Communications

Transition Radiation Caused by a Chiral Plate

NADER ENGHETA AND ALAN R. MICKELSON

Abstract—**A simple calculation is made of the electromagnetic field radiated due to a charged particle traversing a plate of chiral material. The transition radiation from this chiral plate is found to differ from the usual dielectric transition radiation. Discussion is presented placing in evidence the characteristics of the radiation and comments are made concerning the possible applicability of the transition radiation mechanism.**

I. INTRODUCTION

A chiral plate is a plate comprised of a chiral medium, i.e., a medium endowed with handedness. As is discussed in [1], and several references cited therein, such a medium may be constructed of macroscopic (molar) chiral objects (all of the same handedness) embedded in a dielectric. Such well-known macroscopic objects as irregular tetrahedrons, Möbius strips, golf clubs or (metallic-coated) gloves could be employed as the chiral objects in such a construction. Lindman selected wire helices for the chiral objects in his verification [2] of the chirality (operationally defined as optical activity) of such a composite medium. Although the ultimate applicability of composite chiral media to problems such as electromagnetic shielding, remote sensing and improvement of antenna radiation through use of artificial dielectrics has yet to be assayed, the ease of construction of such media as well as their peculiar polarization characteristics justifies further study of composite chiral media.

Although Cerenkov radiation had been experimentally observed [3], [4] and had had its basic mechanism theoretically examined [5] by the mid-1930's, it was not until after the second world war that the closely related phenomenon of transition radiation was considered by Ginzburg and Frank [6]. In spite of the possible applications of this radiative mechanism, the subject has received attention primarily in the Soviet Union. That this is the case is indicated by the extensive reference lists included in review articles [7]-[9] and relevant section of Ter-Mikhaelian's book [10].

Transition radiation refers to the radiation produced by a charged particle which traverses a region endowed with heterogeneous constitutive properties. Therefore a plate of dielectric material illuminated by a beam of charged particles comprises a source of electromagnetic radiation [11]. This source is continuously tunable through variation of the mean velocity of the particle beam. Although the consideration of a continuous medium would seem to limit the validity of the concept of this radiation mechanism to a regime of small velocities and low frequencies in which the microscopic nature of the scatterer is not probed, discussions in the introduction to [10] show that the concept is applicable over broad frequency and velocity ranges when the scatterer is a condensed medium.

Although the transition radiation emitted at the interface of a gyrotropic medium (medium exhibiting optical activity due to its chirally symmetric molecular structure) and vacuum has been calculated [12], the tunable source problem of a chiral (or gyrotropic) plate has yet to be addressed. As is evidenced by recent work on the free electron laser [13], tunable sources of radiation (especially of submillimeter waves) are currently of interest to the scientific community. As a chiral plate and charged particle beam comprise a continuously tunable source, we wish to give attention to the salient features of the radiation emitted by such a configuration.

II. THE CHIRAL PLATE

We now consider the interaction of a charged particle with a plate of chiral medium with consideration to the emitted transition radiation. The plate of thickness l is placed normally to the particle's momentum vector $\mathbf{P}$, which is taken to be the z-axis as in Fig. 1(a). This figure also illustrates the propagation vector $\mathbf{k}$ of the radiated electromagnetic wave which makes an angle θ with the z-axis. Fig. 1(b) illustrates the plate location relative to the coordinate system. The constitutive relations for this heterogeneous space can then be written in the form [14] (in SI units, i.e., where ϵ_0 and μ_0 are the free space permittivity and c is the vacuum velocity of light).

$$\left.\begin{aligned}\mathbf{D} &= \epsilon_0\mathbf{E}\\ \mathbf{H} &= \frac{1}{\mu_0}\,\mathbf{B}\end{aligned}\right\}\qquad |z| > \frac{l}{2} \tag{1}$$

$$\left.\begin{aligned}\mathbf{D} &= \epsilon\mathbf{E} \pm i\chi_c c\mathbf{B}\\ \mathbf{H} &= \frac{1}{\mu}\mathbf{B} \pm i\sqrt{\frac{\epsilon_0}{\mu_0}}\,\chi_c\mathbf{E}\end{aligned}\right\}\qquad |z| < \frac{l}{2} \tag{2}$$

where the quantities ϵ, μ and χ_c are interrelated as in [1] and in particular

$$\epsilon = \epsilon_0(1 + \chi_e) \tag{3a}$$

$$\mu = \frac{\mu_0}{1 - \chi_M} \tag{3b}$$

where the χ_e, χ_M and χ_c are, respectively, the electric, magnetic, and cross susceptibilities of the chiral medium. For simplicity the medium is considered to be a weak scatterer in the sense that

$$0(\chi_e) \sim 0(\chi_M) \sim 0(\chi_c) \sim 0(\chi) \tag{4}$$

where

$$\chi kl \ll 1 \tag{5}$$

where 0 denotes the ordering function, χ a maximal value of susceptibility, and k a maximal value for the magnitude of the propagation vector of the emitted radiation. The charged par-

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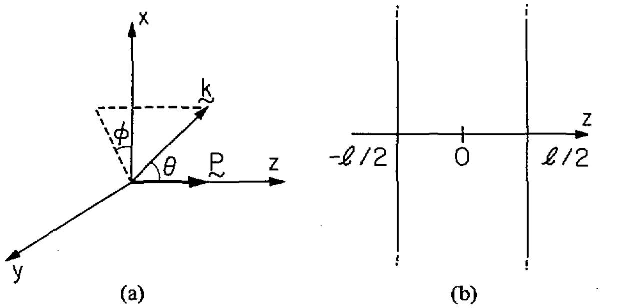

Fig. 1. Coordinates employed in the calculation of the radiated electromagnetic field. (a) Directions of P, the incident particle direction and K, the emitted wave direction. (b) Location of the chiral plate in the coordinate system of (a).

ticle is considered to have a nonrelativistic velocity $v \ll c$ such that only first-order quantities in the velocity need be considered. This "Born approximation" approach is justifiable for two reasons. Firstly, the purpose here is to uncover certain salient features of the radiation process, rather than to burden the reader with complications concerning the details of the relativistic radiation pattern. Secondly, as a chiral medium can be constructed as an artificial dielectric, such a medium could be constructed to be a weak scatterer.

With the above considerations one can replace the particle by an effective dipole moment and consider the radiation to be the result of an induced dipole moment in the medium. For the problem at hand, the effective fields can be written in the form (see Appendix for details)

$$\mathbf{E} = -\frac{ie}{\omega\epsilon_0}\delta(x)\delta(y)e^{i\frac{\omega}{v}z}\hat{e}_z \tag{6a}$$

$$\mathbf{B} = 0\left(\left(\frac{v}{c}\right)^2\right) \tag{6b}$$

where $\mathbf{E}(\mathbf{B})$ is the Fourier component of the electric (magnetic) field vector at angular frequency ω and where $\hat{e}_z$ is the unit vector in the z direction. It follows from (2) and (3) that the polarization density vector $\mathbf{p}$ and magnetization density vector $\mathbf{m}$ in the plate are expressible

$$\mathbf{p} = \epsilon_0\chi_e\mathbf{E} \pm i\chi_c c\mathbf{B} \tag{7a}$$

$$\mathbf{m} = \frac{-1}{\mu_0}\chi_m\mathbf{B} \mp i\sqrt{\frac{\epsilon_0}{\mu_0}}\chi_c\mathbf{E} \tag{7b}$$

where the $+(-)$ in (7a) and $-(+)$ in (7b) refer to a right- (left-) handed medium. Combining (6) and (7), one finds

$$\mathbf{p} = -i\frac{\chi_e e}{\omega}\delta(x)\delta(y)e^{i\frac{\omega}{v}z}\hat{e}_z \tag{8a}$$

$$\mathbf{m} = \mp\frac{\chi_c}{\sqrt{\epsilon_0\mu_0}}\frac{e}{\omega}\delta(x)\delta(y)e^{i\frac{\omega}{v}z}\hat{e}_z\ . \tag{8b}$$

One can calculate the far field of scattered radiation by using the familiar dipole radiation formulas [15]

$$E_\theta(\omega, \mathbf{r}) = -\frac{k^2e^{ikr}}{4\pi\epsilon_0 r}P_z\sin\theta \tag{9a}$$

$$E_\phi(\omega, \mathbf{r}) = \frac{\eta k^2e^{ikr}}{4\pi r}M_z\sin\theta \tag{9b}$$

where

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \tag{10}$$

is the impedance of free space, and P_z and M_z are defined by

$$P_z = \int_S p_z(r')e^{-jkr'}\,d^3r' \tag{11a}$$

$$M_z = \int_S m_z(r')e^{-ikr'}\,d^3r' \tag{11b}$$

where S denotes the source region, e.g., the region in which the beam and plate overlap. Note that the kr' in the exponent is negligible compared with $\omega/vz = k/\beta z$ in the exponent. Therefore, ignoring e^{ikr}, the operations indicated in (11) can be performed to obtain (9) in the form

$$E_\theta(\omega, x) = i\frac{ev\chi_e}{2\pi rc^2\epsilon_0}\sin\theta\sin\frac{kl}{2\beta}e^{ikr} \tag{12a}$$

$$E_\phi(\omega, x) = \mp\eta\frac{ev\chi_c}{2\pi rc}\sin\theta\sin\frac{kl}{2\beta}e^{ikr}. \tag{12b}$$

It is interesting to note that in the case of a chiral medium the emitted radiation is elliptically polarized, with degree of ellipticity independent of propagation direction. This is in contradistinction to the case of a dielectric plate, in which the radiation is linearly polarized, as can be seen from setting χ_c equal to zero in (12).

To conform with standard usage one wishes to compute the energy spectral density of the radiation propagating along a unit vector $\hat{n}$ in a solid angle Ω, denoted by $I(\hat{n}, \Omega)$, which can be defined by

$$\int_\Omega\int_0^\infty d\omega\, d\Omega I(\eta, \Omega) = \int_\Omega\int_{-\infty}^\infty dtr^2\, d\Omega\mathbf{S}(t)\cdot\hat{n} \tag{13}$$

where $\mathbf{S}$ is the real Poynting vector, defined by

$$\mathbf{S}(t) = \mathbf{E}(t, \mathbf{r})x\mathbf{H}(t, \mathbf{r}). \tag{14}$$

With identifications of (12), (13) one finds the result

$$I_\theta = \frac{\eta}{4\pi^3}e^2\chi_e{}^2\beta^2\sin^2\theta\sin^2\frac{kl}{2\beta} \tag{15a}$$

$$I_\phi = \frac{\eta}{4\pi^3}e^2\chi_c{}^2\beta^2\sin^2\theta\sin^2\frac{kl}{2\beta}. \tag{15b}$$

These results are easily generalizable to the case of oblique particle beam incidence. As is well known, a particle with subluminal velocity in a homogeneous medium does not radiate. (Superluminal velocity is eliminated from the consideration here as the nonrelativistic and weakly inhomogeneous assumption preclude it.) However, a charge which moves tangentially to the interface (in the x-y plane) is moving with subluminal velocity in a homogeneous medium (to first order in v/c). Breaking up the particle's motion into components, one sees that the only component of the particle's velocity to contribute to the transition radiation is that component normal to the interface. Therefore, (15) can be generalized to this case of oblique incidence by simply replacing β by the component of β normal to the plate. Denoting the angle the particle beam makes with the normal to the plate by ψ, one can simply make the replacement $\beta \to \beta\cos\psi$ in (15). With this substitution one finds

$$I_\theta = \frac{\eta}{4\pi^3} e^2 \chi_e{}^2 \beta^2 \cos^2 \psi \sin^2 \theta \sin^2 \frac{kl}{2\beta \cos \psi} \tag{16a}$$

$$I_\phi = \frac{\eta}{4\pi^3} e^2 \chi_c{}^2 \beta^2 \cos^2 \psi \sin^2 \theta \sin^2 \frac{kl}{2\beta \cos \psi}. \tag{16b}$$

In the case where χ_c and χ_m vanish (a dielectric plate), it is readily seen that (16) agrees with the results of [10] and [15] in the low velocity (small β) limit.

III. DISCUSSION

There are several notable features about the transition radiation process contained in (16). Firstly, it is evident that the chirality of the plate increases the radiated energy in all directions and at all frequencies by a factor of $(1 + \chi_c{}^2/\chi_e{}^2)$. The increase comes about due to the appearance of a second linear polarization state of strength $\chi_c{}^2/\chi_e{}^2$ that is in quadrature with the linear polarization state generated by a simple dielectric plate. A second feature of interest is the frequency spectrum of the process. The spectrum exhibits a sinusoidal behavior with respect to increasing frequency. The frequencies at which the spectrum achieves maxima are defined by

$$f_N = (2N + 1)f_0 = (2N + 1)v/l \tag{17}$$

or in terms of wavelength

$$\lambda_N = \frac{l}{(2N + 1)\beta}. \tag{18}$$

Although (16) indicates that at each frequency f_N the spectrum should achieve the same maximal value, physical considerations somewhat modify this picture. As the source of the transition radiation can be considered to be a thin filamentary current, results applicable to the theory of thin wire antennas should also be applicable in considering the radiation from this source. In this case, we would expect that the frequencies corresponding most closely to the half-wave dipole length of the plate would radiate most strongly. Higher frequencies correspond to small antenna lengths and therefore nonresonant behavior, whereas lower frequencies correspond to weaker antenna resonances. Also, the wide frequency separation $(2f_0)$ between successive maxima should allow a desired frequency component to be electrically selected from the received spectrum. A third feature of interest is the radiation pattern of the plate, which is formally identical to that of a z-directed dipole antenna. The radiation pattern dictates the direction of maximal radiation is well off the beam direction allowing the radiation to be easily detectable. This ease of detectability should also extend to the regime of relativistic velocities. In this limit the pattern should be modified by the realativistic beaming effect [17], which should narrow the beam to an effective solid angle of approximately $2\sqrt{1-\beta^2}$ and cause the angle of maximum radiation intensity to lie at an angle of approximately $\sqrt{1-\beta^2}$. However, secondary maxima could appear due to the Cerenkov effect. It is known that in the case of a charge incident on a dielectric with dielectric constant sufficiently high so that the particle's velocity is superluminal within the medium the transition radiation one calculates contains the Cerenkov radiation [18], [19], (i.e., it is inextricably intertwined). But the angle of the Cerenkov radiation will in general be different from the angle of the transition radiation and therefore will show up as secondary maxima. Other modifications that will be present in the relativistic regime are somewhat harder to predict, in part due to the fact that effects due to χ_m, absent in (16), come into play in the first order of β in the Cerenkoѷ effect, and to the second order in β in relativistic transition radiation. As is evident from the work of Pafomov [20], the properties of the transition radiation generated by a ferrodielectric plate are quite different from those generated by the simple dielectric plate.

IV. CONCLUSION

The use of a chircal medium can enhance the quantity of radiation generated by a dielectric plate. The increase is effected through the generation of a quadrature, orthogonal polarization state which tansforms the linearly polarized radiation of the simple dielectric plate to an elliptically polarized state. If chiral media with $\chi_c = \chi_e$ could be constructed, however, purely circular polarization could be generated and the resultant radiated energy would be double that generated by simple dielectric media.

Although the frequency spectrum of a dielectric plate is, in general, broadband, the spectrum is comprised of widely spaced maxima separated by nulls. A given frequency maxima can be continuously tuned through variation of the beam velocity as is described by (18). By selecting a given frequency maxima and varying the beam velocity it is therefore possible to employ a chiral plate and particle beam as a continuously tunable source of circularly polarized radiation.

The measurement of the transition radiation generated by plate could be used as a nondestructive test of the properties of the chiral medium. As is manifest in (17)

$$I_\varphi(\omega)/I_\theta(\omega) = \chi_c{}^2(\omega)/\chi_e{}^2(\omega). \tag{20}$$

Evident in (20) is the fact that the degree of chirality (at a frequency ω) of a chiral material can be determined from a single measurement. Such a simple experimental technique could prove useful in the design and fabrication of electromagnetic devices employing chiral media.

APPENDIX

The effective fields of (7) can be obtained by a minor modification of an argument in [4]. To summarize, the current of a constant velocity electron can be expressed in the form

$$\begin{aligned} \mathbf{j}(x, y, z, t) &= -ev\delta(x)\delta(y)\delta(z - vt)\hat{e}_z \\ &= -e\delta(x)\delta(y)\delta\left(\frac{z}{v} - t\right). \end{aligned} \tag{A1}$$

Defining the Fourier transform pair by

$$\hat{f}(\omega) = \int f(t) e^{i\omega t}\, dt \tag{A2a}$$

$$f(t) = \frac{1}{2\pi} \int \hat{f}(\omega) e^{-i\omega t}\, d\omega \tag{A2b}$$

one can find the Fourier transform of the current to be

$$\mathbf{j}_\omega(x, y, z) = -e\delta(x)\delta(y) e^{i\frac{\omega}{v}z} \hat{e}_z. \tag{A3}$$

Recalling that the electric dipole moment can be defined by

$$\mathbf{j}(t) = \frac{d\mathbf{p}}{dt}. \tag{A4}$$

One notes that the dipole moment density of the charge is expressible in the form

$$\mathbf{p}_\omega = \frac{-\mathbf{j}_\omega}{i\omega} = -i\frac{e}{\omega}\delta(x)\delta(y)e^{i\frac{\omega}{v}z}\hat{e}_z. \tag{A5}$$

The total dipole moment of the heterogeneous space defined by (1)-(3) (to 0th order in v/c) can therefore be written in the form

$$\mathbf{p}_\omega = \int_{-\infty}^{\infty} \mathbf{p}_\omega \, dx\, dy\, dz + \chi_e \int_{-l/2}^{l/2} \mathbf{p}_\omega \, dx\, dy\, dz \tag{A6}$$

where the second integral on the right side of (A6) is due to the extra induced dipole moment within the medium. The first integral on the right side of (A6) is easily shown to have the property that

$$\int_{-\infty}^{\infty} \mathbf{p}_\omega \, dx\, dy\, dz \propto \delta(\omega) \tag{A7}$$

and therefore involves only the charge's static field. Therefore, the transition radiation involves only the second integral of (A6). Using (2), therefore, gives that the effective $\mathbf{E}$ field which is responsible for transition radiation can be written in the form

$$\mathbf{E}_{\text{eff}} = \mathbf{p}_\omega/\epsilon_0 = -\frac{ie}{\epsilon_0\omega}\delta(x)\delta(y)e^{i\frac{\omega}{v}z}\hat{e}_z. \tag{A8}$$

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The Effect of Reference's Phase on Radio-Frequency Holographic Imaging

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Abstract—**As electromagnetic waves provide a means of seeing through optically opaque dielectrics, the technique of holographic imaging in the Fresnel zone furnishes a potential application to nondestructive testing. The technique also provides an ability of short-distance continuous wave (CW) imaging radar of moving targets. In this communication, the effect of the phase of the electronic reference on the imaging characteristics is experimentally confirmed. The effect of the reference's phase on the image quality is also examined.**

I. INTRODUCTION

In radio-frequency holographic imaging, the image can be reconstructed from digitized hologram data by using a computer. In this case, the phase relation between the object and reference waves in the hologram plane has an important effect on the characteristics of the reconstructed image. This problem has been pointed out [1], but detailed discussions have not been found.

In computer reconstruction, as opposed to optical reconstruction, it is possible to take only either the real or the imaginary part of the complex field in the reconstructed plane. In the case of Gabor-type holography [2], where a fixed relation between the object and reference waves exists, the object signal appears only in the real part of the reconstructed field. In the case of radar-type holography [3]-[5], where the electronic reference is employed, the phase of reference is adjusted at will with respect to the phase of the object wave, and the reconstructed image field rotates in the complex plane with changing the reference's phase. It is the purpose of this communication to confirm experimentally the effect of the reference's phase on the characteristics of Fresnel-zone imaging.

II. ANALYSIS

A familiar configuration of Gabor-type holography system is shown in Fig. 1. An unit amplitude plane wave $\exp[j(\omega t - kz)]$ illuminates the conducting two-dimensional object $G(x, y)$ defined as

$$\begin{aligned} G(x,y) &= 1, \qquad \text{inside of} \quad g(x,y) = 0, \\ G(x,y) &= 0, \qquad \text{outside of} \quad g(x,y) = 0, \end{aligned} \tag{1}$$

where $g(x, y) = 0$ the peripheral contour of the object G. The field diffracted by the object is easily obtained by applying Babinet's principle [6]: the field is obtained by subtracting

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