DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

A CITIZEN CANDIDATE MODEL WITH PRIVATE INFORMATION AND UNIQUE EQUILIBRIUM

Jens Grosser
Florida State University

Thomas R. Palfrey
California Institute of Technology

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Jens GroBer Thomas Palfrey

Abstract

We study a citizen candidate model where citizen ideal points are private information and each ideal point is an independent draw from a uniform distribution. We characterize the equilibrium as a function of the entry cost, the office-holding benefit, and the number of citizens. In contrast to the standard citizen candidate models, equilibrium is unique. Entry is from the extremes of the distribution. A citizen enters if and only if her ideal point greater than or equal to some critical distance from the expected median. Expected policies are more extreme as entry costs increase or office-holding benefits decrease, and as the number of citizens increases.

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A Citizen Candidate Model with Private Information and Unique Equilibrium[1]

Jens Großer[2] and Thomas R. Palfrey[3]

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[2]Departments of Political Science and Economics, Florida State University

[3]Division of the Humanities and Social Sciences, California Institute of Technology

1 Introduction

We study a citizen candidate model with *private information* about the candidates' preferred policies (or, ideal points). By contrast, in the seminal models of Osborne and Slivinski (OS 1996) and Besley and Coate (BC 1997), and most citizen candidate models that have followed, the candidates' ideal points are assumed to be common knowledge. In the baseline model, a community is about to elect a new leader to implement a policy decision. Each citizen may enter the electoral competition as a candidate at some commonly known cost. Because each candidate's preferred policy is public information, she cannot credible promise any other than this policy in case of being elected. Anticipating this, citizens prefer the candidate whose ideal point is closest to their own ideal point, possibly themselves. OS assume a continuum of citizens (i.e., potential candidates) and sincere voting. That is, citizens vote for the most preferred candidate. BC assume a finite number of citizens and strategic voting (i.e., a Nash equilibrium in undominated strategies for the voting game). They identify a variety of different kinds of equilibria supporting different numbers of entrants, and show how the set of equilibria depends on the distribution of ideal points as well as the entry costs and benefits from holding office. For most environments, there are multiple equilibria. Both median and non-median policy outcomes can be supported in equilibrium.

The citizen candidate model makes an important departure from the Hotelling-Downs model of spatial competition because it provides a framework to address questions of endogenous entry of candidates (or parties) when these candidates have preferences over policy outcomes.[1] Importantly, in this model the configuration of equilibrium candidate policies must resist the potential entry of any citizen as a candidate, given the restrictions on the entry costs and spoils of office. Moreover, standard spatial competition models assume candidates without own ideal points and let them float in the policy space in order to maximize their chances of being elected. By contrast, in the baseline model, citizen candidates have their own ideal points and these coincide with their policy promises.

However, the assumption of common knowledge about citizen candidates' ideal points is restrictive. For example, it seems to be common that candidates' stands on issues that are not yet foreseen (e.g., unexpected outbreaks of conflicts) are uncertain, and the community may observe unexpected policy decisions when these issues come up. Moreover, extremist candidates may have strong incentives to disguise their actual preferences until they are in power. For example, after becoming the leader of the Communist Party of the Soviet Union, Mikhail Gorbachev surprised many of his comrades with a policy that opened up for the West. In this paper we study such uncertainties by introducing private information about citizen candidates' ideal points. This approach has another advantage in that it has sharp predictions: citizen candidate models typically suffer from multiple equilibria and do not have clean empirical predictions.[2]

[1] The citizen candidate models have their roots in the earlier work on strategic entry, models related to Duvergers' law, and models with policy motivated candidates. See for example Palfrey (1984), Wittman (1983), Palfrey (1989), Feddersen et al. (1990), Feddersen (1992), and Osborne (1993).

[2] See for example Roemer (2004), Dhillon and Lockwood (2002), and the references they cite.

In this paper we develop a citizen candidate model with a finite (possibly small) number of citizens whose ideal points are iid draws from a continuous uniform distribution on the policy space and private information. We look at symmetric equilibria in the entry stage of the model, and prove that they always exist and are always unique. There is never a symmetric equilibrium with only 'moderate' types entering (moderate in the sense of smaller distances between their ideal points and the median ideal point, as compared to 'extreme' types). The equilibrium has the property that if a citizen enters if her ideal point is x, then she also enters when her ideal point is more extreme than x. This unique equilibrium implies a unique probability distribution of the number of entrants, and we are able to obtain comparative statics about how this distribution changes with the underlying parameters of the model: community size, entry costs, and benefits from holding office. As the entry costs increase or the benefits from holding office decrease, there are fewer entrants in the sense of first order stochastic dominance, and the candidates are more extreme on average. As the number of citizens increases, candidates are again more extreme on average but the effect on the number or entrants is ambiguous. A more general account of our citizen candidate model with private information is given in Großer and Palfrey (2008), where various symmetric and asymmetric distributions of ideal points and other extensions are analyzed.

Several papers have begun to explore the effects of uncertainty on citizen candidate equilibria, in several different ways. Due to space constraints in this volume, we can only mention some relevant literature. Eguia (2007) allows for uncertain turnout and shows how this can reduce somewhat the set of equilibria in the BC model. Fey (2007) uses the Poisson game approach to study entry where there is an uncertain number of citizens. Brusco and Roy (2007) add aggregate uncertainty, allowing for shifts in the distribution of ideal points. Casamatta and Sand-Zantman (2005) study a model with private information and three types of citizens, and analyze the asymmetric equilibria of the resulting coordination game. Osborne et al. (2000) present a model, though not a citizen candidate model, where extreme types participate in costly meetings.

Section 2 describes our citizen candidate model with private information and a uniform distribution of ideal points. Section 3 characterizes the equilibrium of the model. Section 4 derives comparative statics, and illustrates these using some examples with specific parameter values. Section 5 briefly discusses concave payoff functions. Section 6 discusses possible extensions and concludes.

2 The Model

A community of $n \geq 2$ citizens is electing a leader to implement a policy decision. The policy space is represented by the $[-1, 1]$ interval of the real line. Each citizen, $i = 1, ..., n$, has preferences over policies, which are represented by a utility function that is linearly decreasing in the Euclidean distance between the policy decision and her ideal point, $x_i \in [-1, 1] \subset \mathbb{R}$. An individual's ideal point is private information, so only citizen i knows x_i. Ideal points are uniformly distributed according to the continuous cumulative distribution function F, with $F(x) = \frac{1+x}{2}$, $x \in [-1, 1] \subset \mathbb{R}$, and this distribution is common knowledge. The game for implementing a policy decision proceeds in four stages. In the first stage (*Entry*), all citizens decide simultaneously and independently on whether to bear the entry cost $c \geq 0$ and run for office, $e_i = 1$, or not run, $e_i = 0$. The number of citizen candidates is denoted by $m \equiv \sum_{i=1}^{n} e_i$. In the second stage (*Policy promises*), each candidate publicly announces a non-binding policy promise. If $m = 0$, a default policy, δ, is implemented according to a commonly known (possibly stochastic) procedure, Δ. In the third stage (*Voting*), each citizen i makes a costless decision on whether to abstain from voting or to vote for one of the candidates, possibly for herself. The new leader is determined by simple majority rule (with random tie breaking) and announced publicly. In the final stage (*Policy decision*), the leader implements a policy $\gamma \in [-1, 1] \subset \mathbb{R}$. Then, each citizen i's total payoff in the game is given by

$$\pi_i (x_i, \gamma, e_i, w_i) = - |x_i - \gamma| - ce_i + bw_i, \tag{1}$$

where $w_i = 1$ if citizen i is elected as the new leader, in which case she receives private benefits from holding office, $b \geq 0$. If citizen i is not the new leader, then $w_i = 0$. We assume that citizens maximize their own expected payoffs and face identical entry costs, c, and leadership benefits, b.

3 Political equilibrium

To solve our citizen candidate model with private information, we use sequential equilibrium (henceforth 'political equilibrium') and consider behavioral strategies (Kuhn 1953) for each information set. Our theoretical analysis starts with policy promises, voting, and policy decisions before we proceed in more detail with the citizens' decisions on whether or not to run for office.

Lemma 1 (Policy promises, voting, and policy decisions) *In any political equilibrium, (i) policy promises are 'cheap talk'; (ii) each citizen candidate is elected with equal probability of $\frac{1}{m}$; and (iii) the new leader implements her ideal point, $\gamma^* = x_i$.*

Proof. (iii): The only credible policy choice of a new leader is to implement her ideal point, $\gamma^* = x_i$, yielding her a zero payoff loss, $-|x_i - \gamma^*| = 0$. (i): If there is only one candidate, her policy promise is irrelevant because she will be elected anyway (at least she will vote for herself; see below). If there are two or more candidates, their policy promises are not credible because each candidate has an incentive to increase her chances of being elected by misrepresenting her ideal point (recall that promises are non-binding and preferences are private information). Thus, policy promises are 'cheap talk'. (ii): Then, each non-candidate is indifferent between the candidates because she cannot distinguish among their ideal points. Thus, she either abstains from voting or votes for any of the candidates. Moreover, each candidate prefers herself to any other candidate (whose ideal points she cannot distinguish either). This is because any other's ideal point yields her a strict payoff loss with probability one. Thus, each candidate votes for herself.[3] ■

[3] Note that the *policy promises* and *voting* stages do not demand any particular decision structure. Specifically, lemma 1 holds for any sequence of decisions and information about these decisions. Also note that voting equilibria exist in which some candidates have larger probabilities of being elected than others. However, our model rules out any kind of coordination prior to entry decisions and, hence, ex ante each candidate has an equal probability of becoming the new leader.

Lemma 1 greatly simplifies the equilibrium analysis in the *entry* stage, to which we turn next. We focus on equilibria in *symmetric* cutpoint strategies, defined by

$$\breve{e}_i = \begin{cases} 0 & if \quad |x_i| < \breve{x} \\ 1 & if \quad |x_i| \geq \breve{x}, \end{cases} \tag{2}$$

where the cutpoint $\breve{x}$ represents a pair of cutpoint policies $(\breve{x}_l, \breve{x}_r)$ with $\breve{x}_l \leq 0 \leq \breve{x}_r$ (i.e., the subscripts denote their relative positions *l*eft and *r*ight, respectively) and $\breve{x} = |\breve{x}_l| = \breve{x}_r \in [0, 1] \in \mathbb{R}$ (i.e., the cutpoint policies are symmetric around $x = 0$). In words, the symmetric cutpoint strategy, $\breve{e}_i$, determines that all citizens with ideal points equally and more 'extreme' than $\breve{x}$ run for office, and all citizens with ideal points more 'moderate' than $\breve{x}$ do not run.

To derive the equilibrium cutpoint policy, $\breve{x}^*$, we must compare a citizen i's expected payoffs as both a candidate and a non-candidate, given the equilibrium decisions in subsequent stages. Then, citizen i's expected payoff for *entering,* $\breve{e}_i = 1$, is

$$E[\pi_i \mid \breve{e}_i = 1] = \breve{x}^{n-1}\, b$$

$$+ \sum_{m=2}^{n} \binom{n-1}{m-1} (1 - \breve{x})^{m-1} \breve{x}^{n-m} \left[\frac{b}{m} + \frac{m-1}{m}\, E[-|x_i - \gamma| \mid \breve{x}] \right] - c, \tag{3}$$

where $\Pr(|x_i| \geq \breve{x}) = 1 - \breve{x}$ and $\Pr(|x_i| < \breve{x}) = \breve{x}$, for our $F(x) = \frac{1+x}{2}$, $x \in [-1, 1] \subset \mathbb{R}$. Moreover, assuming without loss of generality that $x_i \geq 0$, i's expected payoff loss if not being elected is given by

$$E\left[|x_i-\gamma| \mid \check{x}\right] = \frac{1}{2}\frac{\int_{-1}^{-\check{x}}\frac{1}{2}|x_i-x|\,dx}{F(-\check{x})}+\frac{1}{2}\frac{\int_{\check{x}}^{1}\frac{1}{2}|x_i-x|\,dx}{1-F(\check{x})} \tag{4}$$

$$= \frac{1}{4(1-\check{x})}\Bigg[-|x_i+\check{x}|^2+|x_i+1|^2+|x_i-1|^2 + |x_i-\check{x}|^2\times\begin{cases}-1 & if \;\; 0\le x_i<\check{x}\\ 1 & if \;\; 0\le\check{x}\le x_i\end{cases}\Bigg]$$

$$for\ \check{x}\in[0,1),$$

which accounts for the possibility that the expected policy decision lies in the left or right direction, γ_l or γ_r respectively, with equal probability of one half for each. Note that the default policy takes effect if $\check{x}=1$. The first term in expression (3) gives the case where i receives b because she is the only candidate, which occurs with probability $\check{x}^{n-1}$. The second term gives the cases where $m-1\ge1$ candidates enter in addition to herself, which occurs with probability $\binom{n-1}{m-1}(1-\check{x})^{m-1}\check{x}^{n-m}$ and yields her expected leadership benefits of $\frac{b}{m}$. The summation accounts for all possible $m=2,...,n$. Moreover, i will not be elected with probability $\frac{m-1}{m}$ and her expected payoff loss for this event is $E\left[|x_i-\gamma| \mid \check{x}\right]$, given in expression (4). Finally, i bears the entry costs, c, independent of how many other candidates enter, which gives the third term in expression (3).

In contrast, citizen i's expected payoff for *not entering*, $\check{e}_i=0$, is

$$E[\pi_i \mid \check{e}_i=0]=\check{x}^{n-1}\;E[-|x_i-\delta| \mid \check{x}] + \sum_{m=2}^{n}\binom{n-1}{m-1}(1-\check{x})^{m-1}\check{x}^{n-m}\;E\left[-|x_i-\gamma| \mid \check{x}\right]. \tag{5}$$

The first term corresponds to the event where, as herself, no other citizen runs for office, which occurs with probability $\check{x}^{n-1}$. In this paper, we assume for simplicity that the default policy $\delta = 0$ takes effect. This leads to a very simple expression for payoff losses in the no-entry event, which is independent of $\check{x}$:

$$E[|x_i - \delta| \mid \check{x}] = |x_i|. \tag{6}$$

Note that for $\check{x} = 0$ the default policy is irrelevant, because all citizens enter. The remaining terms in expression (5) correspond to the events where $m - 1 \geq 1$ other citizens choose to enter. In contrast to expression (3), b does not appear in these terms because i does not enter and therefore never wins.

Finally, it is readily verified that relating expressions (3) and (5) and rearranging yields the best response entry strategy for a citizen with ideal point x_i if all other citizens are using cutpoint strategy $\check{e}$, which is to enter if and only if[4]

$$\check{x}^{n-1}\left[b + |x_i|\right] + \sum_{m=2}^{n} \binom{n-1}{m-1} (1-\check{x})^{m-1} \check{x}^{n-m} \frac{1}{m} \left[b + E\left[|x_i - \gamma| \mid \check{x}\right]\right] \geq c, \tag{7}$$

where the left-hand and right-hand sides (henceforth LHS and RHS, respectively) give citizen i's expected net benefits and costs from running for office, respectively. We can use this condition, and our assumptions stated above, to derive the following proposition:

Proposition 1 (Equilibrium entry) *There always exists a political equilibrium with a unique symmetric cutpoint policy, $\check{x}^*$, where each citizen i with $|x_i| \geq \check{x}^*$ enters the electoral competition as a candidate, $\check{e}_i^* = 1$, and each citizen i with $|x_i| < \check{x}^*$ does not*

[4] We assume, without loss of generality, that indifferent citizen types choose to enter.

enter, $\check{e}_i^* = 0$*. This cutpoint policy is characterized by the following necessary and sufficient conditions:*

(*i*) *If* $c \leq \underline{c} \equiv \frac{1}{n}\left[b + \frac{1}{2}\right]$*, then* $\check{x}^* = 0$ *and* $\check{e}_i^* = 1, \forall i$ *("every citizen enters", or* $m = n$*);*

(*ii*) *If* $c \geq \overline{c} \equiv b + 1$*, then* $\check{x}^* = 1$ *and* $\check{e}_i^* = 0, \forall i$ *("no citizen enters", or* $m = 0$*);*

(*iii*) *If* $\underline{c} < c < \overline{c}$*, then* $\check{x}^* \in (0, 1)$ *and some citizens are expected to enter (or* $m \in [0, n]$*), where* $\check{x}^*$ *is determined by*

$$(\check{x}^*)^{n-1}\left[b + \check{x}^*\right] + \sum_{m=2}^{n}\binom{n-1}{m-1}(1 - \check{x}^*)^{m-1}(\check{x}^*)^{n-m} \times \frac{1}{m}\left[b + \frac{1 + \check{x}^*}{2}\right] = c. \quad (8)$$

Proof. We give a sketch here (details are in Großer and Palfrey 2008). Recall our assumptions $F(x) = \frac{1+x}{2}$, $x \in [-1, 1] \subset \mathbb{R}$, $n \geq 2$, $c \geq 0$, and $b \geq 0$. First, we show that (*i*) to (*iii*) give sufficient conditions for an equilibrium cutpoint strategy, $\check{e}^*$, to exist. To do so, consider $LHS(7)$ and note that a change in x_i may only affect $|x_i|$ and $E\left[|x_i - \gamma| \mid \check{x}\right]$, but no other term. Observe that $LHS(7)$ is strictly increasing in $x_i \in [0, 1]$ unless $x_i = \check{x} = 0$, in which case it does not change in x_i (this is a situation where everyone enters anyway). This proves that (*i*) to (*iii*) provide sufficient conditions for an equilibrium cutpoint strategy, $\check{e}^*$, to exist. In fact, because this holds for *any* $\check{x}$, this establishes that *any* symmetric equilibrium is in cutpoint strategies. This leads to three possible situations.

(*i*): If $LHS(7)$ is equal to or greater than c for all values of x_i and $\check{x}$, then the unique equilibrium is for all n citizens to enter. This corresponds to an equilibrium cutpoint policy $\check{x}^* = 0$. Thus, for this to hold, we simply set $\check{x} = 0$ and $x_i = 0$ and have only to consider the term $m = n$ in $LHS(7)$. The inequality condition (7) reduces to

$$\frac{1}{n}\left(b + E\left[|\check{x}^* - \gamma| \mid \check{x}^* = 0\right]\right) = \frac{1}{n}\left[b + \frac{1}{2}\right] \equiv \underline{c} \geq c.$$

Thus, there is universal entry if and only if $c \leq \frac{1}{n}\left[b + \frac{1}{2}\right]$.

(ii): If $LHS(7)$ is less than or equal to c for all values of x_i and $\check{x}$, then the unique equilibrium is for no citizen to enter. This corresponds to an equilibrium cutpoint policy $\check{x}^* = 1$. Thus, for this to hold, we simply set $\check{x} = 1$ and $x_i = 1$. The inequality condition (7) changes and reduces to

$$b + E[|\check{x}^* - \delta| \mid \check{x}^* = 1] = b + 1 \equiv \overline{c} \leq c.$$

Thus, there is zero entry if and only if $c \geq 1 + b$ (note that the probability that any citizen has an ideal point $x_i = 1$ is equal to zero).

(iii): If neither boundary condition in (i) or (ii) hold, then we have an equilibrium with an interior cutpoint, $\check{x}^* \in (0, 1)$. Note first that equilibrium condition (8) is the same as equation (7), except substituting $x_i = \check{x}$ and noticing that $E\left[|\check{x} - \gamma| \mid \check{x}\right] = \frac{1+\check{x}}{2}$. Next, observe that $LHS(8)$ is continuous on $x \in [-1, 1]$ because ideal points are distributed continuously, and it is strictly increasing in $\check{x} \in (0, 1)$. This proves that $\check{x}^* \in (0, 1)$ is unique, because $LHS(8)$ and $RHS(8)$ can intersect at most once. To see that this equilibrium exists, recall that $\check{x} = 0$ yields $\underline{c}$ and $\check{x} = 1$ yields $\overline{c}$ and note that $\underline{c} = \frac{1}{n}\left[b + \frac{1}{2}\right] < b + 1 = \overline{c}$ for $n \geq 2$ and $b \geq 0$. Thus, for any $c \in (\underline{c}, \overline{c})$ there exists a unique interior equilibrium, $\check{x}^* \in (0, 1)$, according to condition (8). ■

4 Comparative statics

In this section, we turn to the comparative statics regarding the effects of changes in n, c, and b on $\check{x}^*$ when we are in a region of the parameter space where the solution is interior, i.e., $\check{x}^* \in (0,1)$.

Proposition 2 (Comparative statics) *The interior symmetric equilibrium cutpoint policy, $\check{x}^* \in (0,1)$, is strictly increasing in the number of citizens, n, and the entry costs, c; and it is strictly decreasing in the benefits from holding office, b. An increase in $\check{x}^*$ implies that, on average, candidates and policy outcomes become more extreme. It also implies a decrease in expected entry when caused by changes in c and b.*

Proof. See Großer and Palfrey (2008). ■

Proposition 2 states that candidates and policy outcomes are, on average, more extreme in larger communities. It is also a straightforward exercise to show that $\underset{n\to\infty}{lim}\, \check{x}(n) = 1$. That is, in very large electorates, only the most extreme citizens will throw their hat in the ring. Of course, this does not imply there is zero entry! The limiting distribution of the number of entrants is fully characterized in Großer and Palfrey (2008).

Proposition 2 does not give a comparative static result about the expected number of entrants as a function of the number of citizens, n. In contrast to the increases in the expected number of entrants when c and b change, an increase in the number of citizens can yield either more entry or less entry, on average. To see this, notice that there are two effects on entry that result from increasing from n to $n+1$. First, there is the direct effect that the number of *potential* candidates (i.e., citizens) has increased by 1. This effect works to increase entry. The second effect is an indirect equilibrium effect, namely that $\check{x}(n+1,c,b) > \check{x}(n,c,b)$, and this goes in the opposite direction. I.e., there is one more potential entrant, but each citizen now enters with a lower probability. Which term dominates will depend on n, c, and b.

5 Examples

We next use specific parametric examples of the uniformly distributed ideal points, x_i, with $F(x) = \frac{1+x}{2}$, $x \in [-1,1] \subset \mathbb{R}$, to illustrate graphically the key equilibrium properties of our citizen candidate model with private information.

5.1 Variations in the costs of entry, c

To illustrate our comparative statics results for changes in the costs of entry, this example uses $n = 5$ and $b = 0$ and varies the costs between $c = 0.10$, 0.25, 0.45, and 1. Figure 1 gives the cutpoint policies $\check{x}_l \in [-1, 0]$ and $\check{x}_r \in [0, 1]$ on the horizontal axis and the expected net benefits and costs from entering as a candidate on the vertical axis (i.e., $LHS(8)$ and $RHS(8)$, respectively). Expected net benefits are represented by the U-shaped curve and the various costs by horizontal lines.

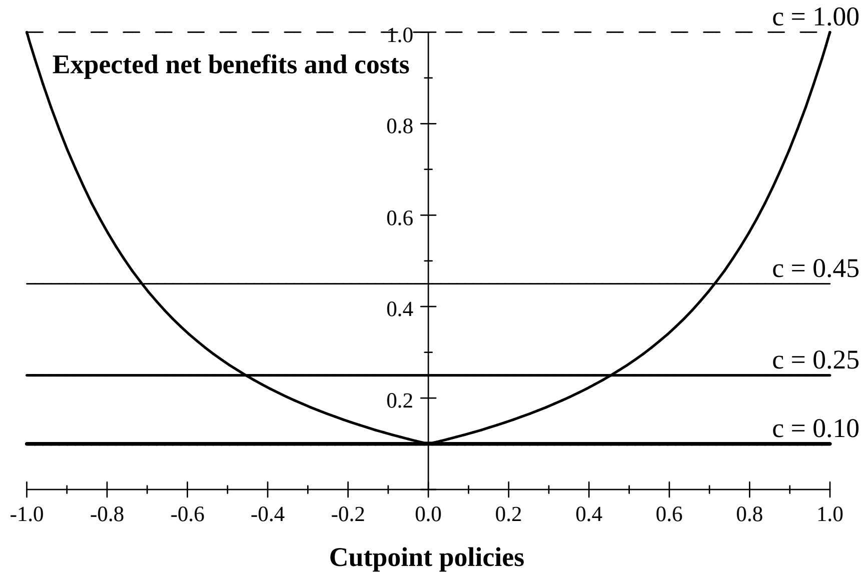

Figure 1: Symmetric cutpoint policy equilibria and variations in entry costs, c, for $n = 5$ and $b = 0$.

The symmetric cutpoint policy equilibria for the various costs, $\check{x}^*(c) \in [0, 1]$, are determined by the intersections of the expected net benefits curve and the respective cost lines. These equilibria are increasing in c, where $\check{x}^*(c = 0.10) = 0$, $\check{x}^*(c = 0.25) = 0.455$, $\check{x}^*(c = 0.45) = 0.713$, and $\check{x}^*(c = 1) = 1$. Note that $\underline{c} = \frac{1}{5}\left[0 + \frac{1}{2}\right] = 0.1$ and $\overline{c} = 0 + 1 = 1$ for the limit cutpoint policies 0 and 1, respectively (recall from proposition 1 (i) and (ii) that everyone enters if $c \leq \underline{c}$ and no one enters if $c \geq \overline{c}$). Finally, as a consequence of the increasing $\check{x}^*$ in c, expected policy outcomes in each direction left and right become more extreme (minus and plus 0.5, 0.727, 0.857, and 1 for our ascending c, respectively) and expected entry decreases (5, 2.726, 1.434, and 0, respectively).[5]

[5]Expected policy outcomes in each direction left and right are derived as $E[\gamma_l \mid \check{x}_l] = \frac{\check{x}_l - 1}{2}$ and $E[\gamma_r \mid \check{x}_r] = \frac{\check{x}_r + 1}{2}$, respectively, and expected entry is derived as $E[m \mid \check{x}] = n(1 - \check{x})$.

5.2 Variations in the spoils of office, b

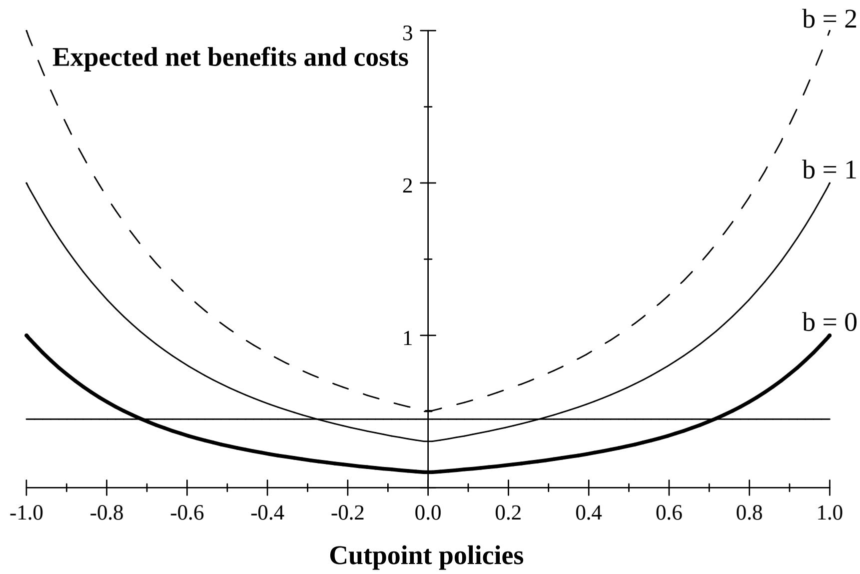

Figure 2: Symmetric cutpoint policy equilibria and variations in the benefits from holding office, b, for $n = 5$ and $c = 0.45$.

Here, we demonstrate our comparative statics results for changes in the benefits from holding office. The example uses $n = 5$ and $c = 0.45$ and varies the spoils between $b = 0, 1$, and 2. Figure 2 shows that $\check{x}^*$ decreases in b, where $\check{x}^*(b = 0) = 0.713$ and $\check{x}^*(b = 1) = 0.276$ (cf. the intersections of the respective net benefits curves and the cost line) and $\check{x}^*(b = 2) = 0$ (because the respective net benefits curve lies above the cost line). Finally, this decrease yields more moderate expected policy outcomes in each direction left and right (minus and plus 0.857, 0.638, and 0.5 for our ascending b, respectively) and raises expected entry (1.434, 3.618, and 5, respectively).

5.3 Variations in the number of citizens, n

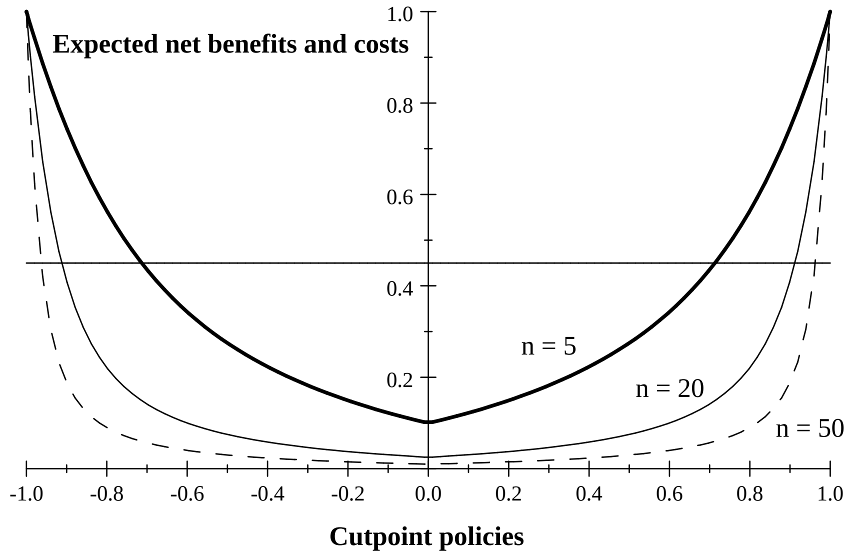

Figure 3: Symmetric cutpoint policy equilibria and variations in the number of citizens, n, for $c = 0.45$ and $b = 0$.

The final example illustrates our comparative statics results for changes in the size of the community. It uses $b = 0$ and $c = 0.45$ and varies the number of citizens between $n = 5, 20$, and 50. Figure 3 shows that $\check{x}^*$ increases in n, where $\check{x}^*(n = 5) = 0.713$, $\check{x}^*(n = 20) = 0.912$, and $\check{x}^*(n = 50) = 0.963$, respectively (once again, cf. the intersections of the respective net benefits curves and the cost line). As a consequence, this increase yields more extreme expected policy outcomes in each direction left and right (minus and plus 0.857, 0.956, and 0.982 for our ascending n, respectively) and raises expected entry (1.434, 1.757, and 1.832, respectively).[6]

[6] Recall that expected entry does not necessarily increase in n, but depends on the specific parameters in this example.

6 Concave payoff functions

The model can also be extended in a straightforward way to allow for a class of utility functions that are a concave function of the Euclidean distance between a citizen's ideal point and the policy outcome. Particularly simple is the special case of power utility functions (which includes the commonly-used specification of quadratic payoffs), and one can think of these utility functions as measuring the risk aversion of the players. The payoff function is

$$\pi_i\left(x_i, \gamma, e_i, w_i, \alpha\right) = \left|x_i - \gamma\right|^\alpha - ce_i + bw_i, \tag{9}$$

where $\alpha \geq 1$.

Formally, the only difference from the piecewise linear utility specification is that the condition for the best response strategy for a voter with ideal point x_i if all other citizens are using cutpoint strategy $\check{e}$ is now to enter if and only if

$$\begin{aligned}&\check{x}^{n-1}\left[b + E[|x_i - \delta|^\alpha \mid \check{x}]\right]\\ &\qquad + \sum_{m=2}^{n}\binom{n-1}{m-1}(1-\check{x})^{m-1}\check{x}^{n-m}\frac{1}{m}\left[b + E\left[|x_i - \gamma|^\alpha \mid \check{x}\right]\right] \geq c.\end{aligned} \tag{10}$$

See Großer and Palfrey (2008) for the equilibrium characterization for strictly concave utility functions.

7 Discussion and conclusions

We presented our basic citizen candidate model with private information. The paper specializes the results of Großer and Palfrey (2008) to the case of uniformly distributed ideal points and the simple default policy, $\delta = 0$. We showed that equilibria with symmetric cutpoint policies always exist *and are always unique*. In these equilibria, all citizens with ideal points equally or more extreme than the cutpoint enter the electoral competition as candidates and all citizens with more moderate ideal points do not enter. And, we showed that the equilibrium cutpoint policy is increasing in the entry costs and the number of citizens, and it is decreasing in the leader's benefits from holding office. Moreover, an increase in the equilibrium cutpoint policy through changes in the entry costs and benefits from holding office decreases the expected number of citizen candidates and the expected policy outcome becomes more extreme. An increase in the equilibrium cutpoint policy through an increase in the number of citizens also results in a more extreme expected policy outcome, however, the effect on the number of expected entrants can be either positive or negative.

The results can be extended and generalized in several directions. First, one can relax the assumption of uniformly distributed ideal points. This assumption made the computations quite easy and allowed us to illustrate the results graphically. In Großer and Palfrey (2008), we obtain similar results for arbitrary symmetric distributions. This allows us to address questions about the effect of polarization of the electorate's preferences on candidate entry. There, we also investigate the effects of distributional asymmetries of ideal points on the characterization of equilibrium entry strategies and more general specifications of the default policy. The model can also accommodate a stochastic default policy, given by a distribution G.

Ideally one would like to endogenize the default policy as part of the equilibrium of the model. Here we used a simple exogenously specified constant default policy. In general, one would expect the default outcome to depend in some way of the electoral process. One way to endogenize this is to have one of the citizens randomly appointed the new leader, in case no one runs as a candidate. This is considered in Großer and Palfrey (2008).

Another interesting possibility for endogenizing the default policy is to allow multiple rounds in the entry stage: if no citizen chooses to run as a candidate in the first entry round, another round starts and this continues until finally there is at least one citizen candidate. Such a model of "default" policy has the virtue of guaranteeing endogenous entry of at least one candidate, provided entry costs are not prohibitively large. The effect of this is that after the first entry round the community can update that there are no ideal points that are equally or more extreme than the equilibrium cutpoint policy in the first round. In the second entry round, the game will be solved as the original, only that the truncated probability distribution is used, and so forth.

Several other directions extending the model could add some additional insights. For example, in the present formulation of the model citizens do not learn anything useful about a candidate's ideal point. As a first step it would be interesting to look at a model where citizens can learn whether this ideal point is to the left or the right of the median ideal point, as might happen for example if there are interest group endorsements or party labels. Along a similar vein, one could introduce nominating procedures or party formation of left and right candidates, with each side nominating one as their running candidate. One could add partial credibility to the policy promises stage, as in Banks (1990). And finally, it might be possible to extend the model to multiple dimensions, for example where the policy space is the closed unit ball. A natural conjecture for well-behaved symmetric distributions is that there will exist a unique equilibrium with similar features to the one-dimensional model: citizens enter if and only if their ideal point is sufficiently far from the origin.

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