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PREFERENCE AGGREGATION UNDER UNCERTAINTY: SAVAGE VS. PARETO

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Abstract

Following Mongin [12, 13], we study social aggregation of subjective expected utility preferences in a Savage framework. We argue that each of Savage's P3 and P4 are incompatible with the strong Pareto property. A representation theorem for social preferences satisfying Pareto indifference and conforming to the state-dependent expected utility model is provided.

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Preference Aggregation under Uncertainty: Savage vs. Pareto

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1 Introduction

Harsanyi's theorem [8] discusses social aggregation of individual preferences in a risky environment. He shows that when all agents' preferences conform to the axioms of expected utility, if social preference also conforms to the axioms of expected utility and satisfies Pareto indifference with respect to agents' preferences, then social preference can be represented as a weighted sum of agents' expected utility functions.¹ An example due to Diamond [5] illustrates that Harsanyi's assumptions on social preference may not be compelling.

Formalizing ideas found in Raiffa [15], Mongin [12, 13] pursues a similar program within Savage's [16] framework of subjective uncertainty. If all agents behave according to Savage's axioms, he shows that it is generally impossible for social preference to jointly satisfy Savage's axioms and the strong Pareto property with respect to individual preferences. Two natural responses to this negative conclusion present themselves, both of which were first suggested by Raiffa [15]. One is to condemn the strong Pareto property in a setting of subjective uncertainty. Gilboa, Samet, and Schmeidler [7] discuss this approach, see also Broome [3]. The other approach is to condemn Savage's axioms for social preference. Implicit in most examples which condemn one of these assumptions is the idea that social preference should satisfy the other of the assumptions. In the words of Raiffa, "These issues can be dramatized as a fight between Group Bayesians and Paretians" ([15], p. 234). For a related result, see Hylland and Zeckhauser [9].

Our approach is to understand the basic structure generating the impossibility results, by studying social and individual preferences at an axiomatic level. At an individual

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¹For more on Harsanyi's theorem, see Mongin and D'Aspremont [14] and Weymark [18].

level, Savage’s axioms are used to deliver a personal notion of probability. The primary axioms in his theory are P2 (*sure-thing principle*), P3 (*eventwise monotonicity*), and P4 (*weak comparative probability*).² In terms of empirical violations of Savage’s theory at an individual level, P2 is often criticized. We have no specific criticism against assuming P2 at a social level.

Instead, we discuss P3 and P4. We show by examples that *each* of these axioms conflicts with the strong Pareto property on its own. Thus, our examples generate impossibility results for a large class of social preferences (for example, social preferences that are probabilistically sophisticated, as defined by Machina and Schmeidler [11]). Informally, each of P3 and P4 carry with them notions of state-independence, which can be understood as the source of the impossibility results.

Our work concludes by studying the most well-known model which generically violates P3 and P4, the state-dependent expected utility model. Arguing that state-independence of social preference is the primary factor driving the impossibility results, Mongin [13] conducts a similar exercise in the Anscombe-Aumann [1] and Karni, Schmeidler, and Vind [10] frameworks. Our theorem gives a counterpart of these results in the Savage framework.

We provide a representation theorem for social preference conforming to the state-dependent model and which satisfies Pareto indifference with respect to individual preferences. Under Pareto indifference, social preference can be represented by a weighted sum of individual utility representations. This result is the natural analogue of Harsanyi’s theorem [8] in the Savage framework. Gilboa, Samet, and Schmeidler [7] obtain a related result by weakening the Pareto property so that it holds only when all agents in society agree about the likelihood of “relevant” events. Whereas we weaken Savage’s axioms as little as possible to obtain a possibility and representation result, they maintain Savage’s axioms and weaken the Pareto property as little as possible to obtain a possibility and representation result.

Section 2 describes the model and provides examples and propositions illustrating the conflict between Savage’s axioms and the Pareto properties. Section 3 provides a representation theorem in a state-dependent expected utility framework. Finally, Section 4 concludes.

2 The model and Savage’s axioms

Let (S, Σ) be a measurable space, where S is a set of **states** and Σ is a σ -algebra of **events**. We use s to denote a generic state and A or B to denote generic events. Let X be a set of **outcomes**. Define the set of (simple) **acts** \mathcal{F} as the set of finite-ranged Σ -measurable mappings $f : S \rightarrow X$. We use f or g to denote a generic act. For $x, y \in X$,

²The terminology is from Machina and Schmeidler [11].

$A \in \Sigma$, define $xAy \in \mathcal{F}$ as

$$xAy \equiv \left(\begin{array}{l} x \text{ if } s \in A \\ y \text{ if } s \in A^c \end{array} \right).$$

With a slight abuse of notation, $x \in X$ also denotes the act whose constant outcome is x .

Savage's axioms apply to binary relations \succeq on \mathcal{F} . Say that $A \in \Sigma$ is **null** for \succeq if for all $x \in X$ and all $f \in \mathcal{F}$,

$$f \sim \left(\begin{array}{l} x \text{ if } s \in A \\ f(s) \text{ otherwise} \end{array} \right).$$

For a set of agents $N \equiv \{1, \dots, n\}$, agent i 's preference is denoted by \succeq_i and social preference is denoted by \succeq_0 . Say that \succeq_0 **satisfies Pareto-indifference with respect to** $(\succeq_1, \dots, \succeq_n)$ if for all $f, g \in \mathcal{F}$, if for all $i \in N$, $f \sim_i g$, then $f \sim_0 g$. Say that \succeq_0 **satisfies the weak Pareto property with respect to** $(\succeq_1, \dots, \succeq_n)$ if for all $f, g \in \mathcal{F}$, if for all $i \in N$, $f \succ_i g$, then $f \succ_0 g$. Say that \succeq_0 **satisfies the strong Pareto property with respect to** $(\succeq_1, \dots, \succeq_n)$ if it satisfies Pareto indifference with respect to $(\succeq_1, \dots, \succeq_n)$ and if for all $i \in N$, $f \succeq_i g$, with strict preference for some $j \in N$, then $f \succ_0 g$.

A function $U : \mathcal{F} \rightarrow \mathbb{R}$ is a **subjective expected utility functional** if there exists a nonatomic, countably additive³ probability measure μ on (S, Σ) and a function $u : X \rightarrow \mathbb{R}$ such that for all $f \in \mathcal{F}$, $U(f) = \int_S u(f(s)) d\mu(s)$. Say a binary relation \succeq is a **subjective expected utility preference** if there exists a subjective expected utility functional U such that for all $f, g \in \mathcal{F}$,

$$f \succeq g \Leftrightarrow U(f) \geq U(g).$$

For any such U representing a nondegenerate subjective expected utility preference, the associated probability measure μ is unique, and the function u is unique up to positive affine transformation.

2.1 P3: Eventwise monotonicity

Savage's axiom P3 states:

P3 For all non-null $A \in \Sigma$, $x, y \in X$, $f \in \mathcal{F}$,

$$x \succeq y \Leftrightarrow \left(\begin{array}{l} x \text{ if } s \in A \\ f(s) \text{ if } s \in A^c \end{array} \right) \succeq \left(\begin{array}{l} y \text{ if } s \in A \\ f(s) \text{ if } s \in A^c \end{array} \right).$$

³Savage's theorem [16] only delivers a finitely additive probability measure. Arrow [2] discusses conditions which guarantee that the probability measure is countably additive.

A social preference that satisfies P3 cannot generally satisfy the weak Pareto property. The following example illustrates this point.

Example 1 Let $N \equiv \{1, 2\}$ and $X \equiv \mathbb{R}_+^2$. The set X represents distributions of wealth amongst the two agents. Let \succeq_0 be a social preference over \mathcal{F} . Assuming each agent likes more wealth to less and cares only about her own wealth, a plausible social ranking is $(0, 100) \sim_0 (100, 0)$. Let $A, A^c \in \Sigma$. Suppose agent one believes A is more likely than A^c and agent two believes A^c is more likely than A . Then for $i = 1, 2$,

$$(100, 0) A (0, 100) \succ_i (0, 100) A (100, 0).$$

By the weak Pareto property,

$$(100, 0) A (0, 100) \succ_0 (0, 100) A (100, 0).$$

Here, \succeq_0 violates P3. To see this, suppose \succeq_0 satisfies P3. As $(100, 0) \sim_0 (0, 100)$, and as \succeq_0 satisfies P3,

$$(100, 0) \sim_0 (100, 0) A (0, 100)$$

and

$$(0, 100) A (100, 0) \sim_0 (0, 100).$$

By transitivity, the ranking $(100, 0) \succ_0 (0, 100)$ holds, a contradiction.

2.2 P4: Weak comparative probability

Savage's axiom P4 states:

P4 For all $\bar{x}, \underline{x}, \bar{y}, \underline{y}$ such that $\bar{x} \succ \underline{x}$ and $\bar{y} \succ \underline{y}$, for all $A, B \in \Sigma$

$$\bar{x}A\underline{x} \succ \bar{x}B\underline{x} \Leftrightarrow \bar{y}A\underline{y} \succ \bar{y}B\underline{y}.$$

Example 2 shows that P4 is also incompatible with the strong Pareto property.

Example 2 Let $N \equiv \{1, 2\}$ and $X \equiv \mathbb{R}_+^2$, where the numerical quantities again represent monetary values. Each agent only cares about the amount she receives. Suppose again that agent one believes A is more likely than A^c , and agent two believes A^c is more likely than A . By strong Pareto, $(100, 0) \succ_0 (0, 0)$ and $(0, 100) \succ_0 (0, 0)$. Thus,

$$(100, 0) A (0, 0) \succ_1 (0, 0) A (100, 0)$$

and

$$(100, 0) A (0, 0) \sim_2 (0, 0) A (100, 0).$$

By the strong Pareto property,

$$(100, 0) A (0, 0) \succ_0 (0, 0) A (100, 0).$$

By a symmetric argument,

$$(0, 100) A (0, 0) \prec_0 (0, 0) A (0, 100).$$

These rankings clearly violate P4. The structure of this example is closely related to an example found in Gilboa, Samet, and Schmeidler [7]. It formally illustrates the reasoning which may lead one to reject the Pareto principle.

2.3 General incompatibility results

The following propositions are general versions of the examples.

The first proposition follows directly from Example 1. For any preference relation \succ satisfying P4, a **likelihood relation** \succeq^l on Σ can be defined as follows: $A \succeq^l B$ if and only if there exist $x, x' \in X$ such that $x \succ x'$ and $xAx' \succeq xBx'$.⁴

Proposition 1: Suppose \succeq_1 and \succeq_2 satisfy P4 and for all $i = 1, 2$, $A \succeq_i^l B \Leftrightarrow B^c \succeq_i^l A^c$. Suppose there exist $\bar{x}, \underline{x} \in X$ such that $\bar{x} \succ_1 \underline{x}$, $\underline{x} \succ_2 \bar{x}$, and $\bar{x} \sim_0 \underline{x}$. If \succeq_0 satisfies P3 and the strong Pareto property with respect to (\succeq_1, \succeq_2) , then $\succeq_1^l = \succeq_2^l$.

The next proposition illustrates a related point. If individuals' "beliefs" are different, then aggregation under the strong Pareto property and P3 is possible only if their "tastes" are the same.

Proposition 2: Suppose \succeq_1 and \succeq_2 satisfy P3 and that there exist $\bar{x}, \underline{x}, \bar{y}, \underline{y}$ such that $\bar{x} \succ_1 \underline{x}$ and $\bar{y} \succ_2 \underline{y}$. Suppose there exist $A \in \Sigma$ such that A is non-null for \succeq_1 and null for \succeq_2 , and $B \in \Sigma$ such that B is non-null for \succeq_2 and null for \succeq_1 . If \succeq_0 satisfies P3 and satisfies the strong Pareto property with respect to (\succeq_1, \succeq_2) , then $\succeq_1 \upharpoonright_X = \succeq_2 \upharpoonright_X$.

Proof: We first show that each of A and B is non-null for \succeq_0 . As \succeq_1 satisfies P3 and A is non-null for \succeq_1 , $\bar{x}A\underline{x} \succ_1 \underline{x}$. As A is null for \succeq_2 , $\bar{x}A\underline{x} \sim_2 \underline{x}$. By the strong Pareto property, $\bar{x}A\underline{x} \succ_0 \underline{x}$, so that A is non-null for \succeq_0 . The proof for B is symmetric.

Suppose the statement of the proposition is false. Thus, $\succeq_1 \upharpoonright_X \neq \succeq_2 \upharpoonright_X$, and without loss of generality, there exist $x, y \in X$ such that $x \succeq_1 y$ and $y \succ_2 x$. As \succeq_1 satisfies P3, $xAy \succeq_1 y$, and as A is null for \succeq_2 , $xAy \sim_2 y$. By the strong Pareto property, $xAy \succeq_0 y$. As A is non-null for \succeq_0 , by P3, $x \succeq_0 y$.

As \succeq_2 satisfies P3 and as B is non-null for \succeq_2 , $y \succ_2 xBy$, and as B is null for agent \succeq_1 , $y \sim_1 xBy$. By the strong Pareto property, $y \succ_0 xBy$. As B is non-null for \succeq_0 , by P3, $y \succ_0 x$. But we previously concluded that $x \succeq_0 y$, a contradiction. ■

The last proposition follows directly from Example 2.

⁴Propositions 1, 2, and 3 are stated for environments with only two agents. Similar results can be established for any number of agents, by partitioning the set of agents into two types, where each type has a preference relation corresponding to one of the two preference relations in the propositions.

Proposition 3: Suppose \succeq_1 and \succeq_2 satisfy P4 and for all $i = 1, 2$, for all $x, x' \in X$, $A, B \in \Sigma$,

$$x \sim_i x' \implies xAx' \sim_i xBx'.$$

Suppose there exist $\bar{x}, \underline{x}, \bar{y}, \underline{y} \in X$ such that $\bar{x} \succ_1 \underline{x}, \bar{x} \sim_2 \underline{x}, \bar{y} \sim_1 \underline{y}, \bar{y} \succ_2 \underline{y}$. If \succeq_0 satisfies P4 and the strong Pareto property with respect to (\succeq_1, \succeq_2) , then $\succeq_1^l = \succeq_2^l$.⁵

3 A possibility result

Each of P3 and P4 can be thought of as requirements of state-independence. Thus, in this section, we show that if social preference is not required to satisfy P3 and P4, Paretian aggregation is possible under the state-dependent expected utility model. To this end, say $U : \mathcal{F} \rightarrow \mathbb{R}$ is a **state-dependent subjective expected utility functional** if there exists a nonatomic, countably additive probability measure μ on (S, Σ) and a function $u : X \times S \rightarrow \mathbb{R}$ such that for all $x \in X$, $u(x, \cdot) : S \rightarrow \mathbb{R}$ is Σ -measurable and μ -integrable and for all $f \in \mathcal{F}$, $U(f) = \int_S u(f(s), s) d\mu(s)$. Say a binary relation \succeq is a **state-dependent subjective expected utility preference** if there exists a state-dependent subjective expected utility functional U such that for all $f, g \in \mathcal{F}$,

$$f \succeq g \Leftrightarrow U(f) \geq U(g).$$

For more information about state-dependent subjective expected utility preferences, see Wakker and Zank [17].

The probability measure component of a state-dependent subjective expected utility functional is not unique. Hence, we cannot refer to society's "beliefs" in an unambiguous way. This is an artifact of the incompatibility of P4 with Paretian aggregation.

Theorem 1: Suppose that for all $i \in N$, \succeq_i is a subjective expected utility preference represented by $U_i : \mathcal{F} \rightarrow \mathbb{R}$. Then \succeq_0 is a state-dependent subjective expected utility preference which satisfies Pareto-indifference with respect to $(\succeq_1, \dots, \succeq_n)$ if and only if there exists a vector $\lambda \in \mathbb{R}^N$ such that for all $f, g \in \mathcal{F}$,

$$f \succeq_0 g \Leftrightarrow \sum_N \lambda_i U_i(f) \geq \sum_N \lambda_i U_i(g).$$

Representations corresponding to stronger Pareto properties can similarly be derived using Lemma 1 of Appendix A and the general representation theorems of DeMeyer and Mongin [4]. Mongin [13] proves a related theorem using the added structure of the Anscombe-Aumann [1] framework. He also showed in [12] that if the probability

⁵We could replace the hypothesis $\bar{x} \succ_1 \underline{x}, \bar{x} \sim_2 \underline{x}, \bar{y} \sim_1 \underline{y}, \bar{y} \succ_2 \underline{y}$ with the hypothesis that $\bar{x} \succ_1 \underline{x}, \bar{x} \preceq_2 \underline{x}, \bar{x} \succ_0 \underline{x}, \bar{y} \preceq_1 \underline{y}, \bar{y} \succ_2 \underline{y}, \bar{y} \succ_0 \underline{y}$. The first of these hypotheses derives the social preference in the second hypothesis from the strong Pareto property.

measures of all agents and society can have atoms, then Theorem 1 does not hold in a Savage setup. Theorem 1 is closely related to Harsanyi’s original result [8], in that it is driven by the sure-thing principle (which is closely tied to the independence axiom of expected utility theory which drives Harsanyi’s result). In fact, Harsanyi’s Theorem should be viewed as a special case of our Theorem 1, where social preference satisfies Savage’s axioms and the likelihood relations of all agents and of society coincide.

The proof of Theorem 1 is in the Appendix. In order to prove Theorem 1, we establish in Lemma 1 that the utility possibilities set for a collection of state-dependent expected utility maximizers is convex, a fact which comes for free in the Anscombe-Aumann model. After establishing this, we apply a general representation theorem of DeMeyer and Mongin [4].

4 Conclusion

Mongin’s [12] negative results make clear that Paretian aggregation is incompatible with the expected utility model for social preference. Going further, he also suggests the idea that the problem with Paretian aggregation is the state-dependence of preferences satisfying Savage’s axioms [13]. Our first contribution is to work at a primitive level, identifying exactly which of Savage’s axioms are the source of the negative results. We determine that *two* of Savage’s axioms are incompatible with Paretian aggregation—P3 and P4. Our second contribution is to demonstrate the possibility of Paretian aggregation without P3 and P4 by providing a representation for social preference conforming to the state-dependent expected utility model.

5 Appendix—Proof of Theorem 1

Lemma 1 establishes that the utility possibility set for a collection of state-dependent expected utility maximizers is convex. This result relies on a generalization of Lyapunov’s Convexity Theorem, due to Dvoretzky, Wald, and Wolfowitz [6].

Lemma 1: Suppose (U_1, \dots, U_m) is a vector of state-dependent expected utility functionals. Then

$$\{(U_1(f), \dots, U_m(f)) : f \in \mathcal{F}\}$$

is convex.

Proof: We need to show that for all $f, g \in \mathcal{F}$, $\alpha \in [0, 1]$, there exists some $h \in \mathcal{F}$ such that for all $i = 1, \dots, m$, $U_i(h) = \alpha U_i(f) + (1 - \alpha) U_i(g)$.

Therefore, let $f^*, g^* \in \mathcal{F}$, $\alpha^* \in [0, 1]$.

Let $X(f^*, g^*) \equiv (\text{range}(f^*)) \cup (\text{range}(g^*))$. As f^*, g^* are simple acts, $|X(f^*, g^*)| < \infty$. Let $\mathcal{F}(f^*, g^*) \subset \mathcal{F}$ be the set of acts whose range lies in $X(f^*, g^*)$.

By assumption, for all $i = 1, \dots, m$, there exists $u_i : X \times S \rightarrow \mathbb{R}$ and μ_i on (S, Σ) such that $U_i(f) \equiv \int_S u_i(f(s), s) d\mu_i(s)$. For all $i = 1, \dots, m$, and for all $x \in X(f^*, g^*)$, let ν_i^x be a measure on Σ defined by

$$\nu_i^x(E) \equiv \int_E u_i(x, s) d\mu_i(s).$$

Then for all $i \in N$, $x \in X(f^*, g^*)$, ν_i^x is countably additive, nonatomic, and finite.⁶ By definition of the integral, for all $i \in N$, $f \in \mathcal{F}(f^*, g^*)$

$$U_i(f) = \sum_{x \in X(f^*, g^*)} \nu_i^x(f^{-1}(x)).$$

Let Π be the set of Σ -measurable ordered $X(f^*, g^*)$ -partitions. Formally, Π is defined as the set of functions $P : X(f^*, g^*) \rightarrow \Sigma$ satisfying *i*) $\bigcup_{X(f^*, g^*)} P(x) = S$ and *ii*) for all $x, y \in X(f^*, g^*)$ such that $x \neq y$, $P(x) \cap P(y) = \emptyset$.

Clearly, there is a bijection ψ between $\mathcal{F}(f^*, g^*)$ and Π , given by for all $f \in \mathcal{F}(f^*, g^*)$ and for all $x \in X(f^*, g^*)$, $\psi(f)(x) \equiv f^{-1}(x)$. In particular,

$$A \equiv \left\{ (\nu_i^x(P(x)))_{i,x} \subset \mathbb{R}^{m \times X(f^*, g^*)} : P \in \Pi \right\}$$

is equal to

$$B \equiv \left\{ (\nu_i^x(f^{-1}(x)))_{i,x} \subset \mathbb{R}^{m \times X(f^*, g^*)} : f \in \mathcal{F}(f^*, g^*) \right\}.$$

By Theorems 1 and 4 of Dvoretzky, Wald, and Wolfowitz [6], it follows that A is convex.⁷ Thus B is also convex.

By summing the columns of the elements of B , we obtain $\{(U_1(f), \dots, U_m(f)) : f \in \mathcal{F}(f^*, g^*)\}$. Convexity is preserved under this summation. Lastly, note that $\{(U_i(f^*))_i, (U_i(g^*))_i\} \subset \{(U_1(f), \dots, U_m(f)) : f \in \mathcal{F}(f^*, g^*)\}$. Thus, there exists some $h \in \mathcal{F}(f^*, g^*)$ such that for all $i = 1, \dots, m$, $U_i(h) = \alpha^* U_i(f^*) + (1 - \alpha^*) U_i(g^*)$. ■

⁶To see that ν_i^x is nonatomic, suppose that it is not. Then there exists some $E \in \Sigma$ such that $\nu_i^x(E) > 0$, and for all $F \subset E$, $\nu_i^x(F) \in \{0, \nu_i^x(E)\}$. Let $\{E_m\}_{m=1}^\infty$ be a sequence such that $E_1 = E$, and for all m , $E_m \subset E_{m-1}$, $\mu_i(E_m) = \frac{1}{2} \mu_i(E_{m-1})$, and $\nu_i^x(E_m) = \nu_i^x(E)$. By countable additivity, $\mu_i(\bigcap_{m=1}^\infty E_m) = 0$ and $\nu_i^x(\bigcap_{m=1}^\infty E_m) = \nu_i^x(E) > 0$, a contradiction. Finiteness follows as $u_i(x, \cdot)$ is μ_i -integrable.

⁷This follows as Theorems 1 and 4 of Dvoretzky, Wald, and Wolfowitz imply that $\left\{ (\nu_i^x(P(y)))_{(i,x),y} \subset \mathbb{R}^{(m \times X(f^*, g^*)) \times X(f^*, g^*)} : P \in \Pi \right\}$ is convex. The set A is a projection of this set on the subspace in which $x = y$.

We now prove the theorem.

Proof: It is obvious that if there exists a vector $\lambda \in \mathbb{R}^N$ such that for all $f, g \in \mathcal{F}$, $f \succeq_0 g \Leftrightarrow \sum_N \lambda_i U_i(f) \geq \sum_N \lambda_i U_i(g)$, then \succeq_0 satisfies Pareto indifference with respect to $(\succeq_1, \dots, \succeq_n)$. We will now show that it is a state-dependent subjective expected utility preference. If for all $f, g \in \mathcal{F}$, $f \sim_0 g$, then the claim is obvious. So, assume there exist $f', g' \in \mathcal{F}$ such that $f' \succ_0 g'$.

For all $i \in N$, U_i has two components, a utility index u_i , and a probability measure μ_i . Let $\mu \equiv \frac{\sum_N \mu_i}{|N|}$. Then μ is a probability measure defined on (S, Σ) . For all $i \in N$, μ_i is absolutely continuous with respect to μ . By the Radon-Nikodym Theorem, for all $i \in N$, there exists a Σ -measurable, μ -integrable function $h_i : S \rightarrow \mathbb{R}$ such that for all $f \in \mathcal{F}$,

$$\int_S u_i(f(s)) d\mu_i(s) = \int_S u_i(f(s)) h_i(s) d\mu(s).$$

Thus, for all $f \in \mathcal{F}$,

$$\sum_N \lambda_i U_i(f) = \int_S \left(\sum_N \lambda_i u_i(f(s)) h_i(s) \right) d\mu(s).$$

Let $u : X \times S \rightarrow \mathbb{R}$ be defined by

$$u(x, s) \equiv \sum_N \lambda_i u_i(x) h_i(s).$$

Thus, for all $x \in X$, $u(x, \cdot)$ is Σ -measurable and μ -integrable, and for all $f \in \mathcal{F}$,

$$\sum_N \lambda_i U_i(f) = \int_S u(f(s), s) d\mu(s).$$

Conversely, suppose that for all $i \in N$, \succeq_i is a subjective expected utility preference. Suppose that \succeq_0 is a state-dependent subjective expected utility preference which satisfies Pareto indifference with respect to $(\succeq_1, \dots, \succeq_n)$.

For all $i \in N$, let $U_i : \mathcal{F} \rightarrow \mathbb{R}$ be a subjective expected utility functional representing \succeq_i , and U_0 a state-dependent subjective expected utility functional representing \succeq_0 . By Lemma 1 above and Proposition 1 of DeMeyer and Mongin [4], there exist $\lambda \in \mathbb{R}^N$ and $K \in \mathbb{R}$ such that for all $f \in \mathcal{F}$,

$$U_0(f) = K + \sum_N \lambda_i U_i(f).$$

Thus, for all $f, g \in \mathcal{F}$, $f \succeq_0 g$ if and only if $K + \sum_N \lambda_i U_i(f) \geq K + \sum_N \lambda_i U_i(g)$. Equivalently, $f \succeq_0 g$ if and only if $\sum_N \lambda_i U_i(f) \geq \sum_N \lambda_i U_i(g)$. \blacksquare

References

- [1] F.J. Anscombe and R.J. Aumann, A definition of subjective probability, *Annals of Mathematical Statistics* **34** (1963), 199-205.
- [2] K. Arrow, "Essays in the Theory of Risk-Bearing," North-Holland, Amsterdam, 1970.
- [3] J. Broome, "Weighing Goods," Basil Blackwood, Cambridge MA, 1991.
- [4] B. DeMeyer and P. Mongin, A note on affine aggregation, *Economics Letters* **47** (1995), 177-183.
- [5] P. A. Diamond, Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility: a comment, *Journal of Political Economy* **75** (1967), 765-766.
- [6] A. Dvoretzky, A. Wald, and J. Wolfowitz, Relations among certain ranges of vector measures, *Pacific Journal of Mathematics* **1** (1951), 59-74.
- [7] I. Gilboa, D. Samet, and D. Schmeidler, "Utilitarian aggregation of beliefs and tastes," manuscript, 2001.
- [8] J.C. Harsanyi, Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility, *Journal of Political Economy* **63** (1955), 309-321.
- [9] A. Hylland and R. Zeckhauser, The impossibility of Bayesian group decision making with separate aggregation of beliefs and values, *Econometrica* **47** (1979), 1321-1336.
- [10] E. Karni, D. Schmeidler, and K. Vind, On state dependent preferences and subjective probabilities, *Econometrica* **51** (1983), 1021-1031.
- [11] M. Machina and D. Schmeidler, A more robust definition of subjective probability, *Econometrica* **60** (1992), 745-780.
- [12] P. Mongin, Consistent Bayesian aggregation, *Journal of Economic Theory* **66** (1995), 313-351.
- [13] P. Mongin, The paradox of the Bayesian experts and state-dependent utility theory, *Journal of Mathematical Economics* **29** (1998), 331-361.
- [14] P. Mongin and C. D'Aspremont, Utility theory and ethics, in "Handbook of Utility Theory, Volume 1, Principles," (S. Barbera, P. J. Hammond, and C. Seidl, Eds.), Kluwer Academic Publishers, London.
- [15] H. Raiffa, "Decision Analysis," Addison-Wesley, Reading, MA, 1968.
- [16] L.J. Savage, "The Foundations of Statistics," John Wiley, New York, 1954.
- [17] P.P. Wakker and H. Zank, State dependent expected utility for Savage's state space, *Mathematics of Operations Research* **24** (1999), 8-34.

- [18] J. Weymark, A reconsideration of the Harsanyi-Sen debate on utilitarianism, *in* “Interpersonal Comparisons of Well-Being,” (J. Elster and J.E. Roemer, Eds.) Cambridge University Press, Cambridge, 1991.