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SCHEDULING AUCTIONS AND PROTO-PARTIES IN LEGISLATURES

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Abstract

We consider the impact of the scarcity of plenary time in legislatures both on the outcome of the legislative bargaining process and the organization of the legislature itself. We do so by developing a novel model that we call *scheduling auctions*. In the model, the legislature is charged with allocating a fixed budget. Members can propose an allocation and the scheduling agent decides which one of the possible proposals will be considered by the entire legislature in plenary session for an up or down vote. We show in this simple setting that deciding which member should be selected as the scheduling agent endogenously induces the creation of nascent political parties that we call *proto-parties*. We also show that the these legislative structures have positive welfare implications.

JEL codes: C78, D44, D71, D72

1. INTRODUCTION

Time is a scarce commodity across most human endeavors. Here we consider the impact of the scarcity of plenary time in legislatures both on the outcome of the legislative bargaining process and the organization of the legislature itself. Plenary time is the period that a legislative body is in plenary session when important bills are considered by formally stated motions that must be voted upon.[1] The standard solution in almost all legislatures is to create offices that manage the scarse floor time by endowing the holders of these offices with special agenda-setting powers (Cox 2006, see also Cox and McCubbins 1993, and Polsby, Gallaher, and Rundquist 1969). For example, in the U.S.

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[1]For example, it is not possible that a legislature could meet in a plenary session for more than 24 hours a day and often there are constitutional limits on the number of days a given legislature can be in session.

House of Representatives most scheduling matters are handled by the Rules Committee, arguably the most powerful committee in the chamber.[2]

We develop a simple formal model of legislative bargaining with such a scheduling agent that we refer to as a *scheduling auction*. In the model, the legislature is charged with allocating a fixed budget. Members can propose an allocation and the scheduling agent decides which one of the possible proposals will be considered by the entire legislature in plenary session for an up or down vote. If the proposal passes then it is implemented, and if not, some exogenous status quo allocation is implemented. The model, therefore, resembles an auction for the lone available legislative slot where the proposals are the bids. This competition for floor time induces the legislators to undercut one-another's proposals in a manner that is quite similar to price competition á lá Bertrand. This leads the scheduling agent to capture much of the rents from the division of the pie.

The question then becomes which of the legislators will be afforded this potentially lucrative role. Instead of pinpointing the specific legislator or procedure by which this is done, we instead identify a specific group of legislators who have a common incentive to select the scheduling agent from amongst themselves. This group, which we call the *proto-party*, is large enough to be able to push forth their choice by majority approval. Members of the proto-party have a common incentive to restrict the choice of the scheduling agent to one of them. Clearly, this increases the likelihood that one of them will be selected for this important role. Additionally, the proto-party indirectly helps members by increasing the likelihood that their preferred proposal is actually enacted.

We show that in most cases, the proto-party is comprised of a majority of legislators. There also exist cases where the proto-party is larger than a majority. Interestingly enough, there are also cases where the proto-party is just shy of a majority, and anecdotal evidence suggests that such cases do occassionally perspire in real-life legislatures.[3]

What is perhaps most surprising is that we are able to provide a simple rationale for the existence of party structures, a key feature of legislative organization, in a one-shot purely-redistributive setting of dividing a budget where legislators' preferences are completely heterogeneous (and adversarial). That is, we provide a complete model of legislative bargaining where parties arise endogenously purely by the need to solve the legislative scheduling problem. Further, we show that when the scheduling agent is selected from a proto-party, such arrangements in most cases lead to higher social welfare relative to a *content-neutral scheduling*, where the likelihood that a bill is considered by the legislature is independent of the bill's content.

[2]In fact, the Rules Committee also gets to choose the rules for the vote, which gives it even more power (see http://www.rules.house.gov/).

[3]That occured, for example in the 1994 elections to the California State legislature. resulted in a narrow win for the GOP, leading to a messy process of electing the Speaker of the House, who has large scheduling powers. Ultimately, one of the Republican legislators declared himself an independent, and voted in favor of the choice promoted by the Democrats. Abstracting from the messy details of the story (see e.g., The Chicago Tribune, December 12, 1994, "California Assembly At A Standstill"), this situation corresponds very tightly to the case where the proto-party is just shy of a majority.

The rationale for political parties presented here is quite different from the explanations in the rich literature on political parties.[4] Typically, models of legislative party formation assume some signalling value to the voters who must elect legislators, see Snyder and Ting (2002) or Levy (2004), but in our model there is no election stage and our setting is one of complete information. There is an alternative thread in the literature that looks at the rise of political parties from the need to overcome some organization efficiency problems within the legislature, see Cox and McCubbins (1993) and (2005).[5] However, these tend to be more empirically motivated and they do not provide a complete model of legislative bargaining that incorporates these structures endogenously. Our approach is motivated by these studies.

The only other model which builds on endogenous incentives within the legislature that we are aware of is Jackson and Moselle (2002). Their explanation is based on overall efficiency gains from the party structure, which is not the case in the purely redistributive setting considered here. Related is also recent work by Diermeier and Vlaicu (2011), whose argument for political parties in a multi-dimensional model stems from the legislators' correlated preferences.[6] Our results, on the other hand, are being driven purely by the procedural requirements to run a legislature with finite legislative time.

The standard models of legislative bargaining either ignore any scarcity of plenary time or deal with it implicitly by a *content-neutral scheduling* mechanism. For example, in Shepsle (1979) the legislators are divided into committees, and each committee is the monopoly supplier of proposals in their given policy jurisdiction with guaranteed access to a floor vote for their proposed bill, but there is no scarcity of floor time in his model. Baron and Ferejohn (1989), on the other-hand, explicitly model the scarcity of time in a dynamic model where the payoff from the ultimate policy enacted is discounted by the length of time to pass the bill, but assume that Nature randomly selects a legislator to make a proposal.

Another strand of the literature models the legislative scheduling problem, but abstracts from either bargaining or scarcity of floor time. For example, Cox and McCubbins (1993) examine the order the scheduling agent would have proposals voted on in a non-strategic multi-armed bandit model, where there is no impact of competition on proposal behavior. McKelvey and Riezman (1992) take a different approach whereby the recognition probability of a member is determined by a seniority rule in order to endogenously generate an incumbency advantage for the members of the legislature, but they assume independent scheduling.

[4] In political science, the literature dates at least as far back as the works by Duverger (1951) and Riker (1962). A few classic on the economic side of this literature are Stiegler (1971), Becker (1983), and Kalt and Zupan (1984).

[5] This approach is the theory of industrial organization applied to legislative design. It motivated by the seminal work of Demsetz (1985), who compares features of political parties to organizational arrangements in market settings.

[6] In the real world, there may also be other, less utilitarian rationales for political parties, see e.g., the work by Krehbiel (1993) and Kiewiet and McCubbins (1991).

Then there have been a number of papers on endogenous agenda formation, such as Banks and Gasmi (1987), Patty and Penn (2008), and Penn (2008), but these are best characterized as determining the amendment tree of a single bill and again assume independent scheduling of the final votes. Related is also work on legislative procedures, such as Diermeier and Feddersen (1998), who study a vote of confidence procedure and show it creates incentives for members of ruling coalitions to vote together. Levy and Razin (2010) analyze a dynamic model of an all-pay contest of agenda formation where the probability to get a proposal on the agenda is increasing in an agent's payment, and there is a probability that the session ends before any proposal is implemented. Diermeier et. at (2011) study allocation of procedural rights by a majority, and Eguia end Shepsle (2013) study allocation of procedural rights in a dynamic bargaining model. Finally, Palmer (2013) gives a rationale for a majority to collectively allocate scheduling rights to one legislator. In short, while there is a rich volume of studies, we are not aware of any theories of legislative bargaining in the literature that would explicitly model the scarcity of plenary time as well as its impact on policy decisions and the structure of the legislature.

On the more technical side, scheduling auctions bears the most similarity to the Baron and Ferejohn (1989) model of legislative bargaining. However, there are crucial differences. First, in a scheduling auction a bill's content affects the chances it is afforded for consideration by the full chamber. Second, the two models differ in terms of their mechanics. In Baron and Ferejohn (1989) the legislative session is modeled as an infinite-horizon legislative bargaining game. Here, the legislative session is modeled as a one-shot game, where in case of an impasse the legislators obtain their shares under some exogenously given status quo distribution. Finally, our model allows for the possible specification of *legislated constraints*, which are some additional constraints that must be observed in the distribution of resources.

In terms of policy implications, our main result (Theorem 1) is that the surplus to the scheduling agent is given as the residual surplus after the *relatively cheapest* majority have been compensated for their status quo shares.[7] In contrast, in Baron and Ferejohn (1989), the surplus to the proposing agent is determined as the residual amount after the minimal, or *cheapest*, winning majority of legislators have been compensated for their expected continuation values that would obtain had they voted against the proposed distribution. Thus, adding legislated constraints to the model explicitly leads to potentially quite different quantitative as well as qualitative predictions. Moreover, depending on the legislated constraints and the status quo, the winning majority need not be minimal, but may in fact be a supra-majority (Proposition 3). This latter implication is observed in real legislative sessions, and cannot be obtained in models that build on the Baron and Ferejohn (1989) framework, e.g., Jackson and Moselle (2002).[8]

[7] A legislator's relative cost is her status quo share minus the legislated constraint specifying the minimal share to that legislator.

[8] This also provides empirically plausible scenarios against Riker's (1962) intuition that winning coalitions should be minimal.

The exogenous specification of the status quo and the legislated constraints might at first blush seem a less desirable feature of our model. However, both of these parameters should be possible to estimate in most practical applications, for example, in many cases the law specifies the division of the budget in case of an impasse. But the status quo can also be thought of as a reduced form of continuation values from some dynamic bargaining procedure, much like the one in Baron and Ferejohn (1989). Using this insight we in fact derive as a special case of an scheduling auction a very simple exposition of the Baron and Ferejohn (1989) model. This immediately yields the uniqueness of the stationary equilibrium distributions in their original model, see also Eraslan (2002). The legislated constraints, on the other hand have a very real impact on the structure of the legislative outcomes, and self-interested groups of individuals might have strong incentives to put such constraints in place.[9] These parameters thus add additional flexibility to the model, making the model more maleable for empiricial analysis, while also making its mechanics simpler than the Baron and Ferejohn (1989) model.[10] It is precisely this tractability of the predictions of our model that makes it amenable to the study of the emergence of political parties and their consequence on aggregate welfare.

The rest of this paper is organized as follows. In the next section, we present a simple model of legislative bargaining. In Section 3 we develop the scheduling auction. We then turn to the endogenous selection of scheduling agent and the formation of proto-parties in Section 4. In Section 5 we present the welfare comparison of the content-neutral scheduling and the scheduling auction. There we also derive the Baron and Ferejohn (1989) model as a special case of the content-neutral scheduling in our setting.

2. THE MODEL

We consider $n+1$ legislators, where we think of these legislators as representatives of some constituencies. The set of legislators is denoted by $N = \{1, ..., n, n+1\}$. In our model of a legislature, all legislators except one, indexed by $\iota \in N$, can make proposals. Apart from representing a constituency, ι has a special role as the *scheduling agent* and we shall explain that role in a moment.

The setting of this paper is one of distributional politics. The kind of political decision that we have in mind is, for example, the decision to split a budget between the different constituencies. In that setting the policy space is given by $\mathbb{R}^{n+1}$, with some additional restrictions of two different sorts. The first kind of restriction is a budgetary restriction. We normalize the size of the budget to 1, so that the set of *all* possible policy outcomes

[9] In fact, our idea is inspired by the work of Stiegler (1971) who argued that groups of self-interested individuals will likely press for legislated constraints that will increase their benefits and will in fact make such groups even more cohesive. While our model is static, and legislated constraints are exogenous, such legislated constraints in our model figure importantly in providing common incentives to a group of legislators.

[10] The experimental study by Frechette et al (2005) suggests that additional degrees of freedom might be necessary for empirical applications of the Baron and Ferejohn (1989) model.

satsifying the budgetary restriction is formally represented by $\{x \in \mathbb{R}^{n+1} \mid \sum_{i=1}^{n+1} x_i \leq 1\}$; dimension i represents the share of the budget that is allocated to the constituency of legislator i. In general, the set of possible budget allocations might also be restricted by some existing legislation. We therefore call the restrictions of this second kind *legislated* restrictions.

Legislated restrictions are some exogenous legislative constraints, for example, a lower or an upper bound on resources that may be allocated to a given constituency. In principle, legislated restrictions might also allow for some constituencies to subsidize others, i.e., for the set of all policy outcomes to be negative on the dimensions of such generous constituencies. We shall exclude such possibilities and assume that the allocation to all constituencies must be non-negative. The set of all policy outcomes is then given by $\bar{B}(0) = \{x \mid \sum_{i=1}^{n+1} x_i \leq 1, x_i \geq 0, \forall i\}$. A more general possibility considered here is that the allocation to different constituencies must be above some vector of lowest possible shares, $b \in B(0)$, in which case the set of all possible policy outcomes is denoted by $\bar{B}(b) = \{x \mid \sum_{i=1}^{n+1} x_i \leq 1, x_i \geq b_i\}$; in general we will denote the set of all policy outcomes by B.[11]

In our model, we also specify a *status quo* allocation to the legislators, $x^{sq} \in \mathbb{R}^{n+1}$. One interpretation of the *status quo* is that it is the split of the budget which is implemented in case of an impasse. For example, x^{sq}, might be "the last-year's budget", or it might be some legally imposed distribution of resources, which comes in effect if the legislative assembly fails to pass any budget proposal. A different interpretation of x^{sq} is that it reflects how different legislators view the shares they would still find acceptable. For example, a legislator with a high share under the *status quo* can be interpreted as "tough" relative to a legislator with a low share. The *status quo* can be viewed as a result of how constituencies elect their representatives: a given constituency might elect a legislator who is tougher or softer at the bargaining table. Finally, the *status quo* can also be viewed as the continuation value that a legislator might receive in some dynamic game (with a scheduling auction as a stage game), were the game not to end in the first stage. In subsequent sections we will discuss the implications of the latter two interpretations in greater detail. We will assume that the point x^{sq} lies within the budget set, $x^{sq} \in B$. In the first static part of this study, we will assume that x^{sq} is an exogenous parameter of the model.

The preferences of each legislator are represented by a utility function u_i, where we assume that each legislator cares only about the allocation to their own constituency. Thus, given a feasible allocation $x \in B$, the utility of legislator i is given by $u_i(x_i)$, where $u_i : \mathbb{R} \to \mathbb{R}_+$ is some differenetiable, increasing and concave function. In our examples we will mostly consider the case where each legislator's utility function is linear in the share of the budget she obtains, $u_i(x_i) = x_i$.

To compare different legislative arrangements we assume that there is some socially-

[11] The most general possibility is that the set of all possible policy outcome is some convex, closed, and bounded set, $B \subset \{x \in \mathbb{R}^{n+1} \mid \sum_{i=1}^{n+1} x_i \leq 1\}$.

optimal feasible allocation of resources, denoted by x^o. For example, this socially-optimal allocation might be a maximizer of the social welfare function $W(x) = \sum_{i=1}^{n+1} u_i(x_i)$, and x^o is then given by,

$$x^o = \arg\max W(x), \text{ s.t., } x \in B.$$

When legislators' utility functions are linear, a welfare comparison between a policy x and x^o is particularly simple – the welfare loss relative to the socially optimal policy is simply the linear distance between x^o and x. Further assumptions regarding x^o will be specified in our examples.[12] The welfare loss associated with the policy x relative to x^o is then,

$$\bar{W}(x, x^o) = \sum_{i=1}^{n+1} |x_i - x_i^o|. \tag{1}$$

For example, when the social optimum is given by the most egalitarian policy, $x^o = (\frac{1}{n+1}, ..., \frac{1}{n+1})$, and that is feasible, i.e., $x^o \in B$, then the welfare loss of policy x is simply,

$$\bar{W}(x, x^o) = \sum_{i=1}^{n+1} |x_i - \frac{1}{n+1}|.$$

3. THE SCHEDULING AUCTION

Our model of legislative decision-making is the *scheduling auction*. The scheduling auction represents a legislative session where the split of the budget must be decided and is modeled as an extensive-form game, which proceeds in three stages. To simplify notation of this section we assume that the scheduling agent ι is fixed as legislator $n+1$, $\iota = n+1$.[13]

- In the first stage, each legislator $i \in \{1, ..., n\}$ makes one proposal, which is a budget proposal of a feasible allocation of resources to all legislators, including the scheduling agent, $n+1$. A proposal of legislator i is thus given by a feasible allocation of the budget $m_i \in B$, $i \in \{1, ..., n\}$. Denote by $m \in B^n$ the vector of all legislators' proposals.

- In the second stage, the scheduling agent ι selects the proposal that she likes best among all the proposed budget allocations. Given the vector of proposals m, the scheduling agent's choice is denoted by $x^A(m) \in \{m_i \mid i \in \{1, ..., n\}\}$.

[12] Given $W(x) = \sum_{i=1}^{n+1} u_i(x_i)$ and $B = \bar{B}(0)$, if the legislators' utilities are linear, then $W(x) = \sum_{i=1}^{n+1} x_i$, and any x, s.t., $\sum_{i=1}^{n+1} x_i = 1$ will be a maximand of W. Thus, any specific social optimum will in that case entail additional assumptions, e.g., that there is some social loss stemming from inequality. In contrast, when legislators' utilities are strictly concave and $u_i = u_j$, $\forall i, j$, then by concavity of u_i, the egalitarian policy is socially optimal under this $W(.)$.

[13] Here we define an scheduling auction where all legislators except the scheduling agent may be proposers. More generally, the set of proposers might be restricted to some subset of the legislators – e.g., the legislators who actually have access to the floor when it comes to budgetary decisions.

- In the third stage, the whole legislative body simultaneously cast votes between the status quo, and the selected proposal $x^A(m)$. The vote is an up or down vote between $x^A(m)$ and x^{sq}. The vote by legislator i is denoted by $d_i(x^A(m), x^{sq}) \in \{aye, nay\}$, $i \in \{1, ..., n+1\}$, and $d = (d_1, ..., d_{n+1})$ denotes the vector of all votes.

The final outcome of the scheduling auction is given by the $q - majority$ voting correspondence, where q is an exogenous parameter, $q \geq \frac{1}{2}$. Therefore, in general, $x^A(m)$ is the outcome of the scheduling auction if, $\frac{|\{i \in N | d_i(x^A(m), x^{sq}) = aye\}|}{n} > q$, and to resolve ties, we assume that the scheduling agent's vote counts only as a tie-breaker.[14] For example, when $q = \frac{1}{2}$, and $n+1$ is an even number the scheduling agent's vote effectively does not count. Since the scheduling agent only has a tie-breaking vote, if $d_{n+1} = aye$, and,

$$\frac{|\{i \in N \setminus \{n+1\} \mid d_i(x^A(m), x^{sq}) = aye\}|}{n} \geq q,$$

then the outcome of the scheduling auction is given by $x^A(m)$, and x^{sq}, otherwise. To illustrate the working of the scheduling auction consider an example.

EXAMPLE 1. Suppose that there are a total of 4 legislators, $n+1 = 4$: 3 other legislators and the scheduling agent $\iota = 4$. Let $q = \frac{1}{2}$, $b = (0.2, 0, 0.2, 0)$, and $x^{sq} = (0.3, 0.2, 0.3, 0.1)$; note that x^{sq} is not Pareto efficient. In the first stage legislators 1,2, and 3 make proposals. Suppose these proposals are $m_1 = (0.4, 0, 0.2, 0.4)$, $m_2 = (0.2, 0.3, 0.4, 0.1)$, $m_3 = (0.3, 0, 0.3, 0.4)$, so that $m = (m_1, m_2, m_3)$. First observe that these proposals are all feasible, i.e., $m \in B^3$, where $B = B(b)$.

In the second stage, the scheduling agent ι selects one of these proposals. If she selects m_1, then in the third stage legislators 1 and ι vote in favor of m_1, while 2 and 3 vote against m_1 in favor of x^{sq}. Since ι only has a tie-breaking vote, the outcome is x^{sq}. If ι selects m_2, $x^A(m) = m_1$, then in the third stage legislators 2,3, and ι vote in favor of m_2 so that the outcome is m_2. Finally, if ι selects m_3, then 2 votes against m_3, ι votes in favor of m_3. Legislators 1 and 3 are indifferent so that the outcome depends on how legislators 1 and 3 resolve their indifference. In particular, if at least one of them votes against m_3, the outcome is x^{sq}, and if both 1 and 3 vote in favor of m_3, then the outcome is m_3.

In the extensive-form game representing the scheduling auction, a strategy s_i (mixed or pure) of each player $i \in N$ is defined in the usual way, by specifying that legislator's history-contingent actions at all points where he is to move. For each legislator other than the scheduling agent, $s_i = (m_i, d_i(.,.))$; and for the scheduling agent, $s_i = (A(.), d_0(.,.))$.

A profile of strategies is a subgame-perfect Nash equilibrium SPNE if, at every history where she is to move, each legislator chooses a plan of action to maximize her utility function, given the strategies of the other players. Thus, in the scheduling auction at stages

[14] In the US Senate, the Vice President has such a tie-breaking vote.

1 and 2, legislators apply backwards induction in order to evaluate the consequences of their actions on the final outcome.[15]

As in any voting model, x^{sq} is always supported as an equilibrium outcome. The reason is that if all legislators vote against the proposed policy $x^A(m)$, then if one of the legislators changed her mind and voted in favor of $x^A(m)$ that would make no difference as the majority would still vote in favor of x^{sq}. On the flip side, whenever there exists a group of legislators comprising a majority who prefer the proposed policy $x^A(m)$, voting "nay" is weakly dominated for any legislator in this group. Hence, to avoid such outcomes where no legislator votes for $x^A(m)$ because nobody else votes for $x^A(m)$, we focus on equilibria in strategies which are weakly undominated at the voting stage. Since voting is only done once, this assumption is equivalent to assuming truthful voting at the voting stage whenever a legislator strictly prefers $x^A(m)$ over the *status quo*. From now on we refer to SPNE in weakly-undominated strategies as *equilibrium*.[16]

Definition 1. *An* equilibrium *of the scheduling-auction game is a SPNE where weakly dominated actions are eliminated at stage 3.*

We denote the equilibrium actions and strategies by a $*$ in the superscript, e.g., $s_0^* = (A^*(.), d_0^*(.,.))$. We also denote a policy outcome resulting from an equilibrium of the scheduling auction by x^{a*}.

The scheduling auction has similar features to Bertrand competition when producers have potentially different costs. Similarly to such settings, all equilibria here in general have the feature that at least one of the voters who weakly prefer $x^A(m)$ over x^{sq} must vote in favor of $x^A(m)$.[17] As an illutration, modify the proposal m_1 in the Example 1 above, so that instead of m_1, legislator 1 makes a proposal $m_1' = m_3$. Suppose that ι selects m_3 and at the voting stage both 1 and 3 break their indifference in favor of m_3. By applying backward induction it can easily be seen that such actions constitute a part of equilibrium strategies. At the voting stage no legislator can make a profitable deviation. Knowing that, at the selection stage the scheduling agent cannot do any better than choosing either m_1' or m_3. Finally, at the proposal stage, no legislator can profitably deviate to making a different proposal. For legislator 1, any other proposal will either not win approval at the voting stage or will yield less surplus to the scheduling agent, and would consequently not be chosen by the scheduling agent; similarly for legislator 3. Legislator 2 also has no profitable deviation. She must either propose at least 0.2 to

[15] It seems plausible that in a legislative setting players are sufficiently sophisticated for SPNE to be an appropriate equilibrium notion.

[16] Dominated actions at the voting stage can be illustrated in the above Example 1. If the chosen proposal is m_2, then voting "nay" is weakly dominated by "aye" for legislators 2 and 3. If, e.g., 2 votes against the proposal then 3 is indifferent between voting "aye" or "nay", and if 2 votes for the proposal then 3 is strictly better off voting "aye" If one did not eliminate weakly-dominated actions at the voting stage, then practically any set of proposals may be justified as equilibrium proposals. In the above example, if the scheduling agent thought that in the third stage all legislators will vote against any move, she could then just as well select proposal m_1, where the final outcome is x^{sq}.

[17] That is evidently the case when the *status quo* policy x^{sq} yields different shares to the legislators, and is in fact always the case, as long as the voting is not by strict consensus (that is $q = 1$).

herself (to vote in favor of her own proposal at the voting stage), in which case, she can propose at most 0.3 to the scheduling agent (since her proposal must lie in $B(b)$), so that the scheduling agent would still choose m_1, or she must propose at least 0.3 to legislators 1 and 3, in which case her proposal must equal m_3 in order to be chosen by the scheduling agent. In this example, an equilibrium policy x^{*a} is thus a move from the status quo, which maximizes the scheduling agent's utility under the constraint that some q-majority of legislators approve the move. In the following Proposition 1 we establish that this is true in general.

Proposition 1. *Let $q \geq \frac{1}{2}$, and let $B = \bar{B}(b)$, $x^{sq} \in B$. Then, in any equilibrium of the scheduling auction, the outcome is given by*

$$x^{a*} \in \arg\max_{x \in B} u_{n+1}(x_{n+1}),$$

$$s.t., \exists \bar{N} \subset \{1, ...n\}, \frac{|\bar{N}|}{n+1} \geq q, \text{ and, } u_j(x_j) = u_j(x_j^{sq}), \forall j \in \bar{N}.$$

Proof. Given a proposal $x \in B$, let $\bar{N}(x) = \{j \in \{1, ..., n\}; u_j(x_j) \geq u_j(x_j^{sq})\}$. At the voting stage, take a proposal $x \in B$, such that $u_{n+1}(x_{n+1}) \geq x_{n+1}^{sq}$, and $\frac{|\bar{N}(x)|}{n+1} \geq q$. Assume first that for any such proposal x, sufficiently many legislators who are indifferent between x^{sq} and x vote in favor of x, so that the outcome of such a vote is x.

Now take a vector of proposals $m \in B^n$, and assume there exists an $i \in \{1, ..., n\}$, such that $\frac{|\bar{N}(m_i)|}{n+1} \geq q$ and $u_{n+1}(m_{i,n+1}) \geq x_{n+1}^{sq}$. Suppose that there exists an $x \in B$, such that $u_{n+1}(x) > u_{n+1}(m_{i,n+1})$, and $\frac{|\bar{N}(x)|}{n+1} \geq q$.

Case 1. If there $\exists i' \in \bar{N}(x) \setminus \bar{N}(m_i)$, then set a proposal by i', $m_{i'} \in B$, such that, $u_j(m_{i',j}) = u_j(x_j^{sq}), \forall j \in \bar{N}(x)$, $m_{i',j} = b_j$, $\forall j \in \{1, ..., n\} \setminus \bar{N}(x)$, and $m_{i',n+1} = 1 - \sum_{j \in \{1,...,n\}} x_j$. Since $x \in B$, it follows that $m_{i'} \in B$, and $u_{n+1}(m_{i'}) \geq u_{n+1}(x)$. Since $u_{i'}(m_{i',i'}) = u_{i'}(x^{sq}) > u_{i'}(m_{i,i'})$. Therefore, i' would have a strict incentive to propose $m_{i'}$, and $n+1$ would also strictly prefer $m_{i'}$ over m_i, so that m_i could not be an outcome of the scheduling auction.

Case 2. If $\bar{N}(x) \subset \bar{N}(m_i)$, then first let $i'' \in \arg\max_{j \in \bar{N}(m_i)} m_{i,j} - x_j^{sq}$.

If $m_{i,i''} - x_{i''}^{sq} > 0$, then let $i' \in \arg\min_{j \in \bar{N}(m_i)} m_{i,j} - x_j^{sq}$; if $\min_{j \in \bar{N}(m_i)} m_{i,j} - x_j^{sq} = \max_{j \in \bar{N}(m_i)} m_{i,j} - x_j^{sq}$, then let i' be any proposer in $\bar{N}(m_i) \setminus \{i''\}$. Next set $\epsilon \in (0, m_{i,i''} - x_{i''}^{sq})$.

If $m_{i,i''} - x_{i''}^{sq} = 0$, then it must be that $\bar{N}(m_i) \setminus \bar{N}(x) \neq \emptyset$; otherwise, since $x, m_{i,n+1} \in B$, it would have to be that $m_{i,n+1} = 1 - \sum_{j \in \bar{N}(m_i)} x_j^{sq} \geq x_{n+1}$, which would contradict $u_{n+1}(x) > u_{n+1}(m_{i,n+1})$. So take a $\hat{i} \in \bar{N}(m_i) \setminus \bar{N}(x)$, set $\epsilon \in (0, x_{\hat{i}}^{sq} - b_{\hat{i}})$, and fix i' to be any proposer $i' \in \bar{N}(x)$.

Now set $m_{i'}$, such that $m_{i',i'} = m_{i,i'} + \epsilon, m_{i',j} = x_j^{sq}, \forall j \in \bar{N}(x)$, $m_{i',j} = b_j, \forall j \in \{1, ..., n\} \setminus \bar{N}(x)$, and $m_{i',n+1} = 1 - \sum_{j \in \{1,...,n\}} m_{i',j}$. As before, it is immediate that $m_{i'} \in B$, $\bar{N}(m_{i'}) = \bar{N}(x)$, i' would have a strict incentive to propose $m_{i'}$, and $n+1$ would strictly prefer $m_{i'}$ over m_i, so that m_i could not be an outcome of the scheduling auction.

Next, suppose that for any proposal $x \in B$, sufficiently many legislators who are indifferent between x and x^{sq} vote in favor of x^{sq}, such that x^{sq} is the outcome of the vote whenever possible. Formally, let $\bar{N}^-(x) = \{i \in \bar{N}(x) \mid u_i(x_i) = u_i(x_i^{sq}), d_i(x, x^{sq}) = \text{“}nay\text{”}\}$ and $\bar{N}^+(x) = \{i \in \bar{N}(x) \mid d_i(x, x^{sq}) = \text{“}aye\text{”}\}$.

Now take an $x \in B$, such that $u_{n+1}(x_{n+1}) > x_{n+1}^{sq}$, $\frac{|\bar{N}(m_i)|}{n+1} \geq q$, and $\frac{|\bar{N}^+(x)|}{n+1} < q$, so that the outcome of a vote between x and x^{sq} is x^{sq}. Then take an x', where relative to x, a small amount of resources is redistributed from $n+1$ to sufficiently many legislators, so that, $\frac{|\bar{N}^+(x')|}{n+1} \geq q$, and $u_{n+1}(x'_{n+1}) > x_{n+1}^{sq}$. Therefore, the outcome of a vote between x' and x^{sq} is x', so that any legislator in $\bar{N}^+(x') \setminus \bar{N}^+(x)$ would have an incentive to propose x' over x, and $n+1$ would also at the second stage choose x' over x, foreseeing the outcome of the vote at the third stage. Hence, no x, such that $u_{n+1}(x_{n+1}) > x_{n+1}^{sq}$ could in that case be the outcome of the vote. But by the same argument as in cases 1 and 2, there would now exist another outcome x'' (by bringing any of the legislators in $\bar{N}^+(x') \setminus \bar{N}^+(x)$ closer to their indifference relative to x'), which would still pass the vote, and be strictly prefered by $n+1$. Hence, in any outcome of the scheduling auction, sufficiently many legislators who are indifferent must vote in favor of the proposal over x^{sq}, which concludes the proof. □

Consider again the equilibrium in Example 1 where legislators 1 and 3 propose $m'_1 = m_3 = (0.3, 0, 0.3, 0.4)$, with the equilibrium outcome $x^{a*} = (0.3, 0, 0.3, 0.4)$. As described in Proposition 1, this policy x^{a*} is a solution to maximizing the scheduling agent's share, under the constraint that a minimal-winning majority is in favor of the proposal (here 1 and 3, along with 4 who only has a tie-breaking vote). But policy x^{a*} is not supported by the *cheapest* winning majority in the sense that, e.g., legislators 1 and 2 could have constituted a cheaper majority supporting the move. However, because of the legislated constraint on legislator 3, such move could yield at most 0.3 to the agenda setter. Nevertheless, x^{a*} is supported by a majority who under x^{sq} obtain minimal shares relative to b. Thus, x^* is supported by a *relatively cheapest majority.* From Proposition 1 we can derive the following Theorem 1, which characterizes equilibria of the scheduling auction as policies which are supported by a relatively cheapest majority and yield maximal share to the scheduling agent. The intuition is that in order for legislated constraints to be satisfied, the scheduling agent's share will be highest when legislators with the smallest difference between their *status quo* shares and their legislated constraints are included in the majority supporting the move.

Theorem 1. *Let $q \geq \frac{1}{2}$, and let $B = \bar{B}(b)$, $x^{sq} \in B$. Then, any outcome of the scheduling auction is given by x^{a*}, such that,*

$$x^{a*} \in \arg \min_{x \in B,\ s.t.,\ \frac{\bar{N}(x)}{n+1} \geq q} \sum_{i \in \bar{N}(x)} (x_i^{sq} - b_i). \tag{2}$$

Moreover, $\frac{|\bar{N}(x^{a*})|}{n+1} \geq q$, $x_i^{a*} = x_i^{sq}$, $\forall i \in \bar{N}(x^{a*})$, $x_i^{a*} = b_i, \forall i \in \{1, ..., n\} \setminus \bar{N}(x^{a*})$, and $x_{n+1}^{a*} = 1 - \sum_{i \in \{1,...,n\}} x_i^{a*}$.

Proof. By Proposition 1, any outcome of the agenda auction must yield the highest possible payoff to the scheduling agent, subject to the constraint that a q-majority of legislators who are indifferent between the proposal and x_{sq} vote in favor of the proposal. Take an outcome of the scheduling auction x, and take the q-majority of legislators voting in favor of x, $\bar{N}(x)$, where $\frac{|\bar{N}(x)|}{n+1} \geq q$. Suppose that $\bar{N}(x)$ included a legislator i, such that $x_i^{sq} - b_i > x_j^{sq} - b_j$, for some legislator $j \in \{1, ..., n\} \setminus \bar{N}(x)$. By Proposition 1, $x_i = x_i^{sq}$ and $x_j = b_j$. Then consider a proposal x', such that $x'_i = b_i$, $x'_j = x_j^{sq}$, $x'_k = x_k$, $\forall k \in \{1, ..., n\} \setminus \{i, j\}$, and $x'_{n+1} = 1 - \sum_{i \in \{1,...,n\}} x'_i$. Hence, $|\bar{N}(x')| = |\bar{N}(x)|$, but $x'_{n+1} = x_{n+1} + (x_i^{sq} - b_i) - (x_j^{sq} - b_j) > x_{n+1}$, which is a contradiction. □

Theorem 1 shows that in order for a policy x^{a*} to be an outcome of the scheduling auction, x^{a*} must be weakly preferred by *a relatively cheapest* q-majority of legislators. Every other legislator j is expropriated up to their legislated constraint b_j. The legislators who are expropriated are those where the *relative gain* from expropriation is highest. This is quite different from the existing models in the literature – in those models, in particular Baron and Ferejohn (1989) the majority voting in favor of an outcome will be a *cheapest* majority, while the prediction here is that the majority approving an outcome will be a *relatively cheapest* majority. This distinction can be qualitatively and quantitatively important.

To illustrate the difference between the absolute and the relative majority, consider the legislative environment depicted in Figure 1 below.

In this example, the voting rule is simple majority, $q = \frac{1}{2}$, relative costs are on the horizontal axis, and absolute costs are on the vertical axis. The scheduling agent is denoted by an asterisk, and there are 10 other legislators denoted by black dots, a total of 11 legislators. Note that in this example, neither the legislated constraints nor the *status quo* are on the Pareto-frontier. The legislators to the left of the vertical dashed line are those that are included in the relatively cheapest majority, while the legislators below the horizontal dashed line are those that are included in the cheapest majority. The scheduling agent is included in any winning majority, and in this example her vote breaks the tie. Clearly, the cheapest majority and the relatively cheapest majority are quite different. The share allocated to the scheduling agent is much larger under the relatively cheapest majority, as the legislators must be compensated by their relative and not absolute costs. Taking into account the legislated constraints can therefore make a difference, both qualitatively as well as quantitatively.

We have the following immediate corollary to Theorem 1.

Corollary 2. *Any equilibrium outcome of the scheduling auction is on the Pareto-efficient frontier.*

From Theorem 1, it is evident that in any outcome of the scheduling auction, the scheduling agent is favorable to the equilibrium outcome over the status quo; it is also immediate that the agenda setter is indifferent between the two only when the two are equal.

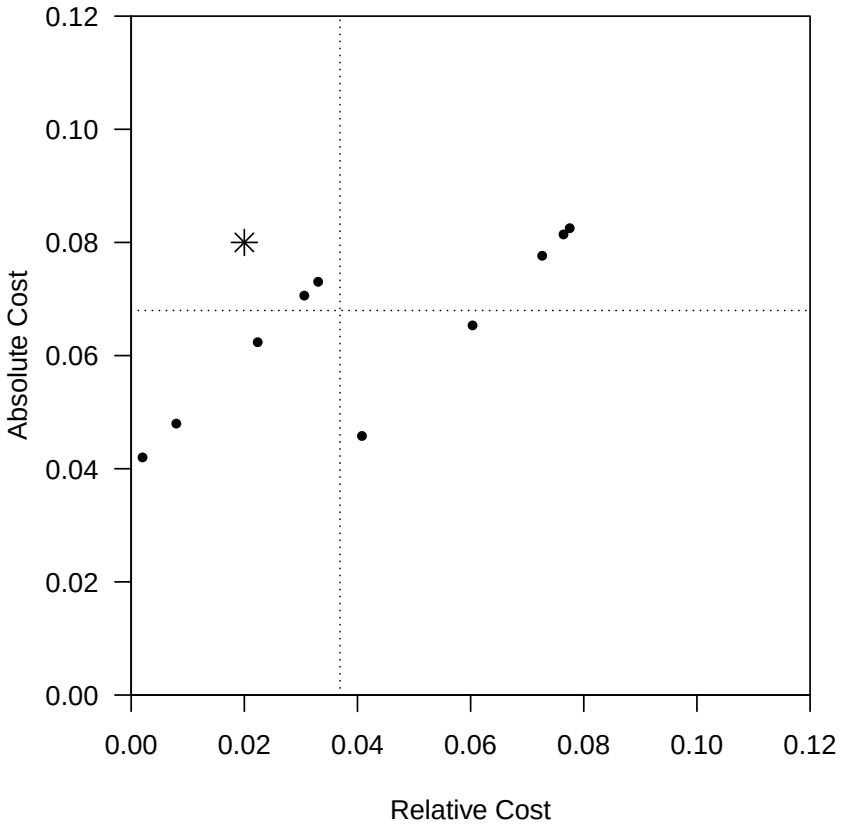

Figure 1: Relatively-cheapest majority.

Corollary 3. *Under the conditions of Theorem 1, the scheduling agent prefers* x^{a*} *to* x^{sq}*, i.e.,* $u_\iota(x_\iota^{a*}) \geq u_\iota(x_\iota^{sq})$*. The scheduling agent is indifferent, i.e.,* $u_\iota(x_\iota^{a*}) = u_\iota(x_\iota^{sq})$*, if and only if,* $x^{sq} = x^{a*}$*.*

From Theorem 1 we can derive several other positive implication. In our previous analysis, we set the scheduling agent as $n+1$, i.e., $\iota = n+1$ to simplify our notation; recall that in general $\iota \in N$, and denote by $x^{\iota*}$ an equilibrium outcome of the scheduling auction with ι as the scheduling agent.

In Proposition 2, we show that the outcome of the agenda auction equals the *status quo*, if and only if, the *status quo* is Pareto efficient, and the *status quo* shares coincide with legislated constraints for all legislators (except perhaps the scheduling agent).

Proposition 2. *Additionally to the conditions of Theorem 1, let* $q \leq \frac{n-1}{n}$*, i.e., approving the move does not require consensus. Then* $x^{\iota*} = x^{sq}$*, if and only if,* x^{sq} *is Pareto efficient and* $b_i = x_i^{sq}, \forall i \in N \setminus \{\iota\}$*.*

Proof. If x^{sq} is on the Pareto-efficient frontier and $b_i = x_i^{sq}, \forall i \in N \setminus \{\iota\}$, then by Theorem 1, $x_i^{\iota *} = x_i^{sq}$, $\forall i \in N \setminus \{\iota\}$, and $x_\iota^{\iota *} = 1 - \sum_{i \in N \setminus \{\iota\}} = x_\iota^{sq}$, so that $x^{\iota *} = x^{sq}$. If either x^{sq} is not on the Pareto-efficient frontier, or there is an $i \in N \setminus \{\iota\}$, such that $x_i^{sq} > b_i$, then since $x^{sq} \in B(b)$, either $x^{sq} = b$ and $\sum_{i \in N} x_i^{sq} < 1$, or $x^{sq} > b$. By Corollary 2, $x^{\iota *}$ is on the Pareto-efficient frontier, so that in the former case, $x^{\iota *} \neq x^{sq}$. In the latter case, we have $\max_{j \in N \setminus \{\iota\}} x_j^{sq} - b_j > 0$. Since by Theorem 1, the winning $q-$majority is comprised of the legislators with minimal difference $x_j^{sq} - b_j$. Since $q \leq \frac{n-1}{n}$ the legislator i^* for whom this maximum is achieved is not in the minimal winning $q-$majority, so that $x_{i*}^{sq} > b_{i*}$, and $x_{i*}^{\iota *} = b_{i*}$. Therefore, $x^{\iota *} \neq x^{sq}$. □

Combining Proposition 2 and Corollary 3 the scheduling agent is therefore indifferent between $x^{\iota *}$ and x^{sq} only in the special case when $x_i^{sq} = b_i, \forall i \in \{1, ..., n\}$, so that under the *status quo*, the agenda setter has already extracted all possible surplus.

When $x_i^{sq} = b_i$ for fewer legislators than a q-majority, then the majority of legislators supporting the proposal is minimal, in the sense that $\frac{|\bar{N}(x^{a*})|}{n+1} \geq q > \frac{|\bar{N}(x^{a*})|-1}{n+1}$. When $x_i^{sq} = b_i$ for more than a minimal majority of legislators, it can happen that a *supra majority* of legislators vote in favor of the new proposed distribution of resources over the *status quo*.

Proposition 3. *Additionally to the assumptions of Theorem 1, assume that $x_i^{sq} = b_i$ for more than a $q-$majority of legislators $i \in N \setminus \{\iota\}$. Moreover, assume that all legislators who are indifferent vote for the proposed move. Then, a* supra majority *of legislators vote in favor of the proposed move.*

Proof. By Theorem 1 a relatively-cheapest minimal majority of legislators obtain their *status quo* shares, which is a subset of legislators with $x_i^{sq} = b_i$. All other legislators obtain a share equal to their legislated constraints, i.e., all legislators for whom $x_i^{sq} = b_i$ are indifferent between the *status quo* and the outcome of the scheduling auction, and this set comprises a *supra* majority. □

For an illustration of Proposition 3, consider Figure 2 below.

In Figure 2, there are a number of legislators with 0 relative cost, and all those legislators are included in the winning coaltion, along with the scheduling agent (again denoted by an asterisk). Since these legislators comprise a supra-majority, the vote in favor of the outcome is therefore approved by a supra-majority. If instead of the relative cost the relevant determinant were the absolute cost, then the majority would here be a minimal majority. The example given in Figure 2 gives a plausible configuration under which the majority approving the outcome need not be minimal but can be a supra-majority. A special case of such a supra-majority approved outcome is when legislated constraints are all 0, and under the *status quo* 0 resources are allocated to a supra-majority of legislators. Such a scenario seems much less plausible than one akin to that in Figure 2, only in such special case would the absolute costs of the winning majority also be 0.

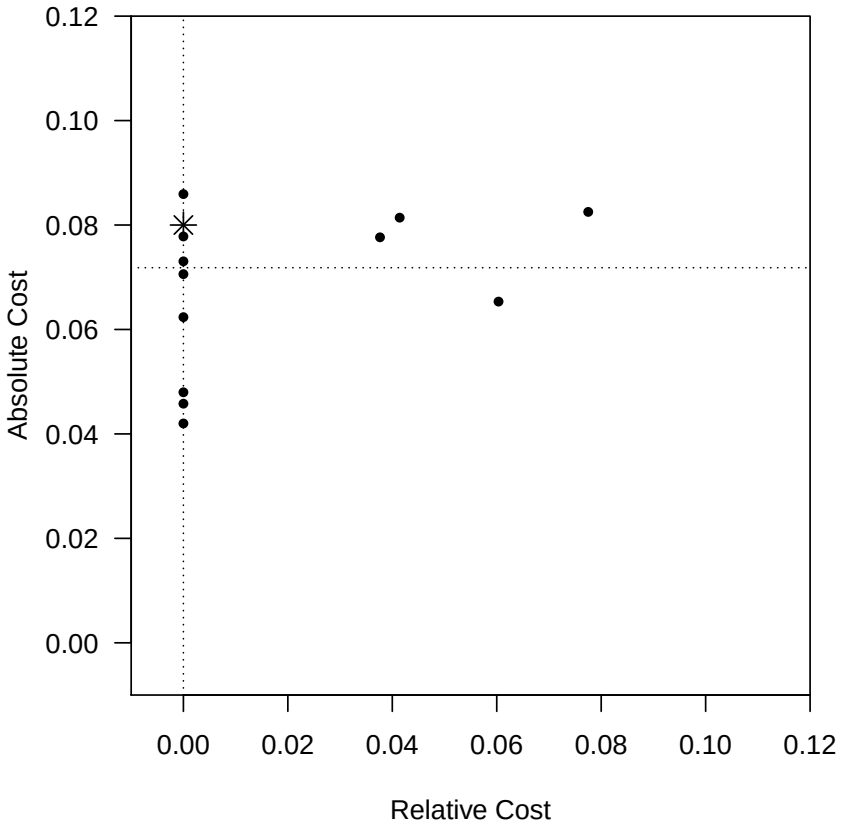

Figure 2: Supra majority.

The pervasive idea in the literature is that the winning majority will be minimal. This idea dates back to Riker (1962) and probably further. In particular, approval by minimal winning majorities obtains in models buliding on the Baron and Ferejohn (1989) model of legislative bargaining.[18] In practice supra-majorities do occur, and while one may attribute such outcomes to the legislators' concern regarding their voting records, Proposition 3 gives conditions under which supra-majorities can occur in the setting of purely redistributive politics with no reputational considerations.

In the scheduling auction the scheduling agent has a positive agenda power in the sense that she chooses which proposal will ultimately be voted against the *status quo*. Through the competition between the proposers, this positive agenda power then ultimately allows

[18]In the case of purely redistributive politics considered here, if x^{sq} is on the Pareto-efficient frontier, then the game between the legislators is zero-sum. In examples of non-redistributive politics, where legislators may have non-consumption related preferences (i.e., spatial preferences over various policy dimensions), the game need not be a zero-sum , but even in those settings, existing models mostly predict exact majorities, e.g., Jackson and Moselle (2002).

the scheduling agent to appropriate substantial rents. The predictions of equilibrium outcomes of the scheduling auction are different in several ways from the prediction of the Baron and Ferejohn (1989) model, and the ensuing literature on legislative bargaining. First, the majority voting in favor of an outcome will be a *cheapest* majority, while the prediction here is that the majority voting in favor will be a *relatively cheapest* majority. Second, in those models the majority approving the outcome is minimal. In contrast, in reality supra-majorities are often observed, and in the present model such supra-majorities are possible, depending on the *status quo* shares and the legislated constraints.

4. SELECTION OF THE SCHEDULING AGENT AND THE PROTO-PARTY

In this section we consider endogenous choice of the agenda setter. To that effect, a loose sense of common interest among certain legislators leads to a natural *proto-party*. We assume that voting is done by simple majority, $q = \frac{1}{2}$, and to not have to consider a variety of cases, we assume that $n+1$ is odd. When it comes to selecting the scheduling agent, each legislator will of course strictly prefer herself to be in that role. That follows directly from Proposition 1: everything else equal, a legislator will obtain the highest share when she is the scheduling agent. Consequently, there cannot be any selection procedure by which the legislators could unanimously agree on whom should be selected as the scheduling agent; there will also be no legislator who can unambiguously win against every other legislator to serve as the scheduling agent in pair-wise majority contests, i.e., there will be no Condorcet winner. Therefore, any specific procedure by which the scheduling agent is chosen will necessarily have some indeterminacy.

Instead of specifying the precise procedure by which the scheduling agent is chosen, we define a set of legislators who might possibly be selected into that role. By appropriately defining this set of legislators, we will show that a majority of legislators will have aligned interests for the scheduling agent to be chosen from within that set, in the sense of expected budgetary gains for their respective constitutencies.[19] Specific procedures will then select the scheduling agent from that set of legislators. In particular, any procedure satisfying what we define as *majoritarian approval* will select the scheduling agent from this set. We call this set of legislators *the proto-party*.

Thus, in the scheduling auction, a loose party structure arises naturally even in the environment of purely redistributive politics, *in the absence of any demagogical preferences*.[20] An added benefit of this approach is that the subsequent results are independent

[19]This notion of a proto-party is in line with the existing ideas in the literature on political parties – the proto-party can be viewed as a loose exogenous structure, which brings benefits to its member by restricting their behavior in a specific way. This is consistent with Krehbiel (1993), and Jackson and Moselle (2002) who distinguish between party-like behavior, and behavior that is effected by the existence of a party.

[20]In the model of Jackson and Moselle (2002), for example, absent a party-structure, the voting outcome can be inefficient and the party effects a Pareto improvement. In the real world, there may be other less directly utilitarian rationales for political parties, for example, brand-recognition and

of the precise identity of the scheduling agent, as long as the scheduling agent belongs to the proto-party. We define the proto-party next, and show that it in most cases includes a majority of legislators; there are also some plausible scenarios under which the proto-party might be just shy of the majority. We then define the notion of *majoritarian approval* and show that choosing the scheduling agent from the proto party satisfies majoritarian approval.

As before, $\iota \in \{1, ...n+1\}$ denote the scheduling agent. Denote by N^ι the set of all other legislators who are included in some minimal winning majority of an equilibrium outcome of the agenda auction when ι is the scheduling agent. That is, $j \in N^\iota$, if $j \neq \iota$, and there exists an $x^{\iota *}$, which is an equilibrium outcome of the scheduling auction when ι is the agenda setter, and j belongs to a minimal-winning majority who voted in favor of $x^{\iota *}$. Denote by N^a the largest set of all legislators who can belong to a minimal-winning majority regardless of what other legislator from N^a is the scheduling agent. Formally,

$$N^a = \max\{I \subset N \mid I \subset \cup_{j \in I} \left(\cap_{i \in I \setminus \{j\}} N^i\right)\}.$$

We call N^a *the proto-party*. Consider the following examples of the proto-party, N^a.

Example 2.a. Suppose that all legislators are *a priori* equal, that is, $b_i = b_j$ and $x_i^{sq} = x_j^{sq}, \forall i, j \in \{1, ..., n+1\}$. In that case it is immediate that $N^a = \{1, ..., n+1\}$.
Example 2.b. Now suppose that there is some set of legislators I, such that $|I| \geq \frac{n}{2} + 1$, and $x_i^{sq} - b_i < x_j^{sq} - b_j$, $\forall i \in I, \forall j \notin I$. Moreover, suppose that I is such that $x_i^{sq} - b_i = x_{i'}^{sq} - b_{i'}$, $\forall i, i' \in I$. In other words, the difference between the *status quo* shares and the legislated constraints is the same for all legislators in I, and this difference is smaller than for any legislator outside I. Since there are more than a majority of legislators in I, by Theorem 1, any winning majority in any equilibrium outcome of the scheduling auction will be comprised of legislators from I. Moreover, since all legislators in I are equally "expensive", any legislator in I can belong to a minimal-winning majority, no matter what other legislator is the scheduling agent. Hence in this case, $N^a = I$.
Example 2.c. Finally, suppose that there are sets of legislators I and I', such that $I \cap I' = \emptyset$, $|I| = \frac{n}{2}$, $|I \cup I'| > \frac{n}{2} + 1$, $x_i^{sq} - b_i < x_{i'}^{sq} - b_{i'} < x_j^{sq} - b_j$, $\forall i \in I, i' \in I', \forall j \notin I \cup I'$, and $x_i^{sq} - b_i = x_{i'}^{sq} - b_{i'}$, $\forall i, i' \in I'$. Thus, the legislators in I are relatively cheapest, but they do not comprise a majority – they are just one legislator short of the majority. Along with somewhat more expensive legislators in I', these two groups together comprise a majority of legislators. Moreover, the difference between the *status quo* and the legislated constraints is the same for all legislators in I'. Again, by Theorem 1, the legislators in I will evidently belong to any minimal-winning majority supporting an equilibrium outcome of the scheduling auction, regardless of what other legislator is the scheduling agent. However, for each legislator in I', when she is not the scheduling agent, she can only belong to the minimal winning majority if the scheduling agent is chosen from I – otherwise, the scheduling agent along with the legislators in I will make up a majority. Therefore, if $\iota \in N \setminus I$, then $I' \cap N^\iota = \emptyset$. Consequently, legislators in I' will be in N^a only when $|I'| = 1$, so that then $N^a = I \cup I'$; if $|I'| > 1$, then whenever $\iota \in I'$, no

reputation, see Kiewiet and McCubbins (1991).

other legislator in I' belongs to a minimal-winning majority, so that in that case $N^a = I$. Hence, in this example N^a may be just short of a majority, whenever the set of *median relative cost legislators* is comprised of more than one legislator.

Example 2.c. describes the case where the proto-party N^a is just shy of a majority of legislators.[21] In the next Proposition 4 we show that this is essentially the only case where the proto-party N^a is comprised of less than a majority of legislators. For $i \in N$, let $I^{i+} = \{j \in N \mid x_j^{sq} - b_j \geq x_i^{sq} - b_i\}$ and $I^{i-} = \{j \in N \mid x_j^{sq} - b_j \leq x_i^{sq} - b_i\}$, i.e., the legislators in I^{i+} are those for whom the difference between the *status quo* and the legislated constraints is at least as large as for i, and for legislators in I^{i-} this difference is weakly smaller. Now define the set of legislators with the median relative difference,

$$N^m = \{i \in N \mid |I^{i-}| \geq \frac{n}{2} + 1, |I^{i+}| \geq \frac{n}{2} + 1\},$$

and note that $x_i^{sq} - b_i = x_j^{sq} - b_j$, $\forall i, j \in N^m$, i.e., the relative difference must be the same for all median legislators. Define the set of legislators for whom the difference between *status quo* and legislated constraints is strictly less than for the legislators in N^m,

$$N^l = \{i \in N \mid x_i^{sq} - b_i < x_j^{sq} - b_j, j \in N^m\}.$$

Note that the set N^l might be empty, e.g., when the relative difference is the same for all legislators in which case $N^m = N$.

In the next Proposition 4 we show that N^a is comprised of a majority of legislators, given by $N^m \cup N^l$, except for the case described in Example 2.c. Thus, in most cases the proto-party is comprised of a majority of legislators with the lowest relative gain from obtaining their *status quo* shares relative to their legislated constraints. When some legislators' relative gains coincide, the proto-party might be comprised of more than a majority of legislators, or in a rather special case of less than a majority of legislators.

Proposition 4. *Let the scheduling auction be given by $B = B(b)$ and $x^{sq} \in B(b)$. If $|N^l| < \frac{n}{2}$, or $|N^m| = 1$, then the proto-party is comprised of a majority of legislators with the lowest differences between* status quo *and legislated constraints, given by $N^a = N^l \cup N^m$. Otherwise, if $|N^l| = \frac{n}{2}$ and $|N^m| > 1$, then the proto-party is just shy of the majority, and is given by $N^a = N^l$.*

Proof. We apply Theorem 1, i.e., that any equilibrium outcome x^{a*} of the scheduling auction is supported by some relatively cheapest majority of legislators. Next, that $|N^l \cup N^m| \geq \frac{n}{2} + 1$. Furthermore, for any equilibrium outcome x^{a*}, every legislator from N^l (appart from the agenda setter) is always included in the relatively cheapest majority of legislators. Hence, $N^l \subset N^a$. Finally, no legislator from $N \setminus (N^l \cup N^m)$ will ever belong to a relatively cheapest majority when she is not the scheduling agent, so that $N^a \subset N^l \cup N^m$. We have therefore established that,

$$N^l \subset N^a \subset N^l \cup N^m. \tag{3}$$

[21] Nevertheless, the legislators in I' would even then have a common interest with the legislators from I to select the scheduling agent from I, rather than from outside I.

Finally, suppose that $\iota \in N^l$. Since $|N^l| \leq \frac{n}{2}$, and all legislators in N^m have the same relative difference, every legislator in N^m then belongs to some relatively cheapest majority supporting any equilibrium outcome. Hence, the issue is, when $\iota \in N^m$, whether every other legislator in N^m belongs to some relatively cheapest majority supporting an equilibrium outcome.

Consider now the case where $|N^l| < \frac{n}{2}$. Then, it must be that $|N^m| \geq 2$, so that if $\iota \in N^m$, then at least one of the legislators in N^m will have to be included in the relatively cheapest majority. Since all legislators in N^m have the same relative difference, every legislator in N^m belongs to some relatively cheapest majority supporting any equilibrium outcome. Therefore, in this case $N^m \subset N^a$, and by (3), $N^a = N^l \cup N^m$.
Next, suppose that $|N^{|l|}| = \frac{n}{2}$ and $|N^m| = 1$. Now it is trivially true that $N^m \subset N^a$.
Finally, the case where $|N^l| = \frac{n}{2}$ and $|N^m| > 1$ has been proven in Example 2.c, where $N^a = N^l$. □

In the rest of this section we give an argument for the proto-party based on the assumption that the scheduling agent has to be approved by *some* majority of legislators. We call this *majoritarian approval*.

In what sense are the incentives of the legislators in the proto-party aligned? The proto-party is the smallest loose alliance which can assure that a legislator from within the proto-party will be assigned into the role of the scheduling agent, and at the same time guarantee to all of its members some prospective gain from participating in the winning majority; when the proto-party consists of more than just a simple majority of legislators, then some of the legislators will be in the winning majority with a positive probability and their prospective gains will be their expecter gains from being selecter the winning majority. Nevertheless, if both, the assignment of the agenda setter from within the proto-party, and the membership in the winning majority are a result of a random draw, then all members of the proto-party have a strict incentive to commit to choosing the agenda setter from within the proto-party. We now construct such an incentive argument for the proto-party.

Given $b \in B(0)$ and $x^{sq} \in B(b)$, let $X^{\iota *}$ be the set of all possible equilibrium outcomes of the scheduling auction when $\iota \in N$ is the scheduling agent, i.e., $X^{\iota *} = \{x^{\iota *} \in B(b) \mid x^{\iota *} \text{ eq. outcome under } \iota\}$. For each $x^{\iota *} \in X^{\iota *}$, denote by $\bar{N}(x^{\iota *})$ the majority who vote in favor of $x^{\iota *}$ over x^{sq} as described in Proposition 1.[22] Finally, denote by $\lambda_{i|\iota}$ the number of outcomes in $X^{\iota *}$, such that i is the member of the winning majority. Now assume that if ι is the scheduling agent, all equilibrium outcomes in $X^{\iota *}$ are equally likely, and let $v_{i|\iota}$ be the expected gain above b_i that a legislator $i \neq \iota$ obtains if $\iota \neq i$ is the scheduling

[22] If there are more than a majority of legislators who are indifferent between a given outcome and the *status quo*, then one can assume that all these legislators are in the winning majority. But this won't matter either way because the only time that can happen is when the *status quo* and the legislated constraints coincide for all legislators in the winning majority including such additional legislators.

agent,

$$v_{i|\iota} = \frac{1}{|X^{\iota *}|}\lambda_{i|\iota}\left(x_i^{sq} - b_i\right).$$

Finally, for $I \subset N$, $|I| \geq \frac{n}{2} + 1$, let $v_{i|I}$ denote the expected gain above b_i, to legislator i, if the scheduling agent is randomly chosen from the set I, and $\iota \neq i$,

$$v_{i|I} = \frac{1}{|I \setminus \{i\}|}\sum_{\iota \in I \setminus \{i\}} v_{i|\iota}.$$

As long as the scheduling agent must be approved by some majority of legislators, then $v_{i|I}$ is positive if and only if i belongs to the proto-party. This is shown in the next proposition.[23]

Proposition 5. *Assume that the selection of the scheduling agent satisfies majoritarian approval and that* $x_i^{sq} - b_i > 0, \forall i \in N$. *Then,*

$$\min_{I \subset N; |I| \geq \frac{n}{2}+1} v_{i|I} > 0 \iff i \in N^a.$$

Proof. Take an $i \in N^a$. By Proposition 4, the proto-party is either composed of a majority in which case, $N^a = N^l \cup N^m$, or when $|N^l| = \frac{n}{2}$ and $|N^m| > 1$, $N^a = N^l$. By Theorem 1, legislators in N^l are included in any winning majority, regardless of what legislator is the scheduling agent. Hence, if $i \in N^l$, then $\min_{I \subset N; |I| \geq \frac{n}{2}+1} v_{i|I} > 0 \iff i \in N^a$. We now treat two separate cases.

Case 1. $N^a = N^l$, so that $|N^a| = \frac{n}{2}$. If $i \in N \setminus N^a$, then precisely all legislators in N^a are included in the winning majority, so that $\min_{I \subset N; |I| \geq \frac{n}{2}+1} v_{i|I} = 0, \forall i \in N \setminus N^a$.

Case 2. $N^a = N^l \cup N^m$, so that $|N^a| \geq \frac{n}{2} + 1$. If $i \in N \setminus N^a$, then i is never included in the winning majority when she is not the agenda setter, so that $\min_{I \subset N; |I| \geq \frac{n}{2}+1} v_{i|I} = 0, \forall i \in N \setminus N^a$. Now take $i \in N^m$. Suppose $\iota \notin N^l$, and take $X^{*\iota}$. As in the proof of Proposition 4, $|N^l| < \frac{n}{2}$, so that for any $x^{*\iota} \in X^{*\iota}$, $\bar{N}(x^{*\iota}) \cap N^m \neq \emptyset$, by Theorem 1. Since $x_i^{sq} - b_i = x_{i'}^{sq} - b_{i'}, \forall i, i' \in N^m$, $\lambda_{i|\iota} > 0$, $\forall i \in N^m$, which implies that $v_{i|\iota} > 0, \forall i \in N^m$, and consequently $v_{i|I} > 0, \forall i \in N^m$. □

Proposition 5 shows that the proto-party rewards its members for supporting the selection of the scheduling agent through (expected) benefits in the subsequent equilibrium allocation of the shares of the budget. A different interpretation of the proto-party is that if the scheduling agent is approved by some majority of legislators, then even in the one-shot environment, the legislators in the proto-party derive benefits from the proto-party in two ways. First, by restricting the choice of the scheduling agent to the proto-party, which increases the likelihood of each legislator in the proto-party to be selected as the scheduling agent, everything else equal. Second, by assuring that the scheduling agent is selected from the proto-party, each legislator in the proto-party gets a positive expected

[23] The assumption that $x_i^{sq} - b_i > 0, \forall i \in N$ is needed in Proposition 5 because otherwise some of the relatively cheapest legislators would get zero relative gain simply by virtue of their *status quo* shares and their legislated constraints being equal.

benefit in the scheduling auction where she might otherwise get 0. More specifically, a legislator in N^l derives benefits from increased likelihood of being the scheduling agent, and no additional benefit when she is not the scheduling agent, while a legislator from N^m derives both kinds of benefits from the proto-party. By Theorem 1, for each legislator, the benefit of restricting the choice of the scheduling agent depends only on the size of the group to which this choice has been restricted. In the next proposition we describe the first kind of benefits, i.e., expected benefits to the members of the proto-party from the outcome of the scheduling auction.

Proposition 6. *Suppose that $N^a = N^l \cup N^m$. If $i \in N^l$, then $v_{i|I} = v_{i|I'}, \forall I, I' \subset N, |I| = |I'| = \frac{n}{2} + 1$. Otherwise, if $i \in N^m$ then for $|I| = \frac{n}{2} + 1$, $v_{i|I}$ is maximized when $I \subset N^a$.*

Proof. For the first part, by Theorem 1, the legislators in N^l belong to every winning majority. For the second part, if the scheduling agent is not chosen from N^a, then one of the legislators from N^m will be excluded from the winning majority. Therefore, when she is not the scheduling agent, each legislator in N^m will derive a higher benefit when the agenda setter is chosen from N^a. □

When $N^a = N^l$, then as long as $\iota \in N^a$, the legislators in N^m will still derive benefits as their expected shares of the budget, so it is conceivable that they might still be willing to support such a choice. One interpretation as to why the proto-party in that case includes only legislators in N^l is precisely that if the scheduling agent were to belong to N^m, no other legislator in N^m would derive any benefits from the outcome of the agenda auction.

The proto-party is thus the smallest group (up to legislators who are identical in their relative costs) that can assure majoritarian approval of the scheduling agent from its ranks through benefits to legislators who provided support for the scheduling agent. In the knife-edge case, when the legislature is split, $N^a = N^l$, and the proto-party is composed of a minority of legislators, some legislators who are not in the proto-party must support the selection of the scheduling agent from the proto-party because it is in their interest. Anecdotal evidence suggests that such situations do in fact perspire in reality.[24] Our analysis here has shown that if the choice of the scheduling agent satisfies majoritarian approval, then the proto-party arises naturally, and the scheduling agent is chosen from the proto-party.

We conclude the section with the following Theorem 4, describing conditions under which the legislature is in a deadlock. If the scheduling agent can be any of the legislators from the proto-party, then all equilibrium outcomes of the scheduling auction coincide with the *status quo* (i.e., for every possible scheduling agent), if and only if, both x^{sq}

[24] As mentioned in the introduction, a situation closely corresponding to the case where the proto-party is just shy of a majority occured in the 1994 the elections to the California House of Representatives – after a narrow win for the GOP, in the process of electing the Speaker of the House, who has large schedulingpowers, one of the Republican legislators ultimately declared himself an independent, and voted in favor of the choice promoted by the Democrats.

and b are on the Pareto-efficient frontier. Hence, that is the case only when the legislated constraints are so severe that $B(b)$ consists of a single possible policy, which must therefore also coincide with x^{sq}. Thus, when the scheduling agent is selected from the proto-party, the only possibility for the outcome of the scheduling auction to invariably be in a deadlock is when the legislated constraints are so restricitive that no policy move at all is possible.

Theorem 4. *Let $b \in B(0)$ and let $x^{sq} \in B(b)$, and suppose $|N^a| \geq 2$. Then $x^{\iota *} = x^{sq}$, $\forall \iota \in N^a$, if and only if, x^{sq} and b are both Pareto efficient.*

Proof. Fix an $\iota \in N^a$. By Proposition 2, all equilibrium outcomes $x^{\iota *}$ coincide with x^{sq} if and only if x^{sq} is Pareto efficient, and $x_i^{sq} = b_i, \forall i \in N \setminus \{\iota\}$. Since $|N^a| \geq 2$, it follows that the claim is true, if and only if x^{sq} is Pareto efficient and $x_i^{sq} = b_i, \forall i \in N$, i.e., b and x^{sq} are both Pareto efficient and coincide. □

5. CONTENT-NEUTRAL SCHEDULING AND WELFARE COMPARISON

A different legislative institution from the scheduling auction is what we call the *content-neutral scheduling*. Content-neutral scheduling is a simpler institution where the decision of what proposal makes it to the floor is independent of proposals' content. Most voting and political-economy literature abstracts from the details of the agenda-setting process by assuming such a content-neutral scheduling procedure. For example, Baron and Ferejohn (1989) and the large ensuing literature on legislative bargaining assume that rationing is probabilistic and that the probability of a proposer being selected is independent of the substance of the proposal. Thus, under the content-neutral scheduling, legislators have no incentive to craft their proposals in any particular, except to their own benefit, and subject to majority approval. In contrast, in the scheduling auction legislators compete in the amount of surplus they allocate to the scheduling agent. In this section we compare the outcomes of the agenda auction to the content-neutral benchmark, where the scheduling agent has no power.

In the content-neutral scheduling game, stages 1 and 3 are the same as in the agenda-auction game; Stage 2 is eliminated. Instead, there is a random move by Nature prior to stage 1, whereby Nature selects a proposer from the set of all legislators according to some probability distribution Pr, where $Pr(i)$ is the probability that legislator i is the proposer. For example, when all legislators are picked with equal probabilities, $Pr(i) = \frac{1}{n+1}$. The key is that these probabilities are indpendent of the legislators' proposals so that in this content-neutral scheduling game, proposers have no incentive to make proposals favorable to the scheduling agent.

As in the agenda-auction, the *equilibrium* in the content-neutral scheduling game is a SPNE, after the elimination of weakly-dominated actions at the voting stage. In the content-neutral scheduling the outcome is generally stochastic since the selection from among the proposals is stochastic. Conditional on the chosen proposer, that is, in each

subgame after Nature has chosen a proposer the outcome is deterministic in any pure-strategy SPNE. For a moment denote by $\tilde{x}^{i*}$ an equilibrium outcome of content-neutral scheduling in the subgame where legislator $i \in N$ has been chosen as the proposer. In each such subgame, the proposer i makes a proposal to allocate the largest possible share of available resources to herself, while satisfying the constraint of majority approval. Thus, this outcome coincides with an equilibrium outcome in the scheduling auction where $\iota = i$, i.e., when i is the agenda setter. We formally state this in the next proposition.

Proposition 7. *An allocation $x^{i*} \in B$ is an outcome of the subgame of the content-neutral scheduling, where i is selected as a proposer, if and only if, x^{i*} is an outcome of the scheduling auction with i as the scheduling agent.*

Proof. Suppose i is selected by Nature to be the proposer. Denote her equilibrium proposal by x^{i*}. When choosing her proposal, i then solves the following optimization program,

$$x^{i*} \in \arg\max_{x \in B} u_i(x_i),$$

$$\text{s.t., } \exists N \subset \{1, ..., n+1\}, \frac{|\bar{N}|}{n+1} > q, \text{ and, } u_j(x_j) = u_j(x_j^{sq}), \forall j \in N.$$

The proof follows by Proposition 1. □

By above Proposition 7, $\tilde{x}^{i*} = x^{i*}$, so that we will occassionally slightly abuse the notation and write simply x^{i*} for the outcome of the content-neutral scheduling when i is selected by Nature. From Proposition 7 and Theorem 1 we can immediately characterize the outcomes of content-neutral scheduling in a subgame where i is chosen as the proposer.

Theorem 5. *Let $q \geq \frac{1}{2}$, and let $B = \bar{B}(b)$, $x^{sq} \in B$. Then, under content-neutral scheduling, any outcome in the subgame where i is chosen is given by x^{i*}, such that,*

$$x^{i*} \in \arg \min_{x \in B, \text{ s.t., } \frac{\bar{N}^i(x)}{n+1} > q} \sum_{j \in \bar{N}^i(x) \setminus \{i\}} (x_j^{sq} - b_j). \tag{4}$$

Then $x_j^{i*} = x_j^{sq}$, $\forall j \in \bar{N}^i(x^{i*}) \setminus \{i\}$, $x_j^{i*} = b_j, \forall j \in \{1, ..., n, n+1\} \setminus \bar{N}^i(x^{i*})$, $x_i^{i*} = 1 - \sum_{j \in \{1,...,n+1\} \setminus \{i\}} x_j^{i*}$, and $\frac{|\bar{N}^i(x^{i*})|}{n+1} > q$.

As an example of content-neutral scheduling consider the legislative bargaining game of Baron and Ferejohn (1989). In that game, there are potentially infinitely many periods. In a given period, a proposer is randomly selected to make a proposal $x \in B$ to the legislative body. If the proposal is accepted the game ends with x and the corresponding payoffs being the outcome, and if the proposal is rejected, the game proceeds to the next period, where a proposer is again randomly selected, and so on, until the game either ends, or it continues forever. The cost of delay between two consecutive periods is represented by a time-discount factor of $\delta < 1$, which is the same for all legislators, and the legislators' payoffs are given by the present discounted share of the pie that is ultimately obtained. Proposition 7 and Theorem 5 allow for a very simple exposition

of the Baron and Ferejohn model.[25] We first interpret the legislators' *status quo* shares as their *expected discounted continuation payoffs* that they would obtain were the game to continue under such a bargaining protocol. Since by propositions 1 and 7 the game will end in the first round, the *status quo* shares are therefore counterfactual payoffs that legislators would obtain were the game to continue. We then use Theorem 1 and Proposition 7 to compute these *status quo* shares as off-equilibirum continuation values. An outcome of such a content-neutral scheduling game can thus be interpreted as an outcome of the Baron and Ferejohn (1989) legislative bargaining game.

EXAMPLE: LEGISLATIVE BARGAINING. In Baron and Ferejohn (1989) there are no legislated constraints. Similarly, we here assume that the only legislated constraints are that the allocation to each legislator must be non-negative, so that $B = \bar{B}(0)$. Denote by $\delta_i < 1$ the discount factor of legislator i, and as in Baron and Ferejohn (1989) assume that $\delta_1 = \delta_2 = ... = \delta_n = \delta_{n+1} = \delta$. Voting is by simple majority, $q = \frac{1}{2}$, and assume that there is an odd number of legislators, i.e., $n+1$ is an odd number. Finally, as in Baron and Ferejohn (1989), assume that Nature chooses a proposer from amongst all legislators with equal probabilities, so that $Pr(i) = \frac{1}{n+1}$. We now compute the *hypothetical* x^{sq}, which is given by the vector of continuation values should the current proposal be rejected; in that case the game would enter the second identical round, with payoffs discounted by δ. The game would then continue either until the proposal in one of the rounds is finally accepted, or it would go on for infinitely many periods, in which case the final payoffs to all legislators would be 0. The *hypothetical* x^{sq} is thus the discounted (by δ) vector of average payoffs before Nature's move, which obtain in an equilibrium of this game. This x^{sq} *hypothetical* since by propositions 1 and 7, in any equilibrium a proposal will be accepted in the first period. Nevertheless, x^{sq} is the vector of discounted payoffs that legislators obtain in an equilibrium and is itself important in determining the equilibrium, i.e., x^{sq} is a fixed point. Since the probability distribution of picking a proposer anew from amongst the legislators is independent of the current proposer, x^{sq} is also independent of whomever the proposer might be. Given a proposal $x \in B$, the minimal majority needed to pass a proposal is picked randomly.[26] For each legislator, let x^p denote the share obtained when they are a proposer, and x^m the share when they belong to a minimal majority. Therefore, we have the two equations, $x_i^{sq} = \delta(\frac{1}{n+1}x^p + \frac{1}{2}x^m) = x^m$, and $x^p = 1 - \frac{n}{2}x^m$, which yield,

$$x_i^{sq} = x^m = \frac{2\delta}{2(n+1) - \delta}.$$

[25] We present the example with linear utility functions as in the original model of Baron and Ferejohn (1989). The extension to more general concave utility functions is also immediate.

[26] The status quo x^{sq} is a result of a similar procedure in the (hypothetical) second stage, i.e., as in Baron and Ferejohn (1989) x^{sq} is the equilibrium amount of resources given to legislator i, on average, before Nature picks the proposer. Then it has to be that when composing a minimal majority to pass the proposal, legislators are selected into that minimal majority with equal probabilities. To see this, suppose that some legislator i had a lower likelihood of being a part of the winning majority. Then i's continuation value would be lower than other legislators' continuation values, i.e., x_i^{sq} would be the smallest. But then i's vote would be effectively cheaper than any other legislators' vote, so that i would in fact be a member of any winning majority, a contradiction.

To conclude this section, we compare the welfare under the content-neutral scheduling and the scheduling auction, when the agenda setter is chosen from the proto-party. To keep this welfare analysis simple, we assume that the proto-party comprises a majority of legislators, so that $N^a = N^l \cup N^m$.

In the welfare comparison, an important determinant is the socially-optimal policy, x^o. For example, the socially-optimal policy x^o might be the egalitarian policy, $x^o = (\frac{1}{n+1}, ..., \frac{1}{n+1})$. Our analysis here is more general, and we assume that the socially optimal policy is not *too unequal*. More precisely, we assume that,

$$x_i^o \leq 1 - \left(\max_{N' \subset N, |N'| = \frac{n}{2}} \sum_{j \in N'} x_j^{sq} + \sum_{j \in N \setminus (N' \cap \{i\})} b_j \right), \forall i \in N. \tag{5}$$

The interpretation of inequality (5) is that when legislator i is the proposer, she is able to obtain a larger than the socially-optimal share of resources. Alternatively, when (5) is not satisfied the socially-optimal allocation is so distorted in favor of some legislator $i \in N$ that even if she were the agenda setter, her share would be still less than under the social optimum. That the social optimum should be so distroted seems in most cases implausible, and condition (refeq:ne) is in that sense relatively mild. When (5) and a mild additional condition are satisfied, the scheduling auction generates a higher than the content-neutral scheduling, i.e., the social-welfare loss is greater unde the content-neutral scheduling. We show this in the next Theorem 6. Recall that $\bar{W}(x, x^o)$ denotes the welfare loss of a policy x relative to the socially optimal policy, and this welfare loss is given by (1). In what follows, denote $N^{ca} = \{1, ..., n+1\} \setminus N^a$.

Theorem 6. *Let $B = B(b)$, $0 \leq b_i \leq \frac{1}{n+1}$, $\forall i \in \{1, ..., n+1\}$, let $x^{sq} \in B$, let the socially-optimal policy be given by x^o, and Suppose the scheduling agent satisfies majoritarian approval so that she is chosen from the proto-party, $\iota \in N^a$. If (5) holds, and additionally,*

$$2 \max_{j \in N^a} (x_j^{sq} - b_j) \geq \max_{j \in N^a, i \in N^{ca}} x_i^o - x_j^o, \tag{6}$$

then, $\bar{W}(x^{a*}, x^o) \leq \bar{W}(x^{c*}, x^o)$, for any outcome x^{a*} of the scheduling auction, and any outcome x^{c*} of the content-neutral scheduling.

Proof. Let x^{i*} be the outcome of content-neutral scheduling when $i \in \{1, ..., n+1\}$ is chosen by Nature as the proposer, and let $x^{\iota*}$ be the outcome of the scheduling auction when $\iota \in N^a$ is the scheduling agent. If $i \in N^a$, then x^{i*} is by Proposition 7 an outcome of the scheduling auction and $\bar{W}(x^{a*}, x^o) = \bar{W}(x^{c*}, x^o)$.

Now take $i \notin N^a$. Note that the legislators in $N^m \subset N^a$ are those with the maximal difference between their *status quo* share and their legislated constraint,

$$N^m = \arg\max_{j \in N^a} (x_j^{sq} - b_j).$$

Let $\bar{N}^i(x^{i*})$ be a minimal winning majority needed for x^{i*} to be voted up against x^{sq}. Fix the scheduling agent $\iota \in N^a$, and let $\bar{N}^\iota(x^{\iota*})$ be some corresponding minimal winning majority needed to pass $x^{\iota*}$. Now observe that by Theorem 1 and Proposition 7, we can take $\bar{N}^i(x^{i*}) = \left(\bar{N}^\iota(x^{\iota*}) \setminus \{j\}\right) \cup \{i\}$, for some $j \in N^{max}$. By Theorem 1, the equilibrium share to ι in the outcome $x^{\iota*}$ is given by,

$$x_\iota^{\iota*} = 1 - \sum_{k \in \bar{N}^\iota \setminus \{\iota\}} x_k^{sq} - \sum_{k \notin \bar{N}^\iota} b_k.$$

By Proposition 7, the equilibrium share to i in the outcome x^{i*} is given by

$$x_i^{i*} = 1 - \sum_{k \in \bar{N}^i \setminus \{i\}} x_k^{sq} - \sum_{k \notin \bar{N}^i} b_k.$$

Since $\bar{N}^i(x^{i*}) = \left(\bar{N}^\iota(x^{\iota*}) \setminus \{j\}\right) \cup \{i\}$, we obtain, $x_i^{i*} = x_\iota^{\iota*} + x_j^{sq} - b_j \geq x_\iota^{sq}$. Therefore,

$$\bar{W}(x^{\iota*}, x^o) - \bar{W}(x^{i*}, x^o) = |x_j^{sq} - x_j^o| - |x_j^o - b_j| + |x_\iota^{\iota*} - x_\iota^o| - |x_\iota^{\iota*} + x_j^{sq} - b_j - x_i^o|$$

By (5), $x_\iota^{\iota*} - x_\iota^o \geq 0$, and $x_\iota^{\iota*} + x_j^{sq} - b_j - x_i^o \geq 0$, so that,

$$\bar{W}(x^{\iota*}, x^o) - \bar{W}(x^{i*}, x^o) = |x_j^{sq} - x_j^o| - |x_j^o - b_j| - x_\iota^o - x_j^{sq} + b_j + x_i^o. \tag{7}$$

If $x_j^{sq} \geq x_j^o$, then (7) becomes,

$$\bar{W}(x^{\iota*}, x^o) - \bar{W}(x^{i*}, x^o) = x_i^o - x_\iota^o - 2(x_j^{sq} - b_j) \leq 0,$$

where the last inequality follows from (6).
If $x_j^{sq} < x_j^o$, then (7) becomes,

$$\bar{W}(x^{\iota*}, x^o) - \bar{W}(x^{i*}, x^o) = 2(b_j - x_j^o) + x_i^o - x_\iota^o \leq 0,$$

where the last inequality follows from (6) and $x_j^{sq} < x_j^o$. □

Theorem 6 shows that under general and intuitive conditions, if the choice of the scheduling agent satisfies majoritarian approval, the scheduling auction is socially preferred over content-neutral scheduling. The first condition states that the socially-optimal outcome should not be too distorted in favor of a particular proposer (who happens to be chosen under the content-neutral scheduling). The second condition states that the variation in the socially optimal outcome should not be too large and should not simultaneously allocate the smallest shares to the proto-party N^a from which the agenda setter is selected.

The intuition behind Theorem 6 is as follows. When the scheduling agent is chosen from the proto-party then the minimal-winning majority (without the scheduling agent) will in most cases be *more expensive in relative terms* than if the scheduling agent were chosen from outside the proto-party. In particular, if the scheduling agent is one of the relatively cheap legislators, then an additional legislator from N^m must belong to

the winning majority; when the agenda setter is a legislator from N^{ca}, then a relatively cheaper legislator can be included in the winning majority. Hence, the relatively cheapest majority is slightly more expensive when the scheduling agent belongs to the proto-party. This implies that the shares then more equally distributed among the legislators. If the legislated constraints are not too unequal, and the social optimum is not too unequal, the equilibrium outcome is then less unequal and hence closer to the social optimum.

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