} dx\right]. \tag{22}$$

6 Discussion and conclusions

We analyze a citizen candidate model with simple majority voting and private information about all citizens' and candidates' ideal points. This is the polar opposite assumption to the seminal models of Besley and Coate (1997) and Osborne and Slivinski (1996), which assume complete information about the ideal points of all citizens before entry decisions are made. We fully characterize the symmetric equilibrium of the Bayesian entry game. These equilibria are in cutpoint strategies and hence are characterized by a unique pair of ideal points, where only citizens more extreme that these cutpoints run for office. In the limiting case as the community becomes very large, only those citizens with ideal points at the extreme boundary of the distribution of ideal points become candidates. We show that this holds for any well-behaved probability distribution of ideal points—symmetric or asymmetric—and it implies substantial political polarization in large populations, independent of the distribution of citizens' ideal points! It only depends on the support of the distribution of ideal points. For example, consider the class of ideal point distributions defined by the set of all Normal distributions truncated (and renormalized) at -1 and 1. Then for all these distributions, the only entrants in large electorate will have ideal points at -1 and 1. Even if the CDF of ideal points is concentrated almost entirely at 0, so there is essentially no polarization of preferences in the general electorate, all candidates (and hence all policies) will be extreme. These properties of our equilibrium cutpoints, uniqueness and the emergence of only extreme candidates, differ from the multiple equilibria derived in previous citizen candidate models, a subset of which typically includes candidates with ideal points at or close to the median ideal point.

We derive comparative statics results for our model, which all follow intuition and are similar to the results of previous citizen candidate models with different informational assumptions (though risk aversion has not been studied in these models). On average, higher entry costs or higher risk aversion yield fewer and more extreme candidates, whereas higher benefits from holding office yield more and less extreme candidates. Moreover, an increase in the number of citizens has ambiguous effects on the number of candidates but they become more extreme on average, and we characterize the limiting equilibrium distribution and expected number of entrants.

There are important examples where the candidates' ideal points on issues are either private or public information. However, for many issues citizens possess information somewhere in-between these two limit cases. A particular relevant piece of information is about a candidate's political leaning, that is, whether she is a "leftist" or "rightist". We show that this directional information does not affect our equilibrium cutpoints at all when ideal points are symmetrically distributed. By contrast, it does bias the cutpoints when ideal points are asymmetrically distributed, because candidates in the direction with more entrants now have a lower chance of winning than those in the opposite direction. However, the main results for our cutpoint equilibrium still goes through. Our model with directional information can, for example, be utilized to study endogenous party formation, since this information can serve as a coordination device for candidates with the same political leaning. We leave the analysis of such a model to future research. Another interesting extension would be to utilize other kinds of information about the candidates' ideal points, such as in the way introduced to the spatial model of competition by Banks (1990) or allowing for polls, media, or other sources of information about candidate preferences.

We also elaborate on the choice of our default policy, which randomly selects one citizen as the new leader if nobody runs for office. This default policy has the advantage that it implements a policy decision as part of the equilibrium. We compared it to the commonly-used fixed default policy (e.g., Besley and Coate 1997; Eguia 2007). If this exogenous policy (e.g., a status quo policy) is a feasible ideal point, the main results for our equilibrium cutpoints are maintained, though they are biased in an intuitive way (i.e., depending on their location). Finally, we discuss an alternative endogenous default policy with multiple entry rounds. That is, if nobody runs for office, another entry round follows and this continues until eventually at least one candidate emerges. This default policy demands additional assumptions to ensure that the process ends with certainty. Comparing the equilibrium cutpoints for the first entry round to those for our citizen candidate model with random default policy, suggests that the first round cutpoints will be more extreme when multiple entry rounds are possible.

On a grander scale, this paper may contribute to our understanding of why we often observe extreme policy decisions and political polarization in democracies (McCarty et al. 2006), in contrast to the classical Downsian predictions of median outcomes. In our model, this phenomenon occurs even in communities where preferences are not polarized at all. Rather, extreme and polarized policies are the outcome of a process where only (the most) extreme citizens find it worth their while to enter the electoral competition as candidates. Interestingly, in this equilibrium the citizens get locked in, meaning that they may vote even though they know there are only extreme candidates. This reduces social welfare, because the community would be better off ex ante if candidates would make truthful policy promises, so that the new leader could be elected according to the distribution of preferences. The informational problem that candidates' true policy intentions when elected are privately known challenges the fundamental democratic idea that policy decisions should reflect the will of the majority. It remains to be shown empirically whether and, if so, to what extent this kind of informational asymmetry combined with endogenous entry of candidates can help explain political polarization.

7 Appendix

7.1 Proof of Lemma 1

Proof. We need to prove that for all $\alpha \geq 1$ the function :

$$B(x_i) = Q_{ne}(n,q)\int_{-1}^{1} f_{ne}(x|\sigma)\left|\tfrac{1}{2}(x_i - x)\right|^{\alpha} dx \ + Q_e(n,q)\int_{-1}^{1} f_e(x|\sigma)\left|\tfrac{1}{2}(x_i - x)\right|^{\alpha} dx$$

is strictly convex in x_i (i.e., $B''(x_i) > 0$) with $B'(x_i) < 0$ for $x_i = -1$ and $B'(x_i) > 0$ for $x_i = 1$. In fact, we prove here something much stronger, to establish the claim in the text that our results apply more broadly than just to power utility function. Consider any utility function $U(x_i, \gamma) = -L(x_i, \gamma)$ where x_i denotes the ideal point and γ is the implemented policy. Assume that L is twice continuously differentiable and satisfies three properties:

$$\begin{aligned} (i)\ \frac{\partial^2 L(x_i,\gamma)}{\partial x_i^2} &> 0 \\ (ii)\ \frac{\partial^2 L(x_i,\gamma)}{\partial \gamma^2} &> 0 \\ (iii)\ \frac{\partial L(x_i,\gamma)}{\partial x_i} &= 0 \text{ if } x_i = \gamma. \end{aligned}$$

In other words, utility functions are concave and single peaked. Then, for any such U (represented by a loss function, L) we can define:

$$B_L(x_i) = Q_{ne}(n,q)\int_{-1}^{1} f_{ne}(x|\sigma)L(x_i,x)dx \ + Q_e(n,q)\int_{-1}^{1} f_e(x|\sigma)L(x_i,x)dx \tag{23}$$

Thus, if we can show $B''_L(x_i) > 0$ with $B'_L(x_i) < 0$ for $x_i = -1$ and $B'_L(x_i) > 0$ for $x_i = 1$ then the result is established for all $\alpha > 1$ (we prove the special case of $\alpha = 1$ separately later).

(Convexity) First, it is straightforward to see that $B''_L(x_i) > 0$:

$$\begin{aligned} B''_L(x_i) &= Q_{ne}(n,q)\int_{-1}^{1} f_{ne}(x|\sigma)\frac{\partial^2 L(x_i,x)}{\partial x_i^2}dx \ + Q_e(n,q)\int_{-1}^{1} f_e(x|\sigma)\frac{\partial^2 L(x_i,x)}{\partial x_i^2}dx \\ &> 0, \end{aligned} \tag{24}$$

because $\frac{\partial^2 L(x_i,\gamma)}{\partial x_i^2} > 0$.

(Relative maximum and unique minimum) Next, for $x_i = -1$ we have

$$B_L'(-1) = Q_{ne}(n,q)\int_{-1}^{1} f_{ne}(x|\sigma)\frac{\partial L(x_i,x)}{\partial x_i}|_{x_i=-1}dx + Q_e(n,q)\int_{-1}^{1} f_e(x|\sigma)\frac{\partial L(x_i,x)}{\partial x_i}|_{x_i=-1}dx < 0$$

where the strict inequality holds for a -1-type because our assumptions on L imply that that $\frac{\partial L(x_i,x)}{\partial x_i} < 0$ if $x_i < x$. Similarly, for $x_i = 1$ we have

$$B_L'(1) = Q_{ne}(n,q)\int_{-1}^{1} f_{ne}(x|\sigma)\frac{\partial L(x_i,x)}{\partial x_i}|_{x_i=1}dx + Q_e(n,q)\int_{-1}^{1} f_e(x|\sigma)\frac{\partial L(x_i,x)}{\partial x_i}|_{x_i=1}dx > 0$$

where the strict inequality holds for a 1-type because our assumptions on L imply that that $\frac{\partial L(x_i,x)}{\partial x_i} > 0$ if $x_i > x$.

Thus, for $\alpha > 1$, the net-benefits of entering are always U-shaped, for any strategy used by the other citizens. This means that for any symmetric (type-dependent) mixed entry strategy, $\sigma_j(x)$, played by all citizens $j \neq i$ there is a unique interior minimum, which we will call at $x_{\min}$—i.e., B is strictly decreasing for $x_i < x_{\min}$, has a derivative of 0 at $x_i = x_{\min}$, and is strictly increasing for $x_i > x_{\min}$—and two relative maxima at $x_i = -1$ and $x_i = 1$.

(Risk neutrality) To cover the case of risk neutrality ($\alpha = 1$) we replace our specific loss function $-U(x_i,\gamma) = \left|\frac{1}{2}(x_i - \gamma)\right|$ with a more general utility function, $\widetilde{L}(x_i,\gamma)$, that includes the linear "tent" preferences. Assume: (i) $\widetilde{L}$ is twice differentiable for all $x_i, \gamma \in [-1,1]$; (ii) $\widetilde{L}(x_i, \gamma = x_i) = 0$; (iii) $\widetilde{L}'_{x_i}(x_i,\gamma) = -K < 0$ if $x_i < \gamma$ and $\widetilde{L}'_{x_i}(x_i,\gamma) = K > 0$ if $x_i \geq \gamma$, where $K > 0$ is a constant (i.e., the value of $\widetilde{L}$ is linearly increasing in the distance between x_i and γ); and (iv) $\widetilde{L}''_{x_i x_i} = 0$. Thus L is single peaked as before, but has a kink at the ideal point. The proof follows the same logic as above for the case of a smoothly convex single peaked L function, and we omit the details.[17] ■

7.2 Proof of Proposition 2

Proof. Recall our assumptions $n \geq 2$, $c \geq 0$, $b \geq 0$, and $\alpha \geq 1$ and that the citizen types, $x_i \in [-1,1] \subset \mathbb{R}$, are distributed according to any continuous cumulative probability function, $F(x)$, strictly increasing and twice differentiable on $[-1,1]$ and with $F(-1) = 0$, $F(1) = 1$, and density $f(x)$ (see Section 2).

Let's first use expressions (4) and (6) to rewrite the best response entry condition (7):

$$(1-p)^{n-1}\left(\tfrac{n-1}{n}\right)\left[b + \frac{\int_{\check{x}_l}^{\check{x}_r} f(x)\left|\frac{1}{2}(x_i - x)\right|^{\alpha}dx}{1-p}\right] + \sum_{m=2}^{n}\binom{n-1}{m-1}p^{m-1}(1-p)^{n-m}\frac{1}{m}$$

[17]The cutpoint best response property is even more general, and applies even if other citizens are not all using the same strategy. However, we are only interested in symmetric equilibria.

$$\times \left[b + \frac{\int_{-1}^{\check{x}_l} f(x) \left| \frac{1}{2} (x_i - x) \right|^{\alpha} dx + \int_{\check{x}_r}^{1} f(x) \left| \frac{1}{2} (x_i - x) \right|^{\alpha} dx}{p} \right] \geq c. \tag{25}$$

Moreover, for this and the following proofs it is helpful to separate the "integral"- and "probability"-terms in condition (25). This yields the following modified best response condition:[18]

$$LHS(7') = P_{ne}(n,p) \int_{\check{x}_l}^{\check{x}_r} f(x) \left| \tfrac{1}{2} (x_i - x) \right|^{\alpha} dx \tag{26}$$
$$+P_e(n,p) \left(\int_{-1}^{\check{x}_l} f(x) \left| \tfrac{1}{2} (x_i - x) \right|^{\alpha} dx + \int_{\check{x}_r}^{1} f(x) \left| \tfrac{1}{2} (x_i - x) \right|^{\alpha} dx \right) + P_b(n,p) b \geq c.$$

The subscript 'ne' in the P_{ne}-term refers to the situation where none of the other citizens enters, and

$$P_{ne}(n,p) \equiv \frac{(n-1)(1-p)^{n-2}}{n} > 0 \ \text{ for } p \in [0,1). \tag{27}$$

And, the superscript 'e' in the P_e-term refers to the situation where at least one other citizen enters, and

$$\begin{aligned}
P_e(n,p) &\equiv \frac{1}{p} \left[\sum_{m=2}^{n} \binom{n-1}{m-1} p^{m-1} (1-p)^{n-m} \frac{1}{m} + (1-p)^{n-1} - (1-p)^{n-1} \right] \\
&= \frac{1}{p} \left[\sum_{m=1}^{n} \binom{n-1}{m-1} p^{m-1} (1-p)^{n-m} \frac{1}{m} - (1-p)^{n-1} \right] \\
&= \frac{1}{p} \left[\frac{1}{np} \sum_{m=1}^{n} \binom{n}{m} p^{m} (1-p)^{n-m} - (1-p)^{n-1} \right] \\
&= \frac{1}{np^2} \left[\sum_{m=0}^{n} \binom{n}{m} p^{m} (1-p)^{n-m} - (1-p)^{n} \right] - \frac{(1-p)^{n-1}}{p} \\
&= \frac{1-(1-p)^n}{np^2} - \frac{(1-p)^{n-1}}{p} > 0 \ \text{ for } p \in (0,1].
\end{aligned} \tag{28}$$

Finally, the superscript 'b' in the P_b-term refers to the benefits from holding office and,

[18] In Section 5, we compare our stochastic default policy, d, with a fixed default policy $\overline{d} \in [-1,1]$. In this case, the best response entry condition (15) can be rewritten as:

$$P_{ne}(n,p) \int_{\check{x}_l}^{\check{x}_r} f(x) \left| \tfrac{1}{2} \left(x_i - \overline{d}\right) \right|^{\alpha} dx$$
$$+ P_e(n,p) \left(\int_{-1}^{\check{x}_l} f(x) \left| \tfrac{1}{2} (x_i - x) \right|^{\alpha} dx + \int_{\check{x}_r}^{1} f(x) \left| \tfrac{1}{2} (x_i - x) \right|^{\alpha} dx \right) + P_b(n,p) b \geq c.$$

The first term on the left-hand side is U-shaped in x_i with a minimum at $\overline{d}$, the second term is U-shaped in x_i (cf. the proof of Lemma 1), and the third term is constant in x_i. Thus, the left-hand side is overall U-shaped with a unique minimum value at $\overline{x}_{\min}(\check{x}_l, \check{x}_r, \overline{d})$. In other words, while $\overline{d}$ affects $\overline{x}_{\min}$, it does not change the U-shape in x_i of the left-hand side of the best response entry condition.

using similar rearrangements as for expression (28),

$$P_b(n,p) \equiv \frac{1-(1-p)^n}{np} - \frac{(1-p)^{n-1}}{n} > 0 \text{ for } p \in [0,1]. \tag{29}$$

We continue by using condition (26) to specify the two best response conditions for citizen types $x_i = \check{x}_l$ and $x_i = \check{x}_r$. To avoid abundant equilibrium characterization, we introduce the notation $\delta \in \{l, r\}$ and the indicator functions

$$F_\delta(x) = \begin{cases} F(x) & if \quad \delta = r \\ F(-x) & if \quad \delta = l \end{cases} \quad \text{and} \quad f_\delta(x) = \begin{cases} f(x) & if \quad \delta = r \\ f(-x) & if \quad \delta = l \end{cases},$$

for $x \in [-1,1] \subseteq \mathbb{R}$. Thus, we consider the mirror images $F(-x)$ and $f(-x)$ of $F(x)$ and $f(x)$, respectively, with $F_{\delta=r}(-1) = F(-1) = 0$ and $F_{\delta=r}(1) = F(1) = 1$, and with $F_{\delta=l}(-1) = F(1) = 1$ and $F_{\delta=l}(1) = F(-1) = 0$.

Using this, we can modify the best response entry condition (26) as follows: if all other citizens $j \neq i$ are using a cutpoint strategy $\check{e}_j$ as defined in expression (2) (see Lemma 1), the best response entry strategy of a citizen type $x_i = \check{x}_\delta$, for $\delta = l, r$ and $\delta \neq -\delta$, is to enter if and only if:

$$\begin{aligned} &P_{ne}(n,p) \int_{\check{x}_{-\delta}}^{\check{x}_\delta} f_\delta(x) \left| \tfrac{1}{2} (\check{x}_\delta - x) \right|^\alpha dx \\ &+ P_e(n,p) \left(\int_{-1}^{\check{x}_{-\delta}} f_\delta(x) \left| \tfrac{1}{2} (\check{x}_\delta - x) \right|^\alpha dx + \int_{\check{x}_\delta}^{1} f_\delta(x) \left| \tfrac{1}{2} (\check{x}_\delta - x) \right|^\alpha dx \right) + P_b(n,p) b \geq c, \end{aligned} \tag{30}$$

where $p = p_{-\delta} + p_\delta$, $p_{-\delta} = F_\delta(\check{x}_{-\delta})$, and $p_\delta = 1 - F_\delta(\check{x}_\delta)$.

(Necessary and sufficient conditions) We can use this best response entry strategy to characterize two necessary and sufficient conditions for a cutpoint equilibrium, $(\check{x}^*_{-\delta}, \check{x}^*_\delta)$, to exist, which must hold simultaneously for types $\check{x}_{-\delta}$ and $\check{x}_\delta$. First, note the important relationship between $LHS(26)$, $LHS(30)$, and Lemma 1. When the "c-line" on $RHS(26)$ intersects the net-benefits curve on $LHS(26)$ at $x_i = \check{x}_l$ and $x_i = \check{x}_r$, it must hold that $x_{\min} \in [\check{x}_l, \check{x}_r]$. Because the net-benefits curve is U-shaped in x_i, this means that the cutpoint strategy $\check{e}$ (see expression (2)) fulfills a necessary condition for the existence of a cutpoint equilibrium. Then, using the two best response strategies (30) for $\delta = l, r$, the following equilibrium characterizations do indeed constitute necessary and sufficient conditions for an entry equilibrium in cutpoint strategies to exist.

(Equilibrium characterization) There are four different equilibrium cases:

Case (i)**:** If $c \leq \underline{c} \equiv \frac{1}{n} \left[b + \int_{-1}^{1} f(x) \left| \frac{1}{2} (\check{x}^*_{\min} - x) \right|^\alpha dx \right]$, then $\check{e}^*_i = 1, \forall i$ ("everybody enters"), where $\check{x}^*_l = \check{x}^*_r = \check{x}^*_{\min} \in (-1,1)$ is determined by $\int_{-1}^{\check{x}^*_{\min}} f(x) \left| \frac{1}{2} (\check{x}^*_{\min} - x) \right|^{\alpha-1} dx$ $= \int_{\check{x}^*_{\min}}^{1} f(x) \left| \frac{1}{2} (\check{x}^*_{\min} - x) \right|^{\alpha-1} dx$.

This case is derived as follows: if $LHS(26)$ is greater than or equal to c for all values of $x_i, \check{x}_l$, and $\check{x}_r$, then the unique equilibrium is for all n citizens to enter. This corresponds to an equilibrium cutpoint $x_{\min} = \check{x}_{\min} = \check{x}_l = \check{x}_r$.[19] Thus, for this to hold, in $LHS(30)$ we simply set $x_{\min} = \check{x}_{\min} = \check{x}_\delta = \check{x}_{-\delta}$ and only consider the case $m = n$ in the P-terms. Then, as stated above, the inequality condition (30) reduces to:

$$\frac{1}{n}\left[b + \int_{-1}^{1} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_{\min} - x\right)\right|^\alpha dx\right] \equiv \underline{c} \geq c \quad \text{for } \delta = l, r, \tag{31}$$

because $p = F_\delta(\check{x}_{\min}) + 1 - F_\delta(\check{x}_{\min}) = 1$, and therefore, $P_e\left[n, p(\check{x}_{\min}) = 1\right] = \frac{1}{n}$ and $P_b\left[n, p(\check{x}_{\min}) = 1\right] = \frac{1}{n}$ (see expressions (28) and (29)). Thus, there is universal entry if and only if $c \leq \underline{c}$.[20] Finally, knowing $x_{\min} = \check{x}_{\min} = \check{x}_l = \check{x}_r$, we can determine $\check{x}_{\min}$ by using the first derivative of the left-hand side of $LHS(26)$ with respect to x_i, setting this equal to zero, and replacing $\check{x}_l, \check{x}_r$, and x_i with $\check{x}_{\min}$. This gives:

$$\frac{\partial LHS(7)'\big|_{x_i=\check{x}_{\min}=\check{x}_l=\check{x}_r}}{\partial x_i} =$$

$$P_e(n, p = 1)\left[\int_{-1}^{\check{x}_{\min}} f_\delta(x)\tfrac{\alpha}{2}\left|\tfrac{1}{2}\left(\check{x}_{\min} - x\right)\right|^{\alpha-1} dx - \int_{\check{x}_{\min}}^{1} f_\delta(x)\tfrac{\alpha}{2}\left|\tfrac{1}{2}\left(\check{x}_{\min} - x\right)\right|^{\alpha-1} dx\right] = 0$$

$$\Leftrightarrow \int_{-1}^{\check{x}_{\min}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_{\min} - x\right)\right|^{\alpha-1} dx = \int_{\check{x}_{\min}}^{1} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_{\min} - x\right)\right|^{\alpha-1} dx, \tag{32}$$

which implicitly determines $\check{x}_{\min}$, as stated above.

Case (ii)**:** If $c \geq \bar{\bar{c}} \equiv \max\left[\bar{c}_l, \bar{c}_r\right]$, where $\bar{c}_l \equiv \frac{n-1}{n}\left[b + \int_{-1}^{1} f(x)\left|\tfrac{1}{2}(-1 - x)\right|^\alpha dx\right]$ and $\bar{c}_r \equiv \frac{n-1}{n}\left[b + \int_{-1}^{1} f(x)\left|\tfrac{1}{2}(1 - x)\right|^\alpha dx\right]$, then $\check{x}_l^* = -1$, $\check{x}_r^* = 1$, and $\check{e}_i^* = 0, \forall i$ ("nobody enters").

This case is derived as follows: if $LHS(26)$ is smaller than or equal to c for all values of $x_i, \check{x}_l$, and $\check{x}_r$, then the unique equilibrium is for no citizen to enter. This corresponds to an equilibrium pair of cutpoints $(\check{x}_l = -1, \check{x}_r = 1)$. Thus, for this to hold, we reverse the inequality sign of condition (30), simply set $\check{x}_{-\delta} = -1$ and $\check{x}_\delta = 1$ in $LHS(30)$, and only consider the case $m = 0$ in the P-terms. Then, as stated above, condition (30) reduces to

$$\bar{c}_\delta \equiv \frac{n-1}{n}\left[b + \int_{-1}^{1} f_\delta(x)\left|\tfrac{1}{2}(1 - x)\right|^\alpha dx\right] \leq c \quad \text{for } \delta = l, r, \tag{33}$$

because $p = p_{-\delta} + p_\delta = F_\delta(\check{x}_{-\delta} = -1) + 1 - F_\delta(\check{x}_\delta = 1) = 0$, and therefore, $P_{ne}(n, p = 0) = \frac{n-1}{n}$ and $P_b(n, p = 0) = \frac{n-1}{n}$ (see expressions (27) and (29)). Thus, there is zero entry if and only if $c \geq \bar{\bar{c}} \equiv \max\left[\bar{c}_l, \bar{c}_r\right]$ (note that the probability of any citizen having type $x_i = -1$ or $x_i = 1$ is equal to zero).

Case (iii)**:** If $\widetilde{c} \equiv \min\left[\widetilde{c}_l, \widetilde{c}_r\right] \leq c < \bar{\bar{c}}$, where $\widetilde{c}_{-\delta} \equiv c(\check{x}_{-\delta} = -1, \check{x}_\delta = \overline{x}_\delta)$ and $\overline{x}_\delta =$

[19] The specification of cutpoints is arbitrary when there is universal entry. Any $\widetilde{x}$ such that $\check{x}_l = \check{x}_r = \widetilde{x}$ implies universal entry.

[20] Note that condition (31) implies that if $c = 0$, there is always universal entry because the left-hand side is greater than or equal to zero for any feasible combination of n, b, and α.

$\check{x}^*_\delta(\check{x}^*_{-\delta} = -1) \in (\check{x}_{\min}, 1)$ for $\delta = l, r$, then there is a unique cutpoint equilibrium where only some more extreme citizen types in one direction are expected to enter. Specifically, if $\widetilde{c} = \widetilde{c}_{-\delta}$ then citizen types $x_i \geq \overline{x}_\delta$ enter, that is, $[\check{x}^*_{-\delta} = -1, \check{x}^*_\delta \in [\overline{x}_\delta, 1)]$, and all other types ($x_i < \overline{x}_\delta$) do not enter. This unique equilibrium is characterized by the two best response conditions

$$\begin{aligned} P_{ne}(n, p_\delta) \int_{-1}^{\check{x}^*_\delta} f_\delta(x) \left| \tfrac{1}{2} \left(\check{x}^*_\delta - x\right) \right|^\alpha dx \\ + P_e(n, p_\delta) \int_{\check{x}^*_\delta}^{1} f_\delta(x) \left| \tfrac{1}{2} \left(\check{x}^*_\delta - x\right) \right|^\alpha dx + P_b(n, p_\delta) b = c \end{aligned} \tag{34}$$

and

$$\begin{aligned} P_{ne}(n, p_\delta) \int_{-1}^{\check{x}^*_\delta} f_\delta(x) \left| \tfrac{1}{2} \left(-1 - x\right) \right|^\alpha dx \\ + P_e(n, p_\delta) \int_{\check{x}^*_\delta}^{1} f_\delta(x) \left| \tfrac{1}{2} \left(-1 - x\right) \right|^\alpha dx + P_b(n, p_\delta) b \leq c, \end{aligned} \tag{35}$$

where $\widetilde{c} = \widetilde{c}_{-\delta}$ and $p_\delta = 1 - F_\delta(\check{x}^*_\delta)$. Note that the probability of any citizen type -1 or $\overline{x}_\delta$ occurring is equal to zero. Moreover, $\overline{x}_\delta = \check{x}^*_\delta(\check{x}^*_{-\delta} = -1)$ is implicitly determined by

$$\begin{aligned} P_{ne}(n, p_\delta) \left[\int_{-1}^{\overline{x}_\delta} f_\delta(x) \left| \tfrac{1}{2} \left(\overline{x}_\delta - x\right) \right|^\alpha dx - \int_{-1}^{\overline{x}_\delta} f_\delta(x) \left| \tfrac{1}{2} \left(-1 - x\right) \right|^\alpha dx \right] \\ + P_e(n, p_\delta) \left[\int_{-1}^{\overline{x}_\delta} f_\delta(x) \left| \tfrac{1}{2} \left(\overline{x}_\delta - x\right) \right|^\alpha dx - \int_{\overline{x}_\delta}^{1} f_\delta(x) \left| \tfrac{1}{2} \left(-1 - x\right) \right|^\alpha dx \right] = 0, \end{aligned} \tag{36}$$

and $\widetilde{c}_{-\delta}$ is determined by replacing $\check{x}^*_\delta$ with $\overline{x}_\delta$ on the left-hand side of condition (34).

This case is derived as follows: if $LHS(26)$ is greater than or equal to c for all values $x_i \geq \overline{x}_\delta$ and smaller than c for all values of $x_i < \overline{x}_\delta$, then the unique equilibrium is for all citizen types larger than or equal to $\overline{x}_\delta$ to enter, and for all other types not to enter. This corresponds to a cutpoint equilibrium $[\check{x}_{-\delta} = -1, \check{x}_\delta \in [\overline{x}_\delta, 1)]$. Thus, for this to hold, for a type $\check{x}_\delta$ we state condition (30) as equality and simply set $\check{x}^*_{-\delta} = -1$ in $LHS(30)\,|_{\check{x}_\delta}$, where the subscript denotes the citizen type whose strategy we investigate (see condition (34)), and for a type $\check{x}_{-\delta}$ we reverse the inequality sign and set $\check{x}^*_{-\delta} = -1$ in $LHS(30)\,\big|_{\check{x}_{-\delta}}$ (see condition (35)). In condition (36), we determine the boundary case $\overline{x}_\delta = \check{x}_\delta(\check{x}_{-\delta} = -1)$—where a type $\check{x}_{-\delta} = -1$ is just indifferent between entering and not entering as a candidate (note that the probability of this type occurring is equal to zero)—by setting the left-hand sides of conditions (34) and (35) equal and making simple rearrangements. Importantly, below we use Envelope Theorem to show that a citizen type $x_{-\delta} = -1$ always prefers not to enter if $c > \widetilde{c}$. Therefore, for $\widetilde{c} \leq c < \overline{\overline{c}}$ we only need condition (34) to compute the interior cutpoint policy $\check{x}_\delta(\check{x}_{-\delta} = -1)$.

Case (iv)**:** If $\underline{c} < c < \widetilde{c}$, then there is a unique equilibrium pair of interior cutpoints, $(\check{x}^*_l, \check{x}^*_r)$, where some more extreme citizen types in both directions are expected to enter.

Specifically, for $\delta = l, r$, if $\widetilde{c} = \widetilde{c}_{-\delta}$ then $[\check{x}^*_{-\delta} \in (-1, \check{x}^*_{\min}),\ \check{x}^*_{\delta} \in (\check{x}^*_{\min}, \overline{x}_{\delta})]$ and if $\widetilde{c} = \widetilde{c}_{-\delta} = \widetilde{c}_{\delta}$ then $[\check{x}^*_{-\delta} \in (-1, \check{x}^*_{\min}),\ \check{x}^*_{\delta} \in (\check{x}^*_{\min}, 1)]$, and in all these cases some citizen types in both directions who are more extreme than or equal to $\check{x}^*_{-\delta}$ or $\check{x}^*_{\delta}$ are expected to enter. This unique interior equilibrium is characterized by the equality condition

$$P_{ne}(n,p) \int_{\check{x}^*_{-\delta}}^{\check{x}^*_{\delta}} f_{\delta}(x) \left|\tfrac{1}{2}\left(\check{x}^*_{\delta} - x\right)\right|^{\alpha} dx \tag{37}$$

$$+P_e(n,p) \left[\int_{-1}^{\check{x}^*_{-\delta}} f_{\delta}(x) \left|\tfrac{1}{2}\left(\check{x}^*_{\delta} - x\right)\right|^{\alpha} dx + \int_{\check{x}^*_{\delta}}^{1} f_{\delta}(x) \left|\tfrac{1}{2}\left(\check{x}^*_{\delta} - x\right)\right|^{\alpha} dx \right] + P_b(n,p)b = c,$$

which must hold simultaneously for $\delta = l, r$, where $p = F_{\delta}(\check{x}^*_{-\delta}) + 1 - F_{\delta}(\check{x}^*_{\delta})$.

This case is derived as follows: if $LHS(26)$ is greater than or equal to c for all values of $x_i \leq \check{x}^*_{-\delta}$ and $x_i \geq \check{x}^*_{\delta}$ and smaller than c for all values of $x_i \in (\check{x}^*_{-\delta}, \check{x}^*_{\delta})$, then the unique equilibrium is for all citizen types who are more extreme than or equal to $\check{x}^*_{-\delta}$ and $\check{x}^*_{\delta}$ to enter, and for all other more moderate types not to enter. This corresponds to a cutpoint equilibrium $[\check{x}_{-\delta} \in (-1, \check{x}_{\min}),\ \check{x}_{\delta} \in (\check{x}_{\min}, \overline{x}_{\delta})]$ for $\widetilde{c} = \widetilde{c}_{-\delta}$. Thus, for this to hold, we simply have to state (30) as equality for both $\delta = l, r$ (see condition (37)), and simultaneously compute the values of the interior cutpoints $\check{x}_{\delta}$ and $\check{x}_{-\delta}$.

(Existence) Next, we prove that a cutpoint equilibrium, $(\check{x}^*_{-\delta}, \check{x}^*_{\delta})$, always exist and is always unique for any cumulative probability distribution of ideal points, $F(x)$, satisfying A1-A3 (see Section 2). We proceed in the following steps. Here, we use Envelope Theorem and Intermediate Value Theorem to show existence. Thereafter, we prove uniqueness using our result that for any given entry probability, p, there is a unique cutpoint equilibrium.

We begin by using Envelope Theorem. Recall that $LHS(26)$ is a continuous function of $x_i, \check{x}_{-\delta}$, and $\check{x}_{\delta}$. Now consider the following value function:

$$\upsilon[\check{x}_{-\delta}, \check{x}_{\delta} \,|f_{\delta}(x), n, \alpha, c, b] = \int_{-1}^{1} f_{\delta}(x) \left[c - LHS(26)[x, \check{x}_{-\delta}, \check{x}_{\delta} \,|f_{\delta}(x), n, \alpha, c, b]\right] dx$$

and the maximization problem

$$\upsilon^*[\check{x}^*_{-\delta}, \check{x}^*_{\delta} \,|f_{\delta}(x), n, \alpha, c, b] \equiv \max_{\check{x}_{-\delta}, \check{x}_{\delta}} \upsilon[\check{x}_{-\delta}, \check{x}_{\delta} \,|f_{\delta}(x), n, \alpha, c, b].$$

However, if both cutpoints are interior (the case with one interior cutpoint will be discussed below), we know from equilibrium condition (37) that a solution to this problem—i.e., a cutpoint equilibrium, $(\check{x}^*_{-\delta}, \check{x}^*_{\delta})$—is implicitly determined by

$$LHS(26)[\check{x}^*_{-\delta}, \check{x}^*_{\delta} \,|f_{\delta}(x), n, \alpha, c, b] \Big|_{x_i = \check{x}^*_{-\delta}} = LHS(26)[\check{x}^*_{-\delta}, \check{x}^*_{\delta} \,|f_{\delta}(x), n, \alpha, c, b] \Big|_{x_i = \check{x}^*_{\delta}} = c$$

which, using Lemma 1, gives:

$$v^*[\check{x}^*_{-\delta}, \check{x}^*_\delta \,|\, f_\delta(x), n, \alpha, c, b] = \int\limits_{\check{x}^*_{-\delta}}^{\check{x}^*_\delta} f_\delta(x) \left[c - LHS(26)[x, \check{x}^*_{-\delta}, \check{x}^*_\delta \,|\, f_\delta(x), n, \alpha, c, b]\right] dx. \quad (38)$$

Here, we are interested in the effects of a marginal change in the entry costs, c, on $v^*[.]$, and on the equilibrium cutpoints in particular. Since the two cutpoints are mutually dependent, let us write the pair as $[\check{x}^*_{-\delta}(\check{x}^*_\delta, c), \check{x}^*_\delta(\check{x}^*_{-\delta}, c)]$. Then, by the chain rule we have

$$\begin{aligned} \frac{dv^*[\check{x}^*_{-\delta}(\check{x}^*_\delta, c), \check{x}^*_\delta(\check{x}^*_{-\delta}, c), c \,|\, f_\delta(x), n, \alpha, b]}{dc} &= \frac{\partial v^*[.]}{\partial c} \\ &+ \frac{\partial v^*[.]}{\partial \check{x}_{-\delta}(\check{x}_\delta, c)} \left[\frac{d\check{x}_{-\delta}(\check{x}_\delta, c)}{dc} + \frac{\partial \check{x}_{-\delta}(\check{x}_\delta, c)}{\partial \check{x}_\delta(\check{x}_{-\delta}, c)} \frac{d\check{x}_\delta(\check{x}_{-\delta}, c)}{dc}\right] \\ &+ \frac{\partial v^*[.]}{\partial \check{x}_\delta(\check{x}_{-\delta}, c)} \left[\frac{d\check{x}_\delta(\check{x}_{-\delta}, c)}{dc} + \frac{\partial \check{x}_\delta(\check{x}_{-\delta}, c)}{\partial \check{x}_{-\delta}(\check{x}_\delta, c)} \frac{d\check{x}_{-\delta}(\check{x}_\delta, c)}{dc}\right], \end{aligned}$$

which, using the first-order equilibrium condition $\frac{\partial v^*[.]}{\partial \check{x}_{-\delta}(\check{x}_\delta, c)} = \frac{\partial v^*[.]}{\partial \check{x}_\delta(\check{x}_{-\delta}, c)} = 0$, yields

$$\frac{dv^*[\check{x}^*_{-\delta}(\check{x}^*_\delta, c), \check{x}^*_\delta(\check{x}^*_{-\delta}, c), c \,|\, f_\delta(x), n, \alpha, b]}{dc} = \frac{\partial v^*[.]}{\partial c} = \int\limits_{\check{x}^*_{-\delta}}^{\check{x}^*_\delta} f_\delta(x) dx > 0 \quad \text{for } p \in [0, 1). \quad (39)$$

Therefore, a marginal change in c affects $v^*[.]$ only directly, but not indirectly through changes in $\check{x}_{-\delta}(\check{x}_\delta, c)$ and $\check{x}_\delta(\check{x}_{-\delta}, c)$ (in other words, the effects on $LHS(26)[.]$ in $v^*[.]$ are negligible and marginal changes in the cutpoints are independent from each other).

This is an important result, and it also informs us about how $\check{x}_{-\delta}$ and $\check{x}_\delta$ change when c changes marginally. Expression (39) shows that an increase in $v^*[.]$ through a marginal increase from c to c' is entirely due to the higher entry costs of each potential citizen type $x \in [\check{x}^*_{-\delta}, \check{x}^*_\delta]$. Among these citizens, for c only types $\check{x}^*_{-\delta}(c)$ and $\check{x}^*_\delta(c)$ enter and all other, moderate types $x \in (\check{x}^*_{-\delta}(c), \check{x}^*_\delta(c))$ abstain. By contrast, for c' the entry costs exceed the net-benefits also for types $\check{x}^*_{-\delta}(c)$ and $\check{x}^*_\delta(c)$, who now abstain too. Therefore, if both equilibrium cutpoints $\check{x}^*_{-\delta}$ and $\check{x}^*_\delta$ are interior (see Proposition 2 (*iv*)), marginally increasing c to c' yields more extreme equilibrium cutpoints, or $\check{x}'^*_{-\delta} < \check{x}^*_{-\delta}$ and $\check{x}^*_\delta < \check{x}'^*_\delta$. As a consequence, the entry probability decreases in both directions (i.e., $p_{-\delta} > p'_{-\delta}$ and $p_\delta > p'_\delta$), and hence decreases overall (i.e., $p > p'$). If only one equilibrium cutpoint is interior, $\check{x}^*_\delta$, and the other is at the boundary, $\check{x}^*_{-\delta} = -1$ (see Proposition 2 (*iii*)), it is readily verified that the value function (38) and its derivative (39) can be used by simply setting $\check{x}^*_{-\delta} = -1$. Then, marginally increasing c to c' yields the interior cutpoint to become more extreme, or $\check{x}^*_\delta < \check{x}'^*_\delta$, while the boundary cutpoint remains unchanged, or $\check{x}^*_{-\delta} = \check{x}'^*_{-\delta} = -1$. As a consequence, the entry probability only decreases in the direction of the interior cutpoint (i.e., $p_\delta > p'_\delta$ and $p_{-\delta} = p'_{-\delta} = 0$), and hence decreases overall (i.e., $p > p'$).

Moreover, importantly, for interior equilibrium cutpoints the net-benefits of citizen types with exactly these cutpoints are larger for c' than for c, respectively. This is because from expression (39) we know that, on the margin, for c' the cutpoints are simply reached by moving upwards along the U-shaped net-benefit curve for c, that is, along $LHS(26)[x_i, \check{x}_{-\delta}, \check{x}_\delta \,|n, \alpha, c, b]$. Finally, note that our results also establish that in any equilibrium it must hold that $\check{x}^*_l \in [-1, \check{x}^*_{\min}]$ and $\check{x}^*_r \in [\check{x}^*_{\min}, 1]$, because cutpoints never get more moderate if c increases. In summary, continuously increasing c creates one or more continuous equilibrium paths $[\check{x}^*_{-\delta}(\check{x}^*_\delta, c), \check{x}^*_\delta(\check{x}^*_{-\delta}, c)]$ with the following properties: the interior cutpoints get more extreme (at the boundary, $\check{x}^*_{-\delta} = -1$ remains), (ii) p decreases; and (iii) $LHS(30)$ increases. The endpoints of any path are at $\underline{c}$ ($p = 1$, where $\check{x}^*_{-\delta} = \check{x}^*_\delta = \check{x}_{\min}$) and at $\bar{\bar{c}}$ ($p = 0$, where $\check{x}^*_{-\delta} = -1$ and $\check{x}^*_\delta = 1$). Using expressions (31) and (33), unless $n = 2 \wedge \alpha = 1 \wedge f_\delta(x) = 1$[21] we have:

$$\underline{c} < \bar{\bar{c}} \Rightarrow \frac{1}{n}\left[b + \int_{-1}^{1} f_\delta(x) \left|\tfrac{1}{2}\left(\check{x}_{\min} - x\right)\right|^\alpha dx\right] < \frac{n-1}{n}\left[b + \int_{-1}^{1} f_\delta(x) \left|\tfrac{1}{2}\left(1 - x\right)\right|^\alpha dx\right],$$

where the strict inequality holds because $\frac{1}{n} < \frac{n-1}{n}$ for $n > 2$ and $\int_{-1}^{1} f_\delta(x) \left|\frac{1}{2}\left(\check{x}_{\min} - x\right)\right|^\alpha dx$ $< \int_{-1}^{1} f_\delta(x) \left|\frac{1}{2}\left(1 - x\right)\right|^\alpha dx$. Therefore, by the Intermediate Value Theorem, at least one equilibrium path $[\check{x}^*_{-\delta}(\check{x}^*_\delta, c),\ \check{x}^*_\delta(\check{x}^*_{-\delta}, c)]$ must exist. Finally, for $c \le \underline{c}$ and $\bar{\bar{c}} < c$, existence (and uniqueness) is readily verified for universal entry and universal abstention, respectively. This completes our proof of existence.

(Uniqueness) Next, we prove uniqueness of $(\check{x}^*_{-\delta}, \check{x}^*_\delta)$. To do so, we show that for any given entry probability, $\overline{p} \in [0, 1]$, at most one pair of cutpoints can simultaneously fulfill the best response condition (37) for $\delta = l, r$ (see Proposition 2 (iv)), or conditions (34) and (35) (see Proposition 2 (iii)). The main idea of the proof is that any continuous equilibrium path must use all $p \in [0, 1]$, and thus, if there is only one cutpoint equilibrium for $\overline{p}$, this would mean there is a unique equilibrium path. Note that keeping $\overline{p}$ constant means that the three $P(n, \overline{p})$-terms in these conditions are not affected when $\check{x}_{-\delta}$ and $\check{x}_\delta$ change (see expressions (27) to (29)). It also means that it can neither be a unilateral change in one cutpoint only, nor a simultaneous change in both cutpoints in opposite directions (i.e., jointly more extreme or jointly less extreme). Note that for a fixed $\overline{p}$ this also holds for equilibria with only one interior cutpoint. Thus, by keeping $\overline{p}$ constant, we need to analyze changes in $\check{x}_{-\delta}$ and $\check{x}_\delta$ in the same direction. Without loss of generality, we focus on increases from $\check{x}$ to $\check{x}'$, that is, $\check{x}_{-\delta} < \check{x}'_{-\delta} \le \check{x}_{\min}$ and $\check{x}_{\min} \le \check{x}_\delta < \check{x}'_\delta$, under the constraint that $\overline{p}(\check{x}_{-\delta}, \check{x}_\delta) = \overline{p}(\check{x}'_{-\delta}, \check{x}'_\delta)$. We account for these increases by modifying the partition of the integrals in $LHS\,(30)$. Then, before the change is implemented, for a

[21] If $n = 2 \wedge \alpha = 1 \wedge f_\delta(x) = 1$ (note that $f_\delta(x) = 1$ gives the uniform distribution of types), we have $\underline{c} = \bar{\bar{c}}$, because $1/n = (n-1)/n$ for $n = 2$ and because the integral terms are equal for $\alpha = 1 \wedge f_\delta(x) = 1$, and there are only two possible equilibria: either both citizens enter or both abstain.

$\check{x}_\delta$-type this gives:

$$
\begin{aligned}
& LHS\,(30)\,\big|_{(\check{x}_{-\delta},\check{x}_\delta),\check{x}_\delta} \\
= \;& P_e(n,\overline{p})\int_{-1}^{\check{x}_{-\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_\delta - x\right)\right|^\alpha dx \qquad (40)\\
& + P_{ne}(n,\overline{p})\int_{\check{x}_{-\delta}}^{\check{x}'_{-\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_\delta - x\right)\right|^\alpha dx + P_{ne}(n,\overline{p})\int_{\check{x}'_{-\delta}}^{\check{x}_\delta} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_\delta - x\right)\right|^\alpha dx \\
& + P_e(n,\overline{p})\int_{\check{x}_\delta}^{\check{x}'_\delta} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_\delta - x\right)\right|^\alpha dx + P_e(n,\overline{p})\int_{\check{x}'_\delta}^{1} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_\delta - x\right)\right|^\alpha dx \\
& + P_b(n,\overline{p})b.
\end{aligned}
$$

Next, we rewrite this expression for a $\check{x}'_\delta$-type, after increasing both cutpoints. Compared to expression (40), note that besides replacing $\check{x}_\delta$ with $\check{x}'_\delta$ in the absolute brackets of the integrals, also the P-terms of the second and fourth terms are affected. This gives:

$$
\begin{aligned}
& LHS\,(30)\,\Big|_{(\check{x}'_{-\delta},\check{x}'_\delta),\check{x}'_\delta} \\
= \;& P_e(n,\overline{p})\int_{-1}^{\check{x}_{-\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx \qquad (41)\\
& + P_{ne}(n,\overline{p})\int_{\check{x}_{-\delta}}^{\check{x}'_{-\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx + P_{ne}(n,\overline{p})\int_{\check{x}'_{-\delta}}^{\check{x}_\delta} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx \\
& + P_e(n,\overline{p})\int_{\check{x}_\delta}^{\check{x}'_\delta} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx + P_e(n,\overline{p})\int_{\check{x}'_\delta}^{1} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx \\
& + P_b(n,\overline{p})b \\
& + \left[P_e(n,\overline{p}) - P_{ne}(n,\overline{p})\right]\int_{\check{x}_{-\delta}}^{\check{x}'_{-\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx \\
& - \left[P_e(n,\overline{p}) - P_{ne}(n,\overline{p})\right]\int_{\check{x}_\delta}^{\check{x}'_\delta} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx,
\end{aligned}
$$

where the last two terms are used to make the first six terms comparable to the six terms in expression (40). Importantly, these terms are strictly larger in expression (41) than in (40). This follows from Lemma 1 by setting $x_i = \check{x}_\delta$ and $x_i = \check{x}'_\delta$, respectively, and using $x_{\min}(\check{x}_{-\delta}, \check{x}_\delta) \leq \check{x}_\delta < \check{x}'_\delta$ (because $\check{x}'_\delta$ moves on the same net-benefits curve as $\check{x}_\delta$).[22]

[22] To see this, we simplify expression (40) and the first six terms of expression (41), the latter of which are equivalent to $LHS\,(30)\,\Big|_{(\check{x}_{-\delta},\check{x}_\delta),\check{x}'_\delta}$. This gives:

$$
\begin{aligned}
LHS\,(30)\,\big|_{(\check{x}_{-\delta},\check{x}_\delta),\check{x}_\delta} &= P_e(n,\overline{p})\textstyle\int_{-1}^{\check{x}_{-\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_\delta - x\right)\right|^\alpha dx + P_{ne}(n,\overline{p})\int_{\check{x}_{-\delta}}^{\check{x}_\delta} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_\delta - x\right)\right|^\alpha dx \\
&\quad + P_e(n,\overline{p})\int_{\check{x}_\delta}^{1} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_\delta - x\right)\right|^\alpha dx + P_b(n,\overline{p})b
\end{aligned}
$$

Next, we examine the last two terms of expression (41). If it holds that

$$\begin{aligned}&[P_e(n,\overline{p})-P_{ne}(n,\overline{p})]\int_{\check{x}_{-\delta}}^{\check{x}'_{-\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx\\ &\qquad\geq [P_e(n,\overline{p})-P_{ne}(n,\overline{p})]\int_{\check{x}_{\delta}}^{\check{x}'_{\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx\end{aligned}$$

$$\Leftrightarrow \int_{\check{x}_{-\delta}}^{\check{x}'_{-\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx \geq \int_{\check{x}_{\delta}}^{\check{x}'_{\delta}} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta - x\right)\right|^\alpha dx, \tag{42}$$

where $P_e(n,\overline{p}) \geq P_{ne}(n,\overline{p})$ for $p \in (0,1]$,[23] then we have shown that $LHS(30)$ always strictly increases for a given $\overline{p}$ if both cutpoints increase. Note that for $\overline{p}=0$, the only feasible pair is $(\check{x}_{-\delta}=-1, \check{x}_\delta=1)$. To see that condition (42) indeed holds, it is sufficient to show that the minimal gain on the left-hand side, $\int_{\check{x}_\delta}^{\check{x}'_\delta} f_\delta(x)dx\left|\frac{1}{2}\left(\check{x}'_\delta-\check{x}'_{-\delta}\right)\right|^\alpha$ (using $\int_{\check{x}_{-\delta}}^{\check{x}'_{-\delta}} f_\delta(x)dx = \int_{\check{x}_\delta}^{\check{x}'_\delta} f_\delta(x)dx$, since $\overline{p}$ is held constant), is equal to or larger than the maximal loss on the right-hand side, $\int_{\check{x}_\delta}^{\check{x}'_\delta} f_\delta(x)dx\left|\frac{1}{2}\left(\check{x}'_\delta-\check{x}_{\delta}\right)\right|^\alpha$. That is, we set the most extreme values constant and multiply them by the equal probabilities. This gives:

$$\begin{aligned}\int_{\check{x}_\delta}^{\check{x}'_\delta} f_\delta(x)dx\left|\tfrac{1}{2}\left(\check{x}'_\delta-\check{x}'_{-\delta}\right)\right|^\alpha &\geq \int_{\check{x}_\delta}^{\check{x}'_\delta} f_\delta(x)dx\left|\tfrac{1}{2}\left(\check{x}'_\delta-\check{x}_{\delta}\right)\right|^\alpha\\ \Leftrightarrow \left|\tfrac{1}{2}\left(\check{x}'_\delta-\check{x}'_{-\delta}\right)\right|^\alpha &\geq \left|\tfrac{1}{2}\left(\check{x}'_\delta-\check{x}_{\delta}\right)\right|^\alpha,\end{aligned} \tag{43}$$

which always holds because $\check{x}'_{-\delta} \leq \check{x}_{\min} \leq \check{x}_\delta < \check{x}'_\delta$. Note that the same things hold when there is one boundary cutpoint, $\check{x}'_{-\delta}=-1$ (this is readily verified by replacing $\check{x}'_{-\delta}$ with -1 in expressions (41) to (43)). Therefore, increasing $\check{x}_{-\delta}$ and $\check{x}_\delta$ while keeping $\overline{p}$ constant yields $LHS\,(30)\left|_{(\check{x}_{-\delta},\check{x}_\delta),\check{x}_\delta}\right. < LHS\,(30)\left|_{(\check{x}'_{-\delta},\check{x}'_\delta),\check{x}'_\delta}\right.$, and also $LHS\,(30)\left|_{(\check{x}_{-\delta},\check{x}_\delta),\check{x}_{-\delta}}\right.$ $> LHS\,(30)\left|_{(\check{x}'_{-\delta},\check{x}'_\delta),\check{x}'_{-\delta}}\right.$ (to understand the latter inequality, consider the reverse decreases from $\check{x}'_{-\delta}$ to $\check{x}_{-\delta}$ and $\check{x}'_\delta$ to $\check{x}_\delta$, which is analyzed analogous to the increases above). However, in equilibrium it must hold for $(\check{x}_{-\delta},\check{x}_\delta)$ that $LHS\,(30)\left|_{(\check{x}_{-\delta},\check{x}_\delta),\check{x}_{-\delta}}\right.$

$< LHS\,(30)\left|_{(\check{x}_{-\delta},\check{x}_\delta),\check{x}'_\delta}\right. = P_e(n,\overline{p})\int_{-1}^{\check{x}_{-\delta}} f_\delta(x)\left|\frac{1}{2}\left(\check{x}'_\delta-x\right)\right|^\alpha dx + P_{ne}(n,\overline{p})\int_{\check{x}_{-\delta}}^{\check{x}_\delta} f_\delta(x)\left|\frac{1}{2}\left(\check{x}'_\delta-x\right)\right|^\alpha dx$

$$+P_e(n,\overline{p})\int_{\check{x}_\delta}^{1} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}'_\delta-x\right)\right|^\alpha dx + P_b(n,\overline{p})b.$$

for $\check{x}_\delta < \check{x}'_\delta$ (see Lemma 1).

[23] Using expressions (27) and (28), this is derived as follows: $P_e(n,p) \geq P_{ne}(n,p)$ for $p \in (0,1]$

$$\Leftrightarrow \quad p\sum_{k=1}^{n}(1-p)^{k-1} \geq np(1-p)^{n-1} + (n-1)\,p^2(1-p)^{n-2} \Leftrightarrow \sum_{k=1}^{n}\frac{1}{(1-p)^{n-k-1}} \geq n-p$$

$$\Leftrightarrow \quad \frac{1}{(1-p)^{n-2}} + \frac{1}{(1-p)^{n-3}} + \ldots + 1 + (1-p) \geq n-p, \text{ since } \frac{1}{(1-p)^{n-k-1}} \geq 1 \text{ for } k=1,\ldots,n-1$$

if $p \in (0,1)$ and $\frac{1}{(1-p)^{n-k-1}} = 1-p$ if $k=n$. Note that $P_e(n,\overline{p}) = P_{ne}(n,\overline{p})$ if $n=2$.

$= LHS\,(30)\left|_{(\check{x}_{-\delta},\check{x}_{\delta}),\check{x}_{\delta}}\right.$, and thus, given the two inequalities it cannot hold simultaneously for $(\check{x}'_{-\delta},\check{x}'_{\delta})$ that $LHS\,(30)\left|_{(\check{x}'_{-\delta},\check{x}'_{\delta}),\check{x}'_{-\delta}}\right. = LHS\,(30)\left|_{(\check{x}'_{-\delta},\check{x}'_{\delta}),\check{x}'_{\delta}}\right.$. Thus, for any given entry probability $\overline{p} \in [0,1]$, there is a unique cutpoint equilibrium. Given the properties of the equilibrium path derived above, this also means that there is a unique cutpoint equilibrium for any given $c > 0$, which completes our proof Proposition 2. ■

7.3 Proof of Proposition 3

Proof. We begin by analyzing the comparative statics effects of changes in the costs of entry, c, the benefits from holding office, b, and the degree of risk aversion, α, on the equilibrium cutpoints, $(\check{x}_l^*, \check{x}_r^*)$, that use at least one interior cutpoint. Thereafter, we show that a change in the number of citizens, n, has ambiguous effects on this equilibrium, and we also derive the cutpoints for very large n, that is, $\lim_{n\to\infty}\check{x}_l^*(n)$ and $\lim_{n\to\infty}\check{x}_r^*(n)$.

The proof uses the best response entry strategy (30). First, note that the three $P(n,p)$-terms (see expressions (27) to (29)) in this condition are not directly affected by a change in c, b, or α, and the three integral terms are (not) directly affected by a change in α $(c, b$, or $n)$. Importantly, if $\check{x}_{-\delta}$ and $\check{x}_{\delta}$ are interior, we know from the proof of Proposition 2 that there is a unique equilibrium path where $LHS(30)\left|_{\check{x}_{-\delta}}\right. = LHS(30)\left|_{\check{x}_{\delta}}\right. = c$ and both cutpoints simultaneously get more extreme if c increases. Moreover, if $\check{x}_{-\delta} = -1$ and $\check{x}_{\delta}$ is interior, there is a unique equilibrium path where $LHS(30)\left|_{\check{x}_{-\delta}=-1}\right. \leq LHS(30)\left|_{\check{x}_{\delta}}\right. = c$ and $\check{x}_{-\delta} = -1$ remains and $\check{x}_{\delta}$ gets more extreme if c increases. These results can be used to derive the following comparative statics effects:

(Costs of entry) $LHS(30)$ is constant in c while $RHS(30)$ is strictly increasing in c for $\delta = l, r$. Because on the unique equilibrium path $LHS(30)$ is strictly increasing if both interior cutpoints $\check{x}_{-\delta}$ and $\check{x}_{\delta}$ get more extreme (if $\check{x}_{-\delta} = -1$ remains and the interior cutpoint $\check{x}_{\delta}$ increases), this implies that on this path $\check{x}_{-\delta}$ strictly decreases (remains) and $\check{x}_{\delta}$ strictly increases if c increases. This implies less entry, in the sense of stochastic dominance, and therefore the expected number of candidates decreases. It also implies that candidates and policy outcomes are more extreme, on average.

(Benefits from holding office) $LHS(30)$ is strictly increasing in b (since $P_b(n,p) > 0$) while $RHS(30)$ is constant in b for $\delta = l, r$. Because on the unique equilibrium path $LHS(30)$ is strictly decreasing if both interior cutpoints $\check{x}_{-\delta}$ and $\check{x}_{\delta}$ get more moderate (if $\check{x}_{-\delta}(\check{x}_{\delta} \geq \overline{x}_{\delta}) = -1$ remains or increases and the interior cutpoint $\check{x}_{\delta}$ decreases), this implies that on this path $\check{x}_{-\delta}$ strictly increases (remains or strictly increases) and $\check{x}_{\delta}$ strictly decreases if b increases. This implies more entry, in the sense of stochastic dominance, and therefore the expected number of entrants increases. It also implies that candidates and policy outcomes are less extreme, on average.

(Risk aversion) $RHS(30)$ is constant in α and $LHS(30)$ is strictly decreasing in α for $\delta = l, r$ (since $P_{ne}(n,p) > 0$, $P_e(n,p) > 0$, and $\partial \int f(x)\left|\frac{1}{2}(\check{x}_{\delta} - x)\right|^{\alpha} dx/\partial\alpha = \int f(x)\left|\frac{1}{2}(\check{x}_{\delta} - x)\right|^{\alpha} \ln\left|\frac{1}{2}(\check{x}_{\delta} - x)\right| dx < 0$, where the inequality holds because $\ln\left|\frac{1}{2}(\check{x}_{\delta} - x)\right| < 0$ for any combination of $\check{x}_{\delta} \in (\check{x}_{\min}, 1), x \in [-1,1]$, and all three definite integrals in condition (30)). Because on the unique equilibrium path $LHS(30)$ is strictly increasing if both interior cutpoints $\check{x}_{-\delta}$ and $\check{x}_{\delta}$ get more extreme (if $\check{x}_{-\delta} = -1$ remains and the interior cutpoint $\check{x}_{\delta}$ increases), this implies that on this path $\check{x}_{-\delta}$ strictly decreases (remains) and $\check{x}_{\delta}$ strictly increases if α increases. This implies less entry, in the sense of stochastic dominance, and therefore the expected number of entrants decreases. It also implies that candidates and policy outcomes are more extreme, on average.

(Community size) Here we show that $\lim_{n\to\infty} \check{x}_l^*(\check{x}_r^*, n) = -1$. The proof that $\lim_{n\to\infty} \check{x}_r^*(\check{x}_l^*, n) = 1$ is identical. Because we are looking at infinite sequences on a compact set, there must exist at least one convergent subsequence so we only need to show $\liminf_{n\to\infty} \check{x}_l^*(\check{x}_r^*, n) = -1$. Suppose to the contrary that $\liminf_{n\to\infty} \check{x}_l^*(\check{x}_r^*, n) > -1$. Then there exists an ϵ and a subsequence $\{n_k\} \to \infty$ and an integer $\overline{k}$ such that for $k > \overline{k}$ the probability a randomly selected citizen enters equals $p_k > \epsilon$. This implies that the equilibrium probability of winning along this subsequence goes to zero. But this in turn implies that nobody will enter, which implies $\check{x}_l^*(\check{x}_r^*, n_k) = -1$, a contradiction. ■

7.4 Proof of Proposition 4

Proof. Here, we derive the expected number of candidates in very large communities. For very large n, the best response condition (30) for $\delta = l, r$ is:

$$\begin{aligned}
&\lim_{n\longrightarrow\infty} \Bigg[P_{ne}(n,p) \int_{\check{x}_{-\delta}}^{\check{x}_{\delta}} f_\delta(x) \left|\tfrac{1}{2}(\check{x}_\delta - x)\right|^\alpha dx \\
&\quad + P_e(n,p) \left(\int_{-1}^{\check{x}_{-\delta}} f_\delta(x) \left|\tfrac{1}{2}(\check{x}_\delta - x)\right|^\alpha dx + \int_{\check{x}_\delta}^{1} f_\delta(x) \left|\tfrac{1}{2}(\check{x}_\delta - x)\right|^\alpha dx \right) + P_b(n,p) b \Bigg] \\
&\geq \lim_{n\longrightarrow\infty} c.
\end{aligned}$$

Using $\lim_{n\to\infty} \check{x}_{-\delta}^*(\check{x}_\delta^*, n) = -1$ and $\lim_{n\to\infty} \check{x}_\delta^*(\check{x}_{-\delta}^*, n) = 1$ (see Proposition 3) and $\lim_{n\to\infty} c = c$, the best response condition for a citizen type $\check{x}_\delta$ can be reduced to:

$$\lim_{n\longrightarrow\infty} \left[P_{ne}(n,p) \int_{-1}^{1} f(x) \left|\tfrac{1}{2}(1 - x)\right|^\alpha dx + P_b(n,p) b \right] \geq c. \tag{44}$$

Moreover, defining $v_\delta \equiv \int_{-1}^{1} f(x) \left|\frac{1}{2}(1-x)\right|^\alpha dx$, and using $\lim_{n\to\infty} \frac{n-1}{n} = 1$ and expressions (27) and (29) gives:

$$\lim_{n\longrightarrow\infty} P_{ne}(n,p) = \lim_{n\longrightarrow\infty} \frac{(n-1)(1-p)^{n-2}}{n} = \lim_{n\longrightarrow\infty} (1-p)^{n-2}$$

and

$$\lim_{n\longrightarrow\infty} P_b(n,p) = \lim_{n\longrightarrow\infty} \left[\frac{1-(1-p)^n}{np} - \frac{(1-p)^{n-1}}{n} \right].$$

Because n is very large and p is very small (as $\lim_{n\to\infty} \check{x}_{-\delta}^*(\check{x}_\delta^*, n) = -1$ and $\lim_{n\to\infty} \check{x}_\delta^*(\check{x}_{-\delta}^*, n) = 1$), we can approximate the binomial distribution by the Poisson distribution using $\binom{N}{k} p^k (1-$

$p)^{N-k} \approx \frac{(Np)^k}{k!}e^{-Np}$. Moreover, let us denote $\tau \equiv \lim_{n\to\infty} E(m) = \lim_{n\to\infty} np$ and $\tau_\delta \equiv \lim_{n\to\infty} E(m_\delta) = \lim_{n\to\infty} np_\delta$, where $p = p_{-\delta} + p_\delta$, $m = m_{-\delta} + m_\delta$, and $\tau = \tau_{-\delta} + \tau_\delta$ for $\delta = l, r$ and $\delta \neq -\delta$. Then, setting $k = 0$ and $N = n-2$ in the Poisson approximation yields:

$$(1-p)^{n-2} \approx \frac{[(n-2)p]^0}{0!}e^{-(n-2)p} = e^{-(n-2)(p_{-\delta}+p_\delta)} \approx e^{-(\tau_{-\delta}+\tau_\delta)},$$

where $e^{-(n-2)p_{-\delta}} \approx e^{-np_{-\delta}}$ and $e^{-(n-2)p_\delta} \approx e^{-np_\delta}$. Similarly, setting $k = 0$ and $N = n$ in the Poisson approximation yields:

$$\frac{1-(1-p)^n}{np} - \frac{(1-p)^{n-1}}{n} \approx \frac{1 - \frac{(np)^0}{0!}e^{-np}}{\tau} - \frac{\frac{[(n-1)p]^0}{0!}e^{-(n-1)p}}{n} = \frac{1-e^{-(\tau_{-\delta}+\tau_\delta)}}{\tau_{-\delta}+\tau_\delta},$$

where $\lim_{n\to\infty} \frac{e^{-(n-1)p}}{n} = 0$ since $e^{-(n-1)p} \leq 1$ for $p \in [0,1]$. Using these results, we can rewrite the best response condition (44) as

$$e^{-(\tau_{-\delta}+\tau_{-\delta})}v_\delta + \frac{1-e^{-(\tau_{-\delta}+\tau_{-\delta})}}{\tau_{-\delta}+\tau_{-\delta}}b \geq c. \tag{45}$$

In the following, we distinguish between conditions (iii) and (iv) of Proposition 2. First, note that

$$\lim_{n\longrightarrow\infty} \underline{c} = \lim_{n\longrightarrow\infty} \frac{1}{n}\left[b + \int_{-1}^{1} f_\delta(x)\left|\tfrac{1}{2}\left(\check{x}_{\min} - x\right)\right|^\alpha dx\right] = 0$$

and

$$\lim_{n\longrightarrow\infty} \overline{c}_\delta \equiv \lim_{n\longrightarrow\infty} \frac{n-1}{n}\left[b + v_\delta\right] = b + v_\delta.$$

Without loss of generality, let us assume that $\widetilde{c} = \widetilde{c}_{-\delta} \leq \widetilde{c}_\delta$ (see the proof of Proposition 2). If $0 < c < \widetilde{c}$ (see Proposition 2 (iv)), stating the "limit" best response condition (45) as equality and rearranging yields the following implicit function for $\delta = l, r$:

$$\tau_\delta = -\tau_{-\delta} + (\tau_{-\delta}+\tau_\delta)e^{-(\tau_{-\delta}+\tau_\delta)}\frac{v_\delta - \frac{1}{2}}{c} + \left[1 - e^{-(\tau_{-\delta}+\tau_\delta)}\right]\frac{b+\frac{1}{2}}{c}, \tag{46}$$

which proves the first part of Proposition 4. If $c \in (\widetilde{c}, \overline{c}_\delta = \overline{\overline{c}}]$ (see Proposition 2 (iii)), then $\check{x}_{-\delta} = -1$ and $p_{-\delta} = 0$ and we simply set $\tau_{-\delta} = 0$ in condition (46). This can be done since the best response condition (44) already accounts for $\check{x}_{-\delta} = -1$ (as $\lim_{n\to\infty} \check{x}_{-\delta}(n) = -1$), and it is readily verified that setting $p_{-\delta} = 0$ and following all steps that led to condition (46) does indeed give:

$$\tau_\delta = \tau_\delta e^{-\tau_\delta}\frac{v_\delta - \frac{1}{2}}{c} + \left(1 - e^{-\tau_\delta}\right)\frac{b+\frac{1}{2}}{c}, \tag{47}$$

which proves the second part of Proposition 4. Finally, note that for symmetric probability distributions of ideal points the equilibrium cutpoints are $(\check{x}_{-\delta} = -\check{x}_\delta, \check{x}_\delta)$ and $\overline{\overline{c}} = \overline{c}_{-\delta} = \overline{c}_\delta$. Then, for $c \in (0, \overline{c})$ we have $\tau_{-\delta} = \tau_\delta = \frac{\tau}{2}$ (recall that $\tau = \tau_{-\delta} + \tau_\delta$) and $v = v_{-\delta} = v_\delta$.

This yields

$$\tau = \tau e^{-\tau} \frac{v - \frac{1}{2}}{c} + \left(1 - e^{-\tau}\right) \frac{b + \frac{1}{2}}{c}. \tag{48}$$

■

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