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Corrigendum: Model for how an accretion disk drives astrophysical jets and sheds angular momentum (2018 *Plasma Phys. Control. Fusion* 60 014006)

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The terms containing $\nabla \cdot (r^{-2}\nabla\psi)$ in equations (9*a*) and (9*c*) had the wrong sign in the paper 'Model for how an accretion disk drives astrophysical jets and sheds angular momentum (2018 *Plasma Phys. Control. Fusion* **60** 014006)'. The corrected form of equation (9) is

$$\frac{\partial}{\partial t}(\rho U_r) + \nabla\cdot(\rho U_r \mathbf{U}) = -\frac{1}{4\pi^2}\left(\frac{1}{\mu_0}\nabla\cdot\left(\frac{1}{r^2}\nabla\psi\right)\frac{\partial\psi}{\partial r} + \frac{\mu_0 I}{r^2}\frac{\partial I}{\partial r}\right) - \frac{\partial P}{\partial r} + \frac{\rho U_\phi^2}{r}, \tag{1a}$$

$$\frac{\partial}{\partial t}(\rho r U_\phi) + \nabla\cdot(\rho r U_\phi \mathbf{U}) = \frac{1}{4\pi^2}(\nabla I \times \nabla\psi\cdot\nabla\phi), \tag{1b}$$

$$\frac{\partial}{\partial t}(\rho U_z) + \nabla\cdot(\rho U_z \mathbf{U}) = -\frac{1}{4\pi^2}\left(\frac{1}{\mu_0}\nabla\cdot\left(\frac{1}{r^2}\nabla\psi\right)\frac{\partial\psi}{\partial z} + \frac{\mu_0 I}{r^2}\frac{\partial I}{\partial z}\right) - \frac{\partial P}{\partial z}. \tag{1c}$$

Because the magnitude of the terms with the wrong sign was shown to be negligible in the discussion of equation (12) this incorrect sign has no impact on the discussion of the jet velocity as presented in section 4.1. However, the incorrect sign does have an impact on the jet collimation discussion presented in section 4.2. The ultimate result remains the same but the logical argument leading to this result needs to be corrected and in particular the argument given in equations (25)–(27) needs to be replaced. This can be done in three equivalent and complimentary ways which will now be presented.

The first way is to restate equation (25) as

$$\begin{aligned}\frac{1}{\mu_0}\nabla\cdot\left(\frac{1}{r^2}\nabla\psi\right) &= \frac{\psi_0}{\mu_0 r^2}\frac{\partial^2}{\partial z^2}\left(\frac{1}{a(z)}\right)^2 \\ &= \frac{\psi_0}{\mu_0 a_0^2 r^2}\frac{\partial^2}{\partial z^2}\exp\left(-2\int_0^z \kappa(z')\mathrm{d}z'\right) \\ &= -2\frac{\psi_0}{\mu_0 a_0^2 r^2}\frac{\partial}{\partial z}\left(\kappa(z')\exp\left(-2\int_0^z \kappa(z')\mathrm{d}z'\right)\right) \\ &= \frac{1}{\mu_0 r^2}\left(-2\frac{\partial\kappa}{\partial z} + 4\kappa^2\right)\psi\end{aligned} \tag{2}$$

so equation (26) with corrected sign becomes

$$-\frac{1}{\mu_0}\left[\nabla\cdot\left(\frac{1}{r^2}\nabla\psi\right)\right]\frac{\partial\psi}{\partial z} \sim \frac{1}{\mu_0 r^2}\left(2\frac{\partial\kappa}{\partial z} - 4\kappa^2\right)\psi\frac{\partial\psi}{\partial z}. \tag{3}$$

Since $\partial\psi/\partial z$ is negative, equation (3) provides a retarding force for sufficiently large $\partial\kappa/\partial z$ and this retarding force will overcome the accelerating force given in equation (27) if

$$2\frac{\partial\kappa}{\partial z} - 4\kappa^2 > \lambda^2. \tag{4}$$

Thus retardation occurs if there is a sudden increase in κ which corresponds to a sharp turning of the poloidal field direction at the jet tip. Specifically, the poloidal field near the z axis is nearly in the z direction in the jet main body but at the jet tip turns abruptly to be in the r direction.

The second equivalent way is to note that there is a bunching up and hence greater density of poloidal flux surfaces at the jet tip. This occurs because the bundle of poloidal magnetic field lines that had been in the main jet body and aligned nearly parallel to the z axis turns to go in the positive

r direction and then at larger r turns again to go in the negative z direction. This bunching up of poloidal field lines at the tip occurs because the jet distends what were initially dipole-like poloidal flux surfaces and stretches these flux surfaces out to the length of the jet. Thus, for a given r the poloidal flux is a slightly decreasing plateau going from $z = 0$ to the instantaneous length of the jet, while at the tip of the jet there is a cliff-like sudden fall-off of the poloidal flux from its plateau value to a much lower value in the vacuum-like region above the tip (i.e., region where z exceeds the jet length). The sudden fall-off region is where the distended poloidal field lines are bunched up. The greatly enhanced poloidal flux density in the cliff-like region implies that at the tip B^2 has a sharp local maximum with respect to z. The curvature term $-B^2\hat{R}/\mu_0 R$ in equation (24) is large and points in the $-z$ direction at the hair-pin-like $180°$ turnaround of the bundle of poloidal field lines near the tip. For z slightly less than the location of maximum B^2 the gradient term $-\nabla_\perp(B^2/2\mu_0)$ is similarly large and also points in the $-z$ direction so the two terms on the right-hand side of equation (24) are additive and give a net retardation force at the tip. However, for z slightly larger than the location of maximum B^2, i.e., just above the tip, the $-\nabla_\perp(B^2/2\mu_0)$ now points in the positive z direction whereas the curvature term $-B^2\hat{R}/\mu_0 R$ continues to point in the negative z direction so above the tip the curvature and gradient terms cancel rather than add. This is essentially a statement that the region above the tip has no current so the left-hand side of equation (24) vanishes, i.e., the curvature and gradient terms on the right-hand side of equation (24) are equal in magnitude and have opposite signs.

The third equivalent, but less precise way is to note that the squeezing together of the poloidal field lines in the main body near the z axis implies there is a positive J_ϕ while the turning of these field lines at the jet tip to go in the positive r direction implies that at the jet tip there is a positive B_r. The axial magnetic force component $-J_\phi B_r$ is thus negative and, if sufficiently large, will overcome the positive $J_r B_\phi$ magnetic force component and result in a retardation.

These arguments indicate that near the tip the jet slows down as was assumed in the remainder of section 4.2 which showed that such slowing down leads to a collimation of the jet.

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