The motional inductances in the two prototype crystal resonators may be equated by scaling the admittance level of the second half-lattice section by n and by scaling the two immittance inverters by $\sqrt{n}$. The network then becomes a cascade of two conjugate half-lattice sections separated by immittance inverters of characteristic immittance $\sqrt{(n)}K_a$. This internal scaling operation has no effect on the transfer characteristics of the overall network.

On bandpassing, the frequency invariant susceptances in the half-lattice may be approximated over a narrow bandwidth by capacitors of value B/ω_c, where ω_c is the centre frequency of the passband.

Where the capacitors turn out to have negative values excess positive capacitance may be added across each arm of the half-lattice sections. This extra shunt capacitance may then be turned out in the usual manner at the input and output of each section.

Conclusion: A simple network equivalence suitable for the design of bandpass elliptic function crystal filters has been presented. The equivalence allows filter designers to tune independently the two transmission zeros associated with each prototype Brune section in the filter. This extra degree of freedom may prove useful in adjusting independently the characteristics of the upper and lower stopbands of a bandpass crystal filter. If the resulting characteristic is asymmetric with respect to the centre frequency small adjustments to the other sections of the filter may be necessary to restore acceptable passband behaviour. Computer analysis of narrowband crystal bandpass filters using this network transformation shows close agreement with theory. Design and realisation of the networks presented here follows normal practice and it is anticipated that the construction and tuning should present no special problems.

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References

1 HOLT, A. G. J., and GRAY, R. L.: 'Bandpass crystal filters by transformation of a lowpass ladder', *IEEE Trans.*, 1968, **CT-15**, pp. 492–494

2 DILLON, C. R., and LIND, L. F.: 'Cascade synthesis of monolithic crystal filters possessing finite transmission zeros', *Int. J. Circuit Theory Appl.*, 1975, **3**, pp. 101–107

3 BELEVITCH, V.: 'An alternative derivation of Brune's cycle', *IRE Trans.*, 1959, **CT-6**, pp. 389–390

4 BELEVITCH, V.: 'Classical network theory' (Holden-Day, San Francisco, 1968)

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SINGLE-CARRIER-TYPE DOMINATED IMPACT IONISATION IN MULTILAYER STRUCTURES

Indexing terms: Semiconductor devices and materials, Photodetectors

A new structure for III-V avalanche photodetectors in which multiplication is dominated by a single-carrier type is proposed. Calculations for a GaAs-AlGaAs detector are reported predicting multiplication dominated by electrons. The reason for this is that electrons are injected into GaAs multiplication layers from high-electric-field AlGaAs layers, while holes are injected into the GaAs layers from low-electric-field AlGaAs layers.

The noise generated by an avalanche photodiode (APD) is dependent on the statistics of the carrier multiplication process, since positive feedback effects, which exist when both electrons and holes produce secondary pairs, can greatly amplify any current fluctuations. Significantly more noise is generated if the electron and hole ionisation rates (α, β) are equal than if only one carrier produces secondary pairs.[1] It is therefore highly desirable to have a detector in which the multiplication process is dominated by one carrier type. Unfortunately, most III-V materials have $\alpha \approx \beta$. Recently, Chin *et al.*[2] proposed a multilayer GaAs-AlGaAs structure designed to enhance the ratio of the ionisation rates α/β. This structure has been fabricated using molecular beam epitaxy (MBE) by Capasso *et al.*[3] The expected enhancement of α/β is due primarily to the fact that the discontinuity of the conduction band is larger than the discontinuity of the valence band. Thus, electrons enter the GaAs multiplication region with more kinetic energy than do holes, and are therefore more likely to produce a secondary pair. In this letter we propose a modification of this structure which we estimate will significantly further increase α/β.

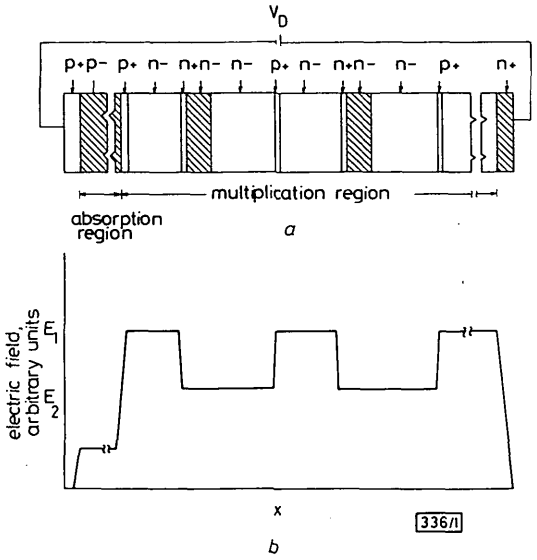

Fig. 1

a Schematic diagram of multilayer ADP structure showing doping of each layer
Shaded regions are material A; white regions are material B
b Electric field in each layer of multilayer structure

To illustrate the basic principles of this photodetector, first consider the hypothetical structure shown in Fig. 1. A unit cell of this multilayer structure consists of five layers and two different materials. Material A has a low ionisation threshold energy and is the material in which the multiplication occurs in this device. Material B has a much larger ionisation threshold energy and negligible multiplication occurs in the layers of material B. The applied voltage is sufficiently large to fully deplete the absorption and multiplication regions. Typically, the layers of the multiplication region are fully depleted even at zero applied voltage. Since the gradient of the electric field in a depleted layer is proportional to the doping, the electric field changes abruptly in the thin, heavily doped layers. In the lightly doped layers, the field is nearly constant. By doping the layers as shown in Fig. 1 the electric field on one side of material A (left side in Fig. 1) can be made larger than the electric field on the other side of A. If the electrons are injected into A from the high-field side and the holes are injected into A from the low-field side, then the fraction of the electrons that are injected with energies above the ionisation threshold can be significantly larger than the fraction of holes that are injected with energies above the ionisation threshold.

To obtain the hypothetical avalanche photodetector just described all that is needed are two materials with sufficiently different ionisation thresholds. Since the ionisation thresholds of semiconductors are generally proportional to the bandgap, any two semiconductors with sufficiently different bandgaps could be used for the materials A and B. For the specific case of the ternary materials $Al_xGa_{1-x}As$, the bandgap increases as x increases. Since GaAs-AlGaAs superlattices can be fabricated using MBE, we expect GaAs ($E_g = 1{\cdot}43$ eV) and

$Al_{0.45}Ga_{0.55}As$ (E_g = 2·0 eV) to be suitable choices for materials A and B, respectively.

Since the electric fields vary significantly over very short distances, the commonly used Baraff theory[4] cannot be readily applied to calculate the ionisation rates of this device. However, to estimate the enhancement of α/β we have applied a simple model of impact ionisation due to Shockley.[5] In this model impact ionisations occur only for the case of a carrier starting from zero energy and accelerating, without suffering any collisions, to an energy above the ionisation threshold. In the Shockley model, the electron and the hole ionisation rates are proportional to $\exp(-D_e(E_i)/L_e)$ and $\exp(-D_p(E_i)/L_p)$, where L_e and L_p are the optical phonon mean free paths for electrons and holes, respectively, and $D_{e,p}(E_i)$ are the distances the electrons and holes must travel without undergoing a collision to accelerate to the threshold energy for ionisation. For this calculation the thicknesses of the GaAs, high-field AlGaAs and the low-field AlGaAs layers were chosen to be 400 Å, 700 Å and 900 Å, respectively. For the ionisation thresholds and optical phonon mean free paths we used the values given in Chin *et al.* These are ionisation thresholds of 2·0 eV for electrons and 1·5 eV for holes and the optical phonon mean free paths of 50 Å for electrons and 40 Å for holes. The results of this calculation are shown in Fig. 2. The curve for $\Delta E = 0$ corresponds to a detector similar to that reported by Capasso *et al.*[3] The main purpose of this calculation is to show that significant enhancement of α/β is expected for $\Delta E \neq 0$. The enhancement of α/β is likely to be somewhat exaggerated for the upper left hand part of Fig. 2 due to shortcomings of the Shockley model. We have also repeated this calculation for other commonly quoted values for the ionisation thresholds and optical phonon mean free paths, which are consistent with experimentally measured ionisation rates of GaAs. The predicted enhancement of α/β was found to be relatively insensitive to which set of parameters we chose. The enhancement of α/β for $\Delta E \neq 0$ is most pronounced for lower electric-field strengths. However, even with higher fields, which means great multiplication per unit cell, the enhancement of α/β is significant. To reduce the complexity of fabricating such a detector it is desirable to have no more than 10–15 unit cells (with an external bias of 6–10 V per cell). A practical detector would therefore probably have electric-field strengths corresponding to the right-hand side of Fig. 2 since higher electric fields would mean that fewer unit cells would be required.

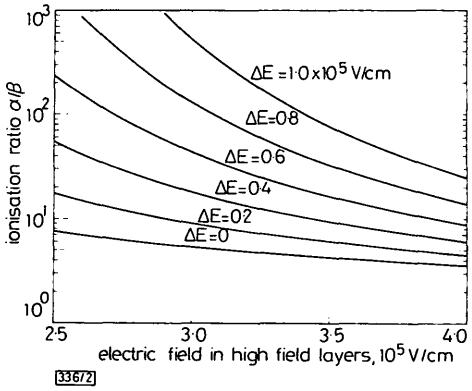

Fig. 2 *Ionisation ratio for various values of electric fields in high- and low-field layers*

To optimise the design of the proposed detector, it would be necessary to calculate the electron and hole energy distribution functions $f_e(\varepsilon)$, $f_h(\varepsilon)$ at each position as the carriers move through the layers of the detector. Although the Shockley model is useful for estimating α/β, it cannot be used to accurately calculate $f_e(\varepsilon)$ and $f_h(\varepsilon)$. However, several qualitative features of the detector design can be stated without precisely knowing the energy distribution functions. First, the $n+$ AlGaAs layers should be as thin as possible so that the hot electrons do not lose much energy in these layers. The high-field AlGaAs layers should have thicknesses and electric fields such that a significant fraction of the electrons are injected into the GaAs layers with enough energy to produce secondary pairs. However, it is undesirable for the fields to be so high that multiplication in the AlGaAs layers becomes significant. To minimise secondary ionisation by holes the low-field layers have to be sufficiently thick that holes can lose (by phonon collision) the kinetic energy gained in the preceding high-field layer. In addition, the difference between the electric fields in the high- and low-field regions should be as large as is practical. A 50 Å-thick $n+$ layer with a doping of 2×10^{18} cm^{-3} will result in a change ΔE in the electric field of approximately $1{\cdot}6 \times 10^5$ V/cm. It is also desirable to have the total number of donors in a unit cell nearly equal to the total number of acceptors, so that the electric-field pattern repeats itself in each unit cell. One final consideration is that electrons that flow across an abrupt interface from GaAs to $Al_{0.45}Ga_{0.55}As$ have to overcome an energy barrier of nearly 0·5 eV. For this reason, it may be desirable to grade the interfaces between the GaAs and the n-$Al_{0.45}Ga_{0.55}As$ layers.

In conclusion a new III-V avalanche photodetector in which the multiplication process is dominated by a single-carrier type has been proposed. For a GaAs-$Al_{0.45}Ga_{0.55}As$ detector of this type α/β has been estimated based on a simple model. The results of this calculation suggest that detectors with greatly enhanced ratios of α/β should be practical. At the present time, work is underway to fabricate such a device and to more precisely calculate the electron and hole ionisation rates.

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References

1 MCINTYRE, R. J.: 'Multiplication noise in uniform avalanche diodes', *IEEE Trans.*, 1966, **ED-13**, pp. 164–167
2 CHIN, R., HOLONYAK, N., STILLMAN, G. E., TANG, J. Y., and HESS, K.: 'Impact ionisation in multilayered heterojunction structures', *Electron. Lett.*, 1980, **16**, pp. 467–469
3 CAPASSO, F., TSANG, W. T., HUTCHINSON, A. L., and WILLIAMS, G. F.: *Appl. Phys. Lett.*, 1982, **40**, pp. 38–40
4 BARAFF, G.: *Phys. Rev.*, 1962, **128**, pp. 2507–2517
5 SHOCKLEY, W.: 'Problems related to P-N junctions in silicon', *Solid State Electron.*, 1961, **2**, pp. 35–67

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SYSTEM REDUCTION BY CAUER CONTINUED-FRACTION EXPANSION ABOUT $s = a$ AND $s = \infty$ ALTERNATELY

Indexing terms: Modelling, Linear systems, Transformations

An algorithm to obtain the Cauer continued-fraction expansion of a given transfer function about an arbitrary point $s = a$ and $s = \infty$ alternately is presented. Its application to system reduction is discussed. The procedure is amenable to digital computation.

Introduction: In Reference 1, Davidson and Lucas have suggested a method of expanding the given transfer function about a general point a; they have shown that this gives a better approximated model than that of Chuang.[3] In their comment,[2] Parthasarathy and Singh have pointed out that the above method does not make use of continued-fraction expansion.

In this letter, we formulate a procedure for implementing the above idea making use of the Cauer continued-fraction technique. We accomplish this by first transforming the given transfer function $G(s)$ to $H(p)$, using the synthetic division technique,[5] under the transformation $s = p + a$; we then