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MINORITIES AND STORABLE VOTES

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Minorities and Storable Votes*

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Abstract

The paper studies a simple voting system that has the potential to increase the power of minorities without sacrificing aggregate efficiency. *Storable votes* grant each voter a stock of votes to spend as desired over a series of binary decisions. By accumulating votes on issues that it deems most important, the minority can win occasionally. But because the majority typically can outvote it, the minority wins only if its strength of preference is high and the majority's strength of preference is low. The result is that with storable votes, aggregate efficiency either falls little or in fact rises. The theoretical predictions of our model are confirmed by a series of experiments: the frequency of minority victories, the relative payoff of the minority versus the majority, and the aggregate payoffs all match the theory.

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1 Introduction

Recent decades have witnessed great efforts at designing democratic institutions, at many levels. New constitutions were created in much of Eastern Europe and the former Soviet Republics, international organizations such as the European Union and the World Trade Organization have been evolving rapidly, and many developing countries have moved from autocratic regimes to regimes based on elected representation with majoritarian principles.

While majoritarian principles may provide a solid foundation for democracy, there are imperfections. This paper focuses on one particular imperfection, which has presented a challenge to designers of democratic institutions for centuries: the *tyranny of the majority*, or the risk of excluding minority groups from representation. At least since Madison, Mill, and Tocqueville, political thinkers have argued that a necessary condition for the legitimacy of a democratic system is for no group with acceptable goals to be disenfranchised. The dangers posed by the tyranny of the majority are not of pure academic interest, as the threat or reality of civil wars around the world makes painfully clear.

According to a leading constitutional law textbook: "This issue is one of the most difficult in political and constitutional theory: how to design political institutions that both reflect the right of "the people" to be self-governing and that also ensure appropriate integration of and respect for the interests of political minorities" (Issacharoff, Karlan and Pildes, 2002, p.673). In the history of US constitutional law, ensuring fair representation to each group is seen as the crucial second step in the evolution of democratic institutions, after granting the franchise: once all individuals are guaranteed the right to participate in the political process, the question becomes the appropriate weights given to each group's political interest. The core of the difficulty is that the two goals seem inherently contradictory.

One possible remedy is recourse to the judiciary system: it amounts to guaranteeing basic rights in the fundamental laws of the country and appealing to the courts when such rights are imperiled. Although this approach can prevent abuses, it does not address the subtler problem of ensuring minority representation when the preferences of the minority, as opposed to its basic rights, are sytematically neglected. For this, the correct design of the political institutions is required. In this paper, we approach the problem from the perspective of voting theory, and propose a simple voting mechanism that, without violating the basic principle of "one-person one-vote," allows the minority to win occasionally. The mechanism is not based on supermajorities, avoiding the costs of inertia and inefficiency they can entail, nor on geographical partitions, with the inevitable arbitrariness and instability of redistricting. But before describing our solution to the tyranny of the majority problem some clarification is useful.

The topic of *minorities* is felt so intensely, and the terms are so emotionally loaded that there is a need to be scrupulously clear in terminology. We define a *minority* as a clearly identifiable group characterized by two features: first, a small numerical size, smaller than the majority; second, preferences that are systematically different from the preferences of the majority. Thus, a minority in this paper is a *political* minority, which may, but need not, correspond to a minority according to racial, ethnic, religious or any other type of considerations. In terms of political decisions, what matters are the coherent and idiosyncratic preferences of the group, as opposed to its sense of identity.

The need to represent minority interests is usually argued on the basis of fairness and legitimacy. But efficiency considerations can be relevant too, and it is on grounds of efficiency that we defend our case here. Chwe (1999) took a similar perspective, arguing for granting "special" voting power to the minority to ensure its participation when voting aggregates diffuse information. We base our analysis on private value considerations - voting in our model aggregates divergent preferences, not diffuse information. But the efficiency rationale remains. A simple example will illustrate why.

Suppose there are just two groups in a polity comprised of 100 citizens. Group A has 55 members and group B has 45 members. There are 3 proposals on the table. All citizens in group A have identical preferences and strictly prefer to pass all proposals; all citizens in group B have identical preferences and strictly prefer the status quo on all 3 issues. Thus, group B fits our definition of a minority. Table 1 gives a specific utility function for each member on each issue, and preferences are assumed to be additive. For each citizen, the utility of the less preferred option is normalized to 0.

Issue	$U_A(pass)$	$U_A(sq)$	$U_B(pass)$	$U_B(sq)$
1	3	0	0	1
2	2	0	0	2
3	1	0	0	3

Note that the *intensity* of preferences varies across the issues, and on a given issue the preference intensity for a group A member may be different from the intensity of a group B member. That is, some issues are "more important" to one group than to the other group - issue 1 is important to group A but not to group B, and issue 3 is important to group B but not to group A.

Now consider what would happen with simple majority rule when issues are decided independently: since group A has a majority, all three proposals pass. Indeed, even if there were a million different issues, group A would always have a majority on all issues, so the B citizens are effectively disenfranchised - the outcome is exactly the same as it would be in a political system where only A citizens were allowed to vote.

Why is this outcome undesirable? First, equity considerations demand that the minority be able to win on at least some issues. But in addition, from a purely utilitarian standpoint, there are plausible welfare criteria according to which the outcome is socially inefficient. In our example, if each individual is treated equally and decisions are evaluated ex ante, before membership into the groups is known, the status quo should prevail on issue 3. Thus, the tyranny of the majority imposes costs both in terms of equity and in terms of efficiency. The equity problem stems from the existence of a smaller group whose preferences are systematically in the *opposite direction* of the larger group's preferences. The efficiency problem stems from differences in the *strength* of preferences of the two groups. But nothing fundamental depends on all citizens in a group having the same intensity of preferences on every issue, a simplification we adopted here to keep the example transparent.[1]

Can the tyranny of the majority problem be solved? In our example, unanimity or any biting supermajority requirement would produce a stalemate and prevent any decision being made. Any solution must deviate from issue-by-issue simple majority voting system. An immediate possibility might be vote trading or some corresponding log-rolling scheme: members of one group could trade their vote on one issue in exchange for votes on other issues. But, in the simple example we constructed above, there are no gains across groups, because every A citizen is already winning on all issues. Any system that allows the minority group to win on even one issue will make all A citizens worse off, and thus would not emerge spontaneously through vote trading. With the perfect correlation of preferences we have posited above, an explicit institution "re-enfranchising" the minority is necessary.

Consider then, endowing every voter with an initial stock of votes, and rather than requiring voters to cast exactly one vote on each issue, allowing them to lump their votes together, casting "heavier" votes on some issues and "lighter" votes on other issues. It is this voting mechanism, called *storable votes*, that we study in this paper. As we prove below, storable votes allow the minority to win some of the time, and in particular, to win when its preferences are most intense. And because the majority generally holds more votes, it is in a position to overrule the minority if it cares to do so: the minority can win only those issues over which its strength of preferences is high *and*, at the same time, the majority's preference intensity is weak. But these are exactly the issues where the minority "should" win from an efficiency viewpoint: the equity gains resulting from the possibility of occasional minority's victory need not come at a cost to aggregate efficiency. In fact, in most of the examples we study in this paper, we find that standard economic measures of aggregate efficiency rise with storable votes. The main contribution of this paper then is not to suggest a new reason to increase minority's representation but to propose a specific voting scheme with the potential to achieve this goal even in the case of a systematic minority, when other voting mechanisms would fail, and to do so without violating the equal treatment of all voters.

Storable votes were initially proposed in Casella (2005) (and independently, in a somewhat different form, in Hortala-Vallve, 2004) in symmetric environments. The desirable efficiency properties of storable votes remain true there, because the basic principle of casting more votes over decisions that matter more continues to apply. The implication is that the probability of obtaining

[1]Nothing fundamental depends on the direction of preferences within the group being perfectly correlated either - there may be some conflicting preferences within groups. We have maintained the assumption throughout the paper, both to avoid complications and to capture the focus of minority advocates on cohesive groups.

the desired outcome shifts away from decisions that matter little and towards decisions that matter more, with positive welfare effects. Storable votes are a particularly natural application of the idea that preferences can be elicited by linking independent decisions through a common budget constraint, an idea that can be exploited quite generally, as shown by Jackson and Sonnenschein (forthcoming).[2] But from a practical point of view it is in the application to minorities that storable votes seem particularly promising, because the potential to increase efficiency is matched by desirable properties on equity grounds. Even when the efficiency gains are not large, the equity considerations loom large.

The observation that storable votes can be useful in increasing minority representation is not surprising. One existing voting system similar to storable votes is *cumulative voting*, a mechanism used in single multi-candidate elections. It grants each voter a budget of votes, with the proviso that the votes can spread or concentrated on as many or few of the candidates as the voter wishes. Cumulative voting has been advocated for the protection of minority rights (Guinier, 1994) and has been recommended by the courts to redress violations of fair representation in local elections (Issacharoff, Karlan and Pildes, 2002). There is evidence, theoretical (Cox, 1990), experimental (Gerber, Morton and Rietz, 1998), and empirical (Bowler, Donovan and Brockington, 2003) that cumulative voting does indeed work in the direction intended. The storable votes mechanism is different in that it is applies to a series of independent binary decisions, but the motivation is similar.

The desirable properties of storable votes are features of the equilibrium of the resulting voting game – they emerge if every voter chooses the correct number of votes, given what he rationally expects others to do. But, in practice there is a need to consider the robustness of the mechanisms. Could the outcome be much worse if voters made mistakes? This is an appropriate concern here because the storable votes game is quite complex: voters need to trade-off the different probabilities of casting the pivotal vote along the full logical tree of possible scenarios, a task further complicated by coordination problems within the two groups. If actual voters were confronted with the problem, what type of decisions would they make?

The second part of the paper presents the results of a set of experiments. In our experiments, the minority does indeed win with some frequency, and both the minority payoff and the aggregate efficiency of the mechanism are quite close to the theoretical predictions. This is less true of individual strategies: the experimental subjects deviate frequently from the equilibrium number of votes. What subjects do consistently, though, is to cast more votes when valuations are higher, a behavior that appears sufficient to take them most of the way towards their equilibrium payoffs. These conclusions are qualified by the different cost

[2] Jackson and Sonnenschein propose a specific mechanism that converges to the first best allocation as the number of decisions grows large. The mechanism allows individuals to assign different priority to different actions but constrains their choices in a tighly specified manner. The design of the correct menu of choices offered to the agents is complex, but the mechanism achieves the first best. Storable votes are simple but in general do not achieve the first best.

of mistakes faced by majority members, who are likely to win anyway, and minority members, whose deviations are particularly costly (and rarer in the data). Previous experiments with storable votes in symmetric environments (Casella, Gelman and Palfrey, forthcoming) had found a similar robustness of efficiency properties to strategic mistakes, but the introduction of minorities complicates the game very significantly. The ability of experimental subjects on the minority side to appropriate a large share of the surplus available to them was far from foretold, and we find it an encouraging sign of the practical viability of the mechanism.

The paper proceeds as follows. The next section presents the basic model. In section 3, we present theoretical results about the possibility of minority victories and its effect on efficiency under storable votes. Section 4 describes the experimental design and section 5 the experimental results. We conclude in section 6. The Appendix discusses some of the proofs.

2 The Model

A committee with n members meets for T consecutive periods to vote over a series of binary proposals $\{P_1, ..., P_T\}$, each of which can either pass or fail. Voter i's preferences over proposal P_t are summarized by a valuation $\mathrm{v}_{it} \in \mathbb{R}$. A positive valuation means that the voter is in favor of the proposal, a negative valuation means that the voter is against, and voter i's payoff from each proposal is given by $|\mathrm{v}_{it}| \equiv v_{it}$ if the outcome of the vote is as he desires, and 0 otherwise. Thus voter i's utility function has the form:

$$U_i(P_1, ...P_T) = \sum_{t-1}^{T} u_{it}(P_t)$$

where

$$\begin{aligned} u_{it}(P_t) &= v_{it} \text{ if}\{.\mathrm{v}_{it} > 0 \text{ and } P_t \text{ passes} \mathrm{v}_{it} < 0 \text{ and } P_t \text{ fails} \\ &= 0 \text{ otherwise} \end{aligned}$$

The magnitude of the valuation, v_{it}, is called the *intensity* of preferences of voter i on proposal t. The profile of intensities, v= $(\mathrm{v}_{11}, ..., \mathrm{v}_{1T}, .., \mathrm{v}_{n1}, ..., \mathrm{v}_{nT})$, is a random variable that is distributed according to the commonly known distribution $\Gamma(\mathrm{v})$.

The committee is composed of two *groups*, the *Majority group* **M**, with M members and the *Minority group* **m**, with $m < M$ members. The two groups differ systematically in their preferences: members of **m** are in favor of all proposals, and members of **M** are against. For all t:

$$\begin{aligned} \mathrm{v}_{it} &> 0 \text{ if } i \in \mathbf{m} \\ \mathrm{v}_{it} &< 0 \text{ if } i \in \mathbf{M} \end{aligned}$$

.

All members of the minority have intensities drawn from a distribution G_m with support $[0,1]$ while all members of the minority have intensities drawn from a distribution G_m with support $[-1,0]$. We assume symmetry in the distribution across groups, so that if we call $G'_M(v)$ defined over the support $[0,1]$ the distribution of the absolute valuations of majority voters, we set $G_m(v) = G'_M(v) \equiv F(v)$. $F(v)$ is common knowledge.

Intensities of preferences are drawn independently *across* the two groups. With respect to the correlation of the intensity of preferences *within* each group, we consider two polar cases. In the first case (case *B)*, intensities are drawn independently for each member of a group; in the second case (case C) intensities are identical for all members within each group. Hence, in the B case members of the same group may have conflicting priorities, while they do not in the C case.The correlation of within group intensities (or lack thereof) is assumed to be common knowledge

The intensity of these preferences v. At the beginning of period t, i privately observes v_{it} but does not observe $v_{it'}$ for $t' > t$: intensities are revealed privately and sequentially. Because draws are independent across issues, voter i's observation of v_{it} does not provide information about $v_{it'}$, and because draws are independent across groups, observation of v_m does not provide information about v_M (and vice versa). Thus, voters do not know the intensity of their own preferences in future periods and do not know the intensity of preferences of the other group. However, in case C, members of the have identical preferences and observation of their own intensity allows them to perfectly infer the preferences of the other members of their group, but gives them no information about the intensities of voters in the other group. In case B, a voter's own intensity provides no information about any other voter's intensities.

2.1 The Storable Votes Mechanism

At the beginning of period 1, each voter is endowed with an account of B_0 "bonus" votes, and for most of our analysis we will think of B_0 as an integer.[3] In the first period, the voter casts his regular vote plus as many bonus votes as he wishes out of his endowment. The bonus votes cast are deducted from his endowment, which is then carried over to the next period. The current endowment of bonus votes for every voter in period t, denoted $B_t = (B_{1t}, ..., B_{nt})$, is common knowledge at the beginning of period t. Thus each voter i independently decides how many votes, x_{it}, to cast after observing his private valuation v_{it} and B_t, subject to $x_{it} \leq 1 + B_{it}$. The proposal passes if there are more votes in favor of the proposal than against, and fails in the opposite case. Ties are resolved randomly. In the next period, $t+1$, voters' valuations over the new proposal are again privately observed, and voting proceeds as before, now subject to the constraint, $x_{it+1} \leq 1 + B_{it+1} = 2 + B_{it} - x_{it}$. Since $x_{it} \geq 1$, this is at least as tight a constraint as in period t. The voting continues in this fashion until the end of period T.

[3] Because we want to study the effect of bonus votes *per se* in strengthening the minority's position, it seems appropriate to give the same initial allocation to all voters.

3 Theoretical results

3.1 Equilibrium

Given F, m, M, B_0, T the storable votes mechanism defines a multistage game of incomplete information. We study the properties of the Perfect Bayesian equilibria of this game, where at each period t and for each possible valuation, v_{it}, and profile of endowments, B_t, individuals choose how many votes to cast so as to maximize expected utility, given the strategies of the other players. Because the sign of each group's preferences is common knowledge and intensities are independent over time, voting decisions cannot be used to manipulate other players' beliefs about future preferences. Assuming, in addition, that players do not use weakly dominated strategies, the direction of each individual vote is always chosen sincerely: all the minority members' votes are cast in favor of each proposal, and all majority votes are cast against each proposal. The *state* of the game at t is defined to be the profile of bonus votes each voter has still available, B_t, and the number of remaining periods, $T - t$. We focus on strategies such that the number of votes each individual chooses to cast each period, x_{it}, depends only on the intensity of preferences at time t, v_{it}, and the state of the game. We denote such strategies by $x_{it}(v_i, B_t, t)$.

3.2 The $C2$ game

When characterizing the equilibria of our model, the correlation of valuations within each group in model C can be a source of complications. But matters can be simplified by a simple observation. Consider the following 2-player storable votes game, which we call $C2$. Voter M has M regular votes each period and a stock of MB_0 bonus votes; his valuation over each proposal is $M\mathrm{v}_{Mt}$ where v_{Mt} is independently drawn from the distribution function F_M with support $[-1, 0]$. Voter m has m regular votes each period and a stock of mB_0 bonus votes; his valuation over each proposal is $m\mathrm{v}_{mt}$ where v_{mt} is independently drawn from the distribution function F_m with support $[0, 1]$. Then the following result holds:

Lemma 1. *If game $C2$ has an equilibrium, then the game described by model C also has an equilibrium. In addition, call $x^*_{Mt}(v_i, B_t, t)$ and $x^*_{mt}(v_i, B_t, t)$ the equilibrium strategies of voter M and voter m in game $C2$, and $\{x^*_{it}(v_i, B_t, t)\}$ the equilibrium strategies in C. If $C2$ has an equilibrium, then there exist equilibrium strategies of model C such that $\sum_{i\in m} x^*_{it}(v_i, B_t, t) = x^*_{mt}(v_i, B_t, t)$ and $\sum_{i\in M} x^*_{it}(v_i, B_t, t) = x^*_{Mt}(v_i, B_t, t)$.*

Proof. See Appendix. ■

Lemma 1 makes a simple point. In model C voters' interests within each group are perfectly aligned; if there is an equilibrium where each group coordinates its strategy so as to maximize the group's payoff, given the aggregate strategy of the other group, then no individual voter can gain from deviating.[4] In the n-person game described by model C, we will call equilibrium

[4] This is the logic exploited by McLennan (1998) to show that "sincere" voting must be a

group strategies the equilibrium individual strategies of the 2-voter game $C2$.[5] Lemma 1 allows us to show:

Lemma 2. *Both model B and model C have an equilibrium in pure strategies. In model B individual equilibrium strategies are monotone cutpoint strategies; in model C, group strategies are monotone cutpoint strategies: at any state* (B_t, t) *and for any* i *with* $k_i = B_i + 1$ *available votes there exists a set of cutpoints* $\{c_{i1}(B_t, t), c_{i2}(B_t, t), \ldots, c_{ik}(B_t, t)\}$, $0 \leq c_{ix} \leq c_{ix+1} \leq 1$, *such that i will cast x votes if and only if* $v_{it} \in [c_{ix}, c_{ix+1}]$, *where* $i \in \{1, .., n\}$ *in model B and* $i \in \{M, m\}$ *in model C.*

Proof. See Appendix. ■

The important point is that storable votes open the possibility of minority victories. We can state:

Theorem 1. *For any* F, M and m and $T > M$, *there always exists a* $\widetilde{B_0}(F, M, m, T)$ *such that for all* $B_0 > \widetilde{B_0}$ *in all equilibria of the storable votes mechanism the minority is expected to win some of the time with strictly positive probability (in both models B and C).*

The intuition is the following. To guarantee itself victory all the time, the majority needs to spread the bonus votes at its disposal over all proposals. If the horizon is sufficiently long and the stock of bonus votes sufficiently large, at least one proposal must exist over which the majority can be overruled with positive probability even by a single minority voter concentrating his bonus votes..

3.3 Efficiency

We measure the efficiency of the storable votes mechanism in terms of ex ante efficiency: a voter's expected utility from all T proposals before any of his valuations is realized, and before knowing whether he belongs to **M** or to **m.** We call our efficiency measure EV_0 and contrast it with the equivalent measure under simple majority voting, denoted by EW_0.[6] Thus, from an efficiency perspective, what is important is that the mechanism induces the minority to win "when it should", i.e. when minority intensities outweigh majority intensities. This will generally be the case due to monotonicity of the equilibrium voting strategies. That is, in any given state, the number of votes cast by each group increases

Nash equilibrium in common value decision problems with information aggregation.

[5] Other equilibria are possible, where no individual voter can gain from deviating, although the group's (and thus each individual's) payoff could be increased by joint group deviation.

[6] An important question is whether the cardinal values and our notion of efficiency force us into comparisons of interpersonal utilities. This is where our assumption of symmetrical distributions of (absolute) values across all voters plays its role. The valuation draws over any specific decision should be read as normalized by a common numeraire. In our model with multiple decisions, the natural numeraire is the individual's mean valuation over the universe of all decisions that could be brought to a vote. In fact, by imposing not only the same mean but the same distribution, we are forcing the voters to adopt an equal scale and to organize the different decisions according to a fixed ordinal ranking, with the same proportion of decisions in any given subinterval of the support. It is this normalization that allows us to avoid interpersonal comparisons. In this model, granting individual voters different distributions would be equivalent to taking a stance on the relative intensity of their preferences.

with the group's intensity of preferences. Thus, the minority will tend to win when its intensity of preferences is suffiently high compared to the majority. The precise argument is complicated by the dynamic nature of the game, the evolving budget constraint, the non-stationary strategies, and, in model B, by the varying intensity of preferences within each group.

The intuition described above applies to both models, but the properties of the voting mechanism are more robust and easier to characterize in model C.

Theorem 2. *In model C, for all F and T, if $m > 2$ and $M < 2m$ there exists a value of B_0 and an equilibrium of the storable votes mechanism such that storable votes are ex ante superior to simple majority voting (i.e. $EV_0 > EW_0$).*
Proof. See Appendix. ■

Both Theorems 1 and 2 rely on sufficient and rather restrictive conditions that allow us to establish results for arbitrary F and T. But when we consider an example, specializing the assumptions about F and T, the functioning of the mechanism becomes more transparent. For this reason, and because the example will guide the parameter choices in our experimental treatments we discuss it below in some detail.

3.4 An Example: Uniform Valuations and Two Periods

We illustrate the results above with the following parametric example, which corresponds to one of our experimental treatments. There are two proposals ($T = 2$); each voter is endowed with two bonus votes in addition to his regular votes ($B_0 = 2$), and the total number n of voters is odd. The distribution $F(v)$ is Uniform. The strategy chosen by each voter is simply the number of bonus votes to cast in period 1 the first proposal, after having learned his valuation for proposal 1.

In model B there exists an equilibrium where all voters, whether in the minority or in the majority, cast both bonus votes over the first proposal if the intensity of their preferences is higher than . $= 0.5$, and none otherwise: $x_{i1} = 1$ if $v_{i1} < 0.5$ and $x_{i1} = 3$ if $v_{i1} > 0.5$ for all i. If $M > 3m$, the majority always wins, but for all $M \leq 3m$ there exists an equilibrium where the minority wins one of the two proposals with positive probability. Ex ante, each of the two proposals has the same probability of a minority victory, and this probability equals $\sum_{s=k}^{m}\left[\sum_{r=0}^{m-s}\binom{M}{r}\binom{m}{r+s}2^{-n}\right] > 0$ where $k \equiv (M - m + 1)/2$.

In model C, if $2M > 3m$, the majority can ensure itself victory every time; if $2M \leq 3m$ it cannot, and there exists an equilibrium in which the minority again wins one proposal with positive probability. In this equilibrium, the minority casts all bonus votes on the first proposal if the intensity of preferences is higher than 0.5, and none otherwise; the majority follows the same strategy if M is large enough, and splits some of its bonus votes otherwise: $x_{m1} = m$ if $v_{m1} < 0.5$ and $x_{m1} = 3m$ if $v_{m1} > 0.5$, while $x_{M1} = \max\{M, m + 3\}$ if $v_{M1} < 0.5$ and $x_{M1} = \min\{3M, 4M - (m + 3)\}$ if $v_{M1} > 0.5$. Again, ex ante each proposal has the same probability of being the one won by the minority, and this probability equals 0.25.[7]

The equilibria and their welfare properties are analyzed in more detail in the Appendix and summarized in Figure 1. Efficiency is maximized when each decision is resolved in favor of the side with higher total valuation, and in the figure we compare equilibrium and ex post efficient outcomes.

Figure 1 here

The figure is drawn for the specific case $M = m + 1$, but its qualitative features hold generally and can easily be interpolated to the generic case $M = m+k$ with k odd. Figure 1a shows, for both models, the probability of a minority victory over either of the two proposals in equilibrium - the black dots - and in the first best - the grey dots. As m increases, this results in an increase in the probability of minority victories. In model B, the equilibrium probability increases smoothly, converging to 0.5 as the number of voters becomes large and the relative difference becomes negligible. The efficient frequency of minority victories is slightly higher than the equilibrium frequency. In model C, the change in the equilibrium probability of minority victories is discontinuous, jumping from 0 to 0.25 (when the majority becomes unable to guarantee victory on both proposals) and then remaining constant at that level. The point at which the jump occurs depends on the absolute difference between the two groups, k. The efficient frequency of minority victories on the other hand increases smoothly with the relative size of the minority and is always higher than the equilibrium frequency.

Figure 1b plots the expected per capita payoff for majority and minority members. With simple majority rule, the respective values are 1 and 0 in both models. With storable votes, the expected payoffs of the two groups are closer, unless the majority can ensure itself victory, although the minority's payoff remains lower than under efficiency (grey dots in Figure 1b,). In model C, equilibrium per capita payoffs remain constant for each group, regardless of m, once the threshold where the majority always wins has been passed.[8]

Figure 1c plots a normalized measure of expected surplus for both models, comparing storable votes and simple majority voting. For both mechanisms, we calculate expected aggregate payoff as a share of the expected first best payoff. Because we want to measure the added value over purely random decision-making (where each proposal is equally likely to pass or fail), we normalize both numerator and denominator by the expected payoff in the random mechanism. Thus if we call EV^* the expected efficient aggregate payoff and R the expected payoff under random decision-making, we define the *normalized aggregate surplus* as $(EV - R)/(EV^* - R)$ with storable votes and $(EW - R)/(EV^* - R)$ with simple majority. Over the two proposals, $EW = M$ and $R = (M + m)/2$ in both models, while EV and EV^* are derived in the Appendix. As the figure

[7]There are multiple equilibria. For example, in model B there is an equilibrium where every voters casts 2 votes each period and the majority always wins. However, this equilibria involves weakly dominated strategies.

[8]In fact, they remain unchanged for any absolute difference between the two groups, once the threshold $3m < 2M$ has been passed. It is the threshold itself that depends on $(M - m)$.

shows, when the number of voters is small and the difference in size between the two groups relatively important, the possibility of minority victories in the storable votes mechanism is accompanied by some loss of efficiency in model B, but not in model C, where efficiency is always at least as high as under simple majority rule. The loss in model B is not large and disappears rapidly as the number of voters and the relative size of the minority increases. For most sizes of the electorate, storable votes allow voters to appropriate a larger share of the total surplus in both models.[9]

4 Experimental design

4.1 Protocol

All sessions of the experiment were run either at the Hacker SSEL laboratory at Caltech, the CASSEL laboratory at UCLA, or the PLESS laboratory at Princeton with enrolled students who were recruited from the campus through the laboratory web sites. No subject participated in more than one session. All sessions focussed on the example described above: subjects voted on two consecutive proposals ($T = 2$) and were allocated 2 bonus votes ($B_0 = 2$), in addition to the regular vote they were required to cast over each proposal. With the exception of one session, committees were composed of 5 voters, divided into two groups of 3 and 2 voters with systematically opposed preferences.[10] The experiment's main treatment variable was the correlation of intensities within each group - the distinction between model B and model C.

After entering the computer laboratory, the subjects were seated randomly in booths separated by partitions and assigned ID numbers corresponding to their computer terminal; when everyone was seated, the experimenter read aloud the instructions, and any questions were answered publicly. The session then began.[11] Subjects were matched randomly into committees and within each committee were assigned randomly to the majority or the minority group. Each

[9]The main difference between the two models emerges in the limit, and is not visible in the figure. In model B, the valuation draws are independent, hence, as the population becomes very large the law of large numbers guarantees that the empirical average intensity of preferences in both groups converges to the mean of the $F(v)$ distribution. This means that random choice, simple majority voting and storable votes all converge to first best efficiency and any efficiency-based argument for protecting the minority disappears. In model C, on the other hand, the valuation draws within each group are perfectly correlated, and the law of large numbers does not apply. As the number of voters increases, the difference in size between the two groups becomes negligible and simple majority voting again converges to random choice, but random choice remains inferior to efficient decision-making and to storable votes. In very large populations, only minorities whose intensities are correlated should be protected on efficiency grounds.

[10]As discussed below, we ran one session with committees of 9 voters, each divided into two opposite groups of sizes 5 and 4.

[11]A sample of the instructions from one of the sessions is reproduced in the Appendix. We used the Multistage Game software package developed jointly between the SSEL and CASSEL labs. This open-source software can be downloaded from http://research.cassel.ucla.edu/software.htm

subject was then shown his valuation for the first proposal and asked to choose how many votes to cast in the first election. Valuations were restricted to integer values and were drawn by the computer, with equal probability, from the support $[-100, -1]$ for majority members, and from $[1, 100]$ for minority members. In both treatments, the valuations were drawn independently for majority and minority members.

In treatment B each member of each group was assigned a valuation drawn independently from the specified support; in treatment C all members of the same group in the same committee were assigned the same valuation (i.e. all majority members in a given committee shared the same valuation, as did all minority members in a committee). The independence of the valuations within each group in treatment B and their perfect correlation in treatment C were common knowledge. After everyone in a committee had voted, the computer screen showed to each subject the number of votes cast by each of the two groups in the subject's committee, whether the proposal has passed or not, and the subject's own payoff from that election. Valuations over the second proposal were then drawn, the remaining votes were automatically cast and the outcome determined.

After the second proposal had been voted upon, subjects were rematched, each was assigned a new budget of bonus votes, and the game was replayed. Experimental sessions consisted of either 20 or 30 such rounds[12], each round a sequence of two consecutive proposals. In the rematching, minority members always remained minority members and majority members always remained majority members, but the composition of each group and of each committee was randomly determined. Subjects were paid privately at the end of each session their cumulative valuations for all proposals resolved in their preferred direction, multiplied by a pre-determined exchange rate and complemented by a fixed show-up payment of $10. Average earnings were about $17 per experiment for minority subjects and about $31 for majority subjects.

4.2 Parameters

The only choice given to our experimental subjects was the number of votes to cast over the first proposal. The equilibrium is described in the previous section for generic M and m, and is reported below for the specific case $M = 3$, and $m = 2$ (and for a robustness control in one experimental section, for $M = 5$, and $m = 4$). Individual equilibrium strategies in treatment B and corresponding equilibrium outcomes are in Table 1. The equilibrium cutpoints - the threshold (absolute) values where individual voters switch from casting 0 to casting 1 bonus vote, and from casting 1 to casting 2 - are reported in row 2 of Table 1 and are denoted c_1 and c_2.[13] Rows 3 and 4 in the table report the expected frequency of minority victories in equilibrium and under ex post efficiency, respectively. Rows 5 and 6 report the expected share of per capita payoff for a minority voter,

[12] With the exception of one session of 15 rounds.

[13] Because the equilibrium cutpoints are identical for minority and majority voters, we use the symbols c_1 and c_2 for both groups.

relative to a majority voter, again in equilibrium and under ex post efficiency. So, for example, in the $\{3, 2\}$ experiment with storable votes a minority subject the minority is expected to win on average 26% of what a majority subject earns, if everybody plays the equilibrium strategy. Finally, the last two rows report the expected share of normalized aggregate surplus appropriated with storable votes (row 7) and with simple majority voting (row 8).[14]

Table 1: Equilibrium strategies and outcomes.

B Treatment

M, m	3, 2	5, 4
c_1, c_2	50, 50	50, 50
% min wins, sv	19	25
% min wins, eff	22.5	28.5
% (min/maj) payoff, sv	26	36
% (min/maj) payoff, eff	35.5	45
% surplus sv	71	61
% surplus nsv	75	62

The qualitative features of these numbers were discussed in the previous section. Notice, once again, that although storable votes here are less efficient from an aggregate point of view than simple majority voting, the efficiency loss is minor, relative to the effect of storable votes on the welfare of minorities.

4.3 Equilibrium

Equilibrium strategies in treatment C pose a coordination problem. As described in the previous section, if the two groups are of size $\{3, 2\}$, in equilibrium the minority uses no bonus votes if its absolute valuation is smaller than 50, and all its bonus votes if it is above; the majority casts a total of 5 votes if its absolute valuation is smaller than 50, and 7 votes if it is larger than 50.[15] Any individual strategy compatible with these group strategies is an equilibrium. Hence, each minority voter has a simple symmetrical strategy that aggregates to the equilibrium group strategy: vote 1 if the valuation is below 50 and 3 if

[14]As remarked earlier, there is a second equilibrium where minority voters use weakly dominated strategies: every subject always uses 1 bonus vote, and the majority always wins. We found no evidence of this equilibrium in the data and do not discuss it further.

[15]When the two groups are of size $\{3, 2\}$, the majority has other equilibrium strategies, but all are payoff-equivalent, and we treat them as identical when reporting the experimental results. All equilibrium strategies satisfy: cast 0, 1, or 2 bonus votes with probabilities p_0, p_1, p_2 if the absolute valuation is smaller than 50, and 4, 5, or 6 bonus votes with probabilities q_0, q_1, q_2 if the absolute valuation is larger than 50, where $p_2 \geq q_2$ and $p_1 = q_1$. The strategy described in the text corresponds to $p_0 = p_1 = 0$, and $q_1 = q_2 = 0$.

The majority could equivalently cast 4 votes for valuations below 50, and 8 for valuations above 50, with no effect on payoffs. Because the two strategies are payoff-equivalent, we treat them as identical when reporting the experimental results.

the valuation is 50 or above. But the aggregation problem for majority voters is more difficult. The group strategy described above cannot be supported by *symmetric* individual strategies, and coordination on asymmetric strategies is hampered by the random rematching in our experimental design. In fact, for our experimental environment, not only is there no symmetric individual strategy that aggregates to the equilibrium group strategy, but there is no asymmetric strategy that each majority voter can adopt consistently and that would always aggregate to the equilibrium group strategy, for any possible rematching.

We know that a symmetrical equilibrium exists (by standard fixed point arguments)[16], but we have not been able to characterize it, and we doubt that our experimental subjects, confronted with a new game and under time pressure, could be much more successful. In practice, our basic C treatment is then a test of the robustness of storable votes' outcomes to coordination problems. To evaluate the role of coordination more precisely, we designed two additional treatments that replicate model C but where coordination problems are absent by construction.

In treatment $C2$ ("correlated valuations, coordinated voting") a single subject represented the whole group. Half of the experimental subjects were randomly assigned to represent majority groups, and half minority groups. Each majority group's representative had 3 indivisible regular votes to cast on each of the two proposals and 6 bonus votes to cast as desired. Each minority group's representative had 2 indivisible regular votes to spend on each of the two proposals and 4 bonus votes. A committee was then formed by one pair of experimental subjects, one subject randomly drawn from all those representing a minority group, and the other from all those representing a majority group. In each committee, and for each proposal, valuations were drawn independently with equal probability, from the support $[-100, -1]$ for the majority representative, and from $[1, 100]$ for the minority one. The timing of the game proceeded as described earlier. After each two-proposal round, partners were rematched, but all minority representatives remained minority representatives for the whole experimental session, as did all majority representatives. When we discuss experimental payoffs from this treatment, we multiply the minority representative's payoff by 2 and the majority's by 3, so as to make them comparable to the theoretical predictions and to the experimental payoffs for the C case and to the following treatment, which we call $CChat$.

In treatment $CChat$ ("correlated valuations, chat option") we replicated the C treatment, with each group composed of multiple individual subjects, adding a "chat option". Before the vote on the first proposal, each group member is allowed to send messages via computer to other members of his own group. Subjects are instructed not to identify themselves, and the messages are anonymous but otherwise unconstrained. In particular, they allow subjects to coordinate on their preferred group strategy. Everything else in the experiment - the stochastic properties of the valuation draws, the timing, the random re-matching - follows exactly the C treatment.

[16] Taking into account that the set of types is finite in our experimental treatment.

Equilibrium group strategies and expected outcomes are identical in the three C treatments - C, $C2$, and $CChat$. They are reported in Table 2, where g_L and g_H denote the cutpoints where both groups switch from casting 0 bonus votes to casting 2, and from casting 2 to casting 4.[17]

Table 2: Equilibrium group strategies and outcomes.

C Treatments	
M, m	3, 2
g_L, g_H	50, 50
% min wins, sv	25
% min wins, eff	33
% (min/maj) payoff, sv	38.5
% (min/maj) payoff, eff	52
% surplus sv	60
% surplus nsv	53

As discussed in the previous section, the outcome is more favorable to the minority in model C than in model B, both in terms of the expected frequency of minority victories and of its expected payoff, relative to the majority. Notice also that storable votes outperform simple majority voting in terms of aggregate efficiency.

The experimental design is summarized in Table 3. In all experiments the majority was formed by 3 subjects and the minority by 2, with the exception of session b_3 where the number of subjects in each group was 5 and 4 respectively. Session b_3 serves us as a control on the sensitivity of the experimental results to the size of the groups.

Table 3: Experimental Design

Session	Groups size	Subject pool	# Subjects	Rounds
b1	3,2	CIT	15	30
b2	3,2	UCLA	20	30
b3	5,4	UCLA	27	30
c1	3,2	UCLA	15	30
c2	3,2	PU	15	20
c3	3,2	PU	10	20
c21	3,2	CIT	12	30
c22	3,2	UCLA	16	30
c23	3,2	PU	12	20
cchat1	3,2	PU	10	20
cchat2	3,2	PU	15	15

[17]Again, we use the same symbols for both groups because the equilibrium cutpoints are identical for the minority and the majority.

5 Experimental Results

The experiments have two purposes. First, we want to verify the extent to which the experimental outcome match the theoretical predictions: are minority subjects able to win some of the votes, and are they able to do so without losing too much aggregate efficiency, as the theory predicts? Second, focussing now on subjects' behavior, do their strategies replicate the theoretical equilibrium strategies?

5.1 Outcomes and Efficiency

5.1.1 How often did the minority groups win?

The diagram on the left of Figure 2a summarizes the answer to this question. The vertical axis is the percentage of times the minority prevailed in the experimental sessions, and the horizontal axis is the percentages of times it would have prevailed if all subjects had played the equilibrium strategy, given the valuations drawn during the experiments. Different treatments are indicated by different symbols, as described in the figure's legend.

Figure 2 here

The figure can then be read in several ways. The vertical height tells us that the minority won between 20 and 25 percent of the time in C, $C2$, and $CChat$, with little dispersion among them; it won less frequently in the B sessions (around 15 percent of the time) with the exception of the one experiment of size $\{5, 4\}$ where the minority won about 23 percent of the time.

Clearly, storable votes help the minority win. The difference in this effect across treatments matches the theoretical predictions, as is evident from the way the points align along the 45-degree line. The closer to the line a point is, the closer the experiment's results are to the equilibrium predictions. If we estimate a simple regression line, the hypotheses of a unitary slope parameter and a zero constant term cannot be rejected at standard confidence values.[18] On average, the frequency of minority victories in the experiments differs from the equilibrium predictions by 3 percentage points, without clear outliers and without systematic treatment effects. We find this surprising because the complexity of the individual equilibrium strategies in the basic C treatment (as opposed to $C2$ and $CChat$) would suggest a larger discrepancy from equilibrium predictions in that specific treatment, a discrepancy the data do not show.

5.1.2 Did the experimental payoff to the minority match the theoretical predictions?

The diagram on the right of figure 2a plots per capita minority payoff as percentage of per capita majority payoff in the experiments on the vertical axis,

[18]The estimated parameters are: 0.76 for the slope (standard error 0.23), and 3.4 for the constant term (standard error 5.8).

and in equilibrium on the horizontal axis, using the symbols of the previous figure to identify the different experimental sessions. In all C, $C2$ and $Cchat$ treatments the relative minority payoff was higher than in any B treatments, as predicted by the theory, ranging between 33 and 45 percent of the average majority payoff, versus 16 to 20 percent in the B treatments of size $\{3, 2\}$ and 30 percent in the B treatment of size $\{5, 4\}$. Again, the effect of the voting mechanism in raising the minority's payoff was significant. Out of eleven experimental sessions, all but two are below the 45-degree line, suggesting that the minority was unable to fully exploit the opportunity presented by the voting mechanism. But the discrepancy is not large - the average distance from the 45-degree line is 5 percentage points, again without clear outliers[19] or treatment effects, which is small in comparison to the differences across treatments. Again, if we estimate a regression line, we cannot reject the hypotheses of unitary slope and zero constant.[20]

5.1.3 At what cost to the majority were the minority's gains? At what cost to overall efficiency?

The left-hand side of figure 2b plots the normalized total surplus in each session on the vertical axis, against the equilibrium predictions on the horizontal axis. The equilibrium predictions are calculated using the actual intensity values in the experiment. Points on the 45 degree line indicate a perfect match to the theory. The mean distance from the 45 degree line is only 7 percentage points, again with little evidence of outliers, versus a mean equilibrium surplus share of 60 percent. As in the previous figures, we cannot reject a regression line with unitary slope and zero constant, although the fit is poorer.[21]

The central question is how the efficiency of storable votes compares to the efficiency of alternative voting systems - in our case against simple majority voting. In the diagram on the right of figure 2b, the vertical axis is again the normalized total surplus in each session, now plotted against the equivalent measure with simple majority voting calculated from the actual intensity values. Theory predicts that data from C, $C2$ and $CChat$ sessions should lie above the 45-degree line, while B data should lie below. The prediction is confirmed by the C and by the B experiments. Surprisingly it is the "easier" treatments with coordination, $C2$ and $Cchat$, that fall short of the prediction. Once again, two of the three most significant losses relative to non-storable votes occur in $C2$ sessions. Pooling all C, $C2$ and $CChat$ data, the mean difference in normalized surplus is $+2$ percentage points, compared to the theoretical prediction of $+7$. Pooling all B data, the mean difference was approximately -10 percentage points, compared with the theoretical prediction of -4.

[19]Note that a plausible range of values in Figure 2b is between 0 (the outcome with simple majority voting) and 100 (the expected outcome with random decision-making). In figure 2a, the corresponding range is between 0 and 50.

[20]The estimated parameters are: 1.03 for the slope (standard error 0.19), and -6.2 for the constant term (standard error 7.1).

[21]The estimated parameters are: 0.7 for the slope (standard error 0.40), and 14.1 for the constant term (standard error 24.1).

The data from our experiment can be summarized in three main points. First, storable votes help minorities substantially, both in terms of the frequency with which minorities won decisions and in terms of the resulting benefits. Second correlation of intensities works to the advantage of the minority. Third, the efficiency costs associated with the increased representation of minority interests were small in magnitude. Without correlation, storable votes induces (small) aggregate welfare losses, but with perfectly correlated intensities, storable votes produced welfare *gains* over simple majority voting.

5.2 Behavior

We begin by studying individual behavior in the treatments that did not allow group members to coordinate their strategies (B and C). We thus focus naturally. Later we turn to group behavior and discuss the effects of explicit coordination (treatments $C2$ and $CChat$).

5.2.1 Individual behavior

Storable votes allow voters to express intensity of preference by casting more votes, at any given state, when they have stronger preferences. Hence, *monotonicity of voting strategies is at the core of the mechanism*, and it is natural to analyze subject behavior in our experiments by studying this property first.

To obtain a measure of monotonicity of individual behavior, we estimate *monotonicity violations* and *cutpoints* for each subject. For each subject we have K pairs of observations, where K equals either 20 or 30 depending on the session[22]. Each pair consists of a first proposal intensity value and the number of votes cast for (or against) the first proposal. The number of votes cast is always 1, 2, or 3. A perfectly monotone strategy is one for which we can find two cutpoints, $c1 \leq c2$ such that whenever the subject's first period valuation was below $c1$ the subject cast 1 vote, whenever the subject's first period valuation was above $c2$, the subject cast 3 votes, and for intermediate values between $c1$ and $c2$ the subject cast 2 votes. We calculate the number of monotonicity violations as the minimum number of voting choices that would have to be changed, for each subject, to make the strategy perfectly, monotonic. We then identify the pair of cutpoints that is consistent with such monotonic strategy. In some cases, multiple cutpoints are consistent with the same number of monotonicity violations; when this happens, we select the pair that is closest to the equilibrium cutpoints.

Figure 3a presents histograms of individual monotonicity violations in treatments B and C. The horizontal axis is divided into deciles representing the percentage of violations over the total number of voting decisions, and the vertical axis reports the fraction of subjects that belong to each decile.

Figure 3 here

[22]With the exception of session **cchat**2, with 15 rounds.

In the B treatment, 50 percent of the subjects have 3 or fewer violations out of 30 voting decisions (10 percent). In the C treatment, 57 percent of subjects had violation rates less than or equal to 10 percent. As comparison, a voter choosing randomly whether to cast 0, 1, or 2 bonus votes would have a violation rate converging to 67% as the number of decisions becomes very large.[23] The comparison makes clear that, although there is some noise, individual choices indeed tend to be monotonic for most subjects.

The estimated cutpoints for all individual subjects in the B and C sessions are displayed in figures 3b. Each point represents one subject's estimated pair of cutpoints, with $c1$ on the horizontal axis and $c2$ on the vertical axis. All cutpoints lying on the 45 degree line involve no splitting of bonus votes: always casting either both or neither of the bonus votes over the first decision. Moving to the upper left corner of the graph are cutpoints that involve more and more splitting of bonus votes, i.e. using one bonus vote in each period for a range of values that increases as one approaches the corner. The upper left corner of the graph, at $(0, 100)$ corresponds to always casting one bonus vote. Cutpoints for subjects in the minority group are in the left graph and cutpoints for the subjects in the majority group are in the right graph. The rates of monotonicity violations are indicated by shading the points, with the darkest points having the fewest monotonicity violationst.

In the B treatments, the equilibrium cutpoints for both majority and minority subjects are $(50, 50)$: if everyone played the equilibrium strategies all points would be on the 45 degree line at 50. In the C treatments, $(50, 50)$ remains an equilibrium for individual minority subjects, but not for subjects in the majority, whose asymmetrical strategies are contingent on the behavior of the other members of the group and cannot be identified unambiguously in the figure.

Two features of the distribution of cutpoints appear in both treatments. First, the minority cutpoints do cluster around $(50, 50)$, and on average minority subjects whose cutpoints are closer to equilibrium have lower violation rates. Second, bonus votes are much more frequently split by majority voters, with little difference between the two treatments in spite of the different theoretical predictions. Intuitively, even in model B, majority voters have less to lose from splitting their bonus votes - their larger number implies that they are guaranteed to always win one of the two decisions, and one single vote more or less plays a smaller role than in the case of the minority. Consider the parameter values used in the experiments and a committee of size $(3, 2)$. The expected loss to a voter deviating from his equilibrium strategy and always casting one bonus vote over each proposal is 15 percent in model B and 50 percent in model C for a minority voter, versus 4 percent in model B and 8 percent in model C for a majority voter (relative to the expected equilibrium payoff)[24] The difference in

[23] To account for the smaller number of violations that would result from the small sample and the free cutpoints, we simulated random behavior with 21 subjects and 30 rounds. We found that no subjects had violation rates less or equal to 30 percent; 2 subjects were in the fourth decile; 8 in the fifth, and 11 in the sixth.

[24] Supposing that all other voters play the equilibrium strategy. In model C, we consider the case where the individual majority voter's deviation causes the majority group strategy

the cost of splitting one's bonus votes in the two models may play some role in the more pronounced clustering of the minority cutpoints around the 45 degree line, and particularly around $(50, 50)$ in the C treatment.

5.2.2 Group behavior

The monotonicity of the individual strategies provides only a partial picture. Efficiency requires *group* strategies to be monotonic in the group value. In the B treatment the notion of "group value" is not clearly defined because different subjects within a group have different values. But we can check for "group monotonicity" in the C treatment, that is, we can check whether the sum of the votes by members of one group is monotone in their (common) value. If there is heterogeneity in behavior, monotonicity at the individual level need not imply monotonicity at the group level because individuals are continuously rematched. The problem is particularly severe for the majority whose individual equilibrium strategies are asymmetric.[25]

The histograms in the first row of Figure 4a illustrate the difficulty that groups had in the C treatment. More than 40 percent of the groups had error rates above 20 percent, compared to only 10 percent of individual subjects in the same sessions (see Figure 3a). As expected, and as shown by the histogram on the right, most errors are associated with the majority, where more than 60 percent of the groups had more than 20 percent error rates.

Figure 4 here

A comparison of these results to monotonicity violations in the $C2$ and $CChat$ treatments allows us to study the role of explicit coordination. According to the histograms in the second row of Figure 4, the open communication in $CChat$ reduced group violations dramatically: *all* minority groups and 2 out of 5 of the majority groups had fewer than 10 percent violations. More surprising is the poor performance of the $C2$ treatment, where perfect coordination is imposed by the experimental design.[26]

These results leave us with a puzzle: if the aggregate group behavior of the experimental subjects in sessions C often violates monotonicity, why did the outcomes of these experiments - in terms of minority victories and efficiency - still conform to the theory? Why did these sessions outperform, on average, the $C2$ sessions with apparently comparable record of monotonicity violations. The answer comes from the underlying monotonicity of the *individual* behavior in treatment C. Intuitively, because individual subjects did cast their vote monotonically, the violations resulting from the uncoordinated aggregation of the votes are numerous, but not large: they tend to be concentrated around

to switch from casting either 5 or 7 votes to always casting 6.

[25] We identify a group by the label in the experiment (group 1, group 2, etc.), but rematching implies that the composition of each group continues to change. Note that if equilibrium strategies were symmetrical, the changing composition of the group would not matter.

[26] This appears to be the result of a single experimental session: session **c2**2 conducted at UCLA (where 25 percent of the subjects had a rate of violations approaching 50 percent).

the cutpoints values. To verify this, the histograms in figure 4b summarize the distribution of the average distance of "mistaken" (i.e. non-monotonic) voting choices from the cutpoints, as percentage of the expected distance if voting choices were random.[27] The $CChat$ experiments show the greatest consistency: with one exceptional outlier, all groups have error distances below 20 percent of the random case. But it is the comparison between the C and the $C2$ treatments that is particularly revealing in explaining the differences in experimental outcomes: one-fourth of all $C2$ groups have error distances that are closer to the purely random case than *any* of the C groups. As mentioned, this reflects mostly one outlier session, **c2**2, and how much of an outlier **c2**2 is is made clear in the diagram on the right, in the bottom row of figure 4b. Almost half of all groups in this session have error distances that are closer to the purely random case than *any* of the C groups, and less than one fifth have distances that are less than 10 percent of the random case, a very different result from the other two $C2$ sessions. This explains why the aggregate experimental payoff of session **c2**2 falls short both of the theoretical prediction and of the payoff with simple majority. As shown in figure 2b, these few cases were sufficient to exact a cost in terms of efficiency, lowering the overall performance of the $C2$ treatment. Why the treatment proved difficult to our subjects is an open question, although we can speculate that the problem may come from the larger size of the individual strategy space: each minority voter had 5 different choices of how many votes $(2, 3, 4, 5, 6)$ to use in the first period, and each majority voter had 7 different choices $(3, 4, 5, 6, 7, 8, 9)$.

As in the analysis of individual behavior, the monotonicity analysis generates cutpoints estimates.[28] Group cutpoints are depicted in Figure 5, with minority cutpoints on the left and majority cutpoints on the right. In line with the equilibrium predictions, we can summarize the strategies of each group through two cutpoints, represented by a point in the diagrams and equal to $(50, 50)$ for both the minority and the majority.[29]

Figure 5 here

The first row of diagrams in Figure 5 refers to C treatments; the second row to $C2$ and the last to $CChat$. As in Figure 3b, darker points indicate

[27]Following this logic, these cutpoints are estimated so as to minimize the average distance (both in the experimental data and in the theoretical random case). With a very large number of random voting choices, the two cutpoints that minimize the expected errors' distance are $(50, 50)$. The frequency of error is $2/3$, with an average distance of 25, yielding an expected distance of $50/3$. The corresponding number in the experimental data is, for a given pair of cutpoints, the sum of all errors' distances, divided by K, the number of rounds in the experiment.

[28]The cutpoints estimates that minimize the number of monotonicity violations need not be identical to those that minimize the errors' distance. In practice, they differ mostly in the case of those subjects with more random behavior. The substance of the results does not change, and we report here the cutpoints the minimize the number of violations, for consistency with the discussion of individual behavior.

[29]For the majority groups, we treat as identical all payoff-equivalent strategies, i.e. voting either 3, or 4, or 5 below g_l, and voting either 7, or 8, or 9 above g_h.

fewer monotonicity violations. Coordination affects the cutpoints of the minority groups: none of the estimated cutpoints in treatments $C2$ and $CChat$ lies outside the 45 degree line, as opposed to what we observe in treatment C. Thus in treatments $C2$ and $CChat$, in accordance with equilibrium the behavior of all minority groups is best described as voting either 2 (at lower values) or 6 (at higher values), with some dispersion around the equilibrium cutpoints $(50, 50)$. The majority's behavior, on the other hand, is best described as splitting the bonus votes for some intermediate range of values. In addition, the light shading of most points in the majority figures reflects the relatively large number of monotonicity violations for any estimate of cutpoints. The relatively greater deviation from equilibrium by the majority groups may reflect their relative low cost of such deviations. With a single coordinated strategy, the expected percentage loss to the majority from always splitting the bonus votes is about 8 percent when the minority plays the equilibrium strategy.[30] For the minority, on the other hand, splitting the bonus votes can be very costly: a minority always casting 4 votes *always* loses against a majority casting 5 votes at valuations below 50 and 7 at valuations above 50.

6 Conclusions and Discussion

We draw three main conclusions from the experiment. First, our results confirm the importance of monotonic voting behavior in realizing the potential efficiency of storable votes. As in previous experiments in symmetrical environments (Casella, Gelman and Palfrey, forthcoming), it is this more intuitive requirement, relative to the full discipline of equilibrium behavior, that keeps the experimental outcomes in line with the theoretical predictions. Second, the results on group behavior in treatment C allow us to propose a stronger conjecture: for the most part, the efficiency of the mechanism is preserved even in the presence of "some" violations of monotonicity, as long as these violations are not large. What matters is that on average more votes are cast at higher values. When this requirement is not satisfied, as in the outlier session **c2**2 in treatment $C2$, the efficiency loss is clear. Third, deviations from equilibrium are particularly costly to the minority, whose payoff, relative to the majority, falls short of the equilibrium prediction in all but two sessions (figure 2a). The advantage of coordination in inducing the minority towards the equilibrium strategy has a counterpart in figure 2a, where $CChat$ and $C2$ treatments almost always have smaller deviations from the theoretical predictions than treatments B and C.

Majoritarian principles are a fundamental ingredient of democratic institutions. But they carry with them the risk of disenfranchising minority groups and endangering the stability of the system, by violating principles of both equity and efficiency. In a well-designed democracy, a judicial system protecting the rights of minority groups needs to be supplemented by political remedies

[30]In fact, in this model the majority's maximin strategy entails splitting the bonus votes. It corresponds to cutpoints $(25, 100)$: cast no bonus votes for values below 25, but split the bonus votes for all values above 25.

that ensure the minority a voice through the daily, ordered exercise of political rights. This paper has analyzed the potential of a simple voting system - storable votes - to fulfill this function. By granting voters a stock of votes to be divided as desired over a series of multiple binary decisions, storable votes allow the minority to cumulate votes on specific issues and to win sometime. Because the minority wins only if its strength of preferences is high, and the majority's is low, the gains in terms of equity have little if any cost in terms of efficiency.

We have studied two related models where two groups of different size have consistently opposite preferences. In our "correlated" model, C, all members of a group - whether the majority or the minority - agree not only on on the direction of their preferences but also on the strength of their preferences. If we think in terms of political parties, these would be parties with strong discipline; more generally, the model is best suited to represent groups with some level of organization, sufficient to agree on the set of priorities. In our "basic" model, model B, on the other hand, all members of a group agree on the direction of their preferences, and the two groups have opposite preferences, but within a group the members' priorities may differ. The groups are not organized.

Although storable votes help in minority in both models, both the theory and the experiments support the intuition that the minority fares better when its members agree on priorities. The voting system is decentralized and coordination can be a problem even when preferences are perfectly correlated - and the minority does better in the experimental treatments with more coordination - but the larger effect comes from the agreement on priorities. The minority can only win if a sufficient number of its members all vote heavily on a given issue. Agreeing on priorities is a very useful first step in achieving that goal. The literature on cumulative voting had conjectured a similar effect: Guinier (1994) states that cumulative voting favors well-organized minorities, and in fact considers only well-organized minorities as deserving of special protection.

For both models, our experimental results confirm the theoretical predictions on voting outcomes: the frequency of minority victories, the payoff to the minority relative to the majority, the aggregate payoff to all voters and the comparison to the aggregate payoff under simple majority. They do not match the theory in terms of behavior: especially among majority voters, we observe equilibrium strategies only rarely. However, the monotonicity of voting strategies - more votes are cast when the strength of preferences is higher - is almost always respected. Where it cannot be by design (in the aggregate majority group vote of treatment C), monotonicity still characterizes individual voting choices, with the result that deviations at the aggregate level, though not infrequent, are not large. The efficiency costs from these deviation appear small. These findings reinforce most of our tentative conclusions from an earlier set of storable votes experiments with identical voters (Casella, Gelman and Palfrey, in press), suggesting robustness to asymmetric environments.

There are many directions for further research. We limit ourselves to mentioning two. First, it would be interesting to compare storable votes to a larger set of alternative mechanisms, both theoretically and experimentally. These alternative mechanisms should include vetoes, serial dictatorship and potentially first-best mechanisms a la Jackson and Sonnenschein (forthcoming). Storable votes are more flexible but more complicated than vetoes, and less flexible and less complicated than the Jackson and Sonnenschein mechanism. Serial dictatorship requires a secondary mechanism to allocate decisions to specific individuals or groups in a somewhat efficient fashion. What can the theory tell us, and how would all compare experimentally?[31] Second, the sensitivity of storable votes to agenda manipulation is an open question. The agenda setting procedure should be part of the overall game, and voters will decide how many votes to cast knowing how new issues are brought to a vote. A priori it is not clear whether problems will arise: having multiple votes that can be shifted across proposals may make the order of the proposals more important, but also increase the ability to resist possible manipulations of this order. The additional consideration of political minorities may exacerbate possible problems, either because majority losses are particularly expensive in terms of efficiency or because the minority may end up unable to ever control any outcome.

[31]Two recent experimental analyses are Engelmann and Grimm (2006) on the Jackson Sonnenschein mechanism, and Kagel et al. (2005) on veto power. Neither paper compares different mechanisms.

7 References

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8 Appendix

Proof of Lemma 1. Suppose that $x^*_{Mt}(v_i, B_t, t)$ and $x^*_{mt}(v_i, B_t, t)$ exist. Consider candidate equilibrium strategies $\{x'_{it}(v_i, B_t, t)\}$ for model C, where $\sum_{i\in m} x'_{it}(v_i, B, t) = x^*_{mt}(v_i, B, t)$ and $\sum_{i\in M} x'_{it}(v_i, B, t) = x^*_{Mt}(v_i, B, t)$. Because preferences between the two groups are always opposed, at any state only the aggregate voting choice of the opposite group affects voters' payoffs. In addition, because in model C preferences within each group are always perfectly correlated, by definition $\{x'_{it}(v_i, B, t)\}, i \in m$ maximize the expected payoff of each individual minority member, given $x^*_{Mt}(v_i, B, t)$ (and similarly for $\{x'_{it}(v_i, B, t)\}, i \in M$, given $x^*_{mt}(v_i, B, t)$). It follows that no individual deviation from the prescribed strategies can be profitable and $\{x'_{it}(v_i, B_t, t)\}$ must be equilibrium strategies. Note that in general the equilibrium will not be unique: any permutation of individual strategies that leaves the aggregate vote for the group unchanged, at given state, is an equilibrium. □

Proof of Lemma 2. (i) *Existence of equilibrium in pure strategies.* Milgrom and Weber (1985) discuss conditions for existence of an equilibrium in distributional strategies. In particular, conditional on a publicly observed variable, individual types are required to be independent. The publicly observed information in our case is each voter's membership in one of the two groups, and hence the support of the distribution from which valuations are drawn. Conditional on such support, individual valuations are independent in case B. The arguments in Casella (2005), showing that the game satisfies all conditions required by Milgrom and Weber remain applicable here. Hence an equilibrium in pure strategies exists for model B. Conditional on public information on the support of each distribution, valuations are independent in the two-voter version of model C. Again, the arguments in Casella (2005) apply, and an equilibrium in pure strategies exists. But since such an equilibrium must be an equilibrium of the n-voter C game, it follows that an equilibrium in pure strategies of the n-voter C game exists. (ii) *Monotonicity of the equilibrium strategies.* Call a strategy *monotonic* if, at a given state, the number of votes cast is monotonically increasing in the intensity of preferences v_{it}. The argument in Casella, Gelman and Palfrey (forthcoming) shows that at any given state all individual best response strategies must be monotonic when members of each group do not play correlated strategies. Thus the argument applies immediately to equilibria of model B. It also applies to the two-voter version of model C, and hence to group strategies, as opposed to individual strategies, in the equilibrium we focus on in the n-voter C game. If, at any given state, all best response strategies must be monotonic and an equilibrium exists, it follows that equilibrium strategies must be monotonic. Because there is a continuum of types and a finite set of strategies, then it must be that monotonic equilibrium strategies must take the form of monotone cutpoint strategies. □

Proof of Theorem 1. Consider any candidate equilibrium where the minority is expected to lose with probability 1 over each decision. A minority member cannot be worse off by cumulating all his bonus votes on one decision.

Over all decisions, there must be at least one where with positive probability the majority casts no more than MB_0/T bonus votes, and since the minority can never cast fewer than m total votes, a deviating minority member can always find a decision where with positive probability the difference in votes cast is at most $M(1 + B_0/T) - m$. Thus with positive probability the outcome of that decision changes and deviation is profitable if $M(1 + B_0/T) \leq m + B_0$, or $B_0(1 - M/T) \geq M - m$. This condition requires $T > M$, and in this case becomes $B_0 \geq T(M - m)/(T - M)$. Note that the condition is sufficient and applies to both models B and C. $\square$

Proof of Theorem 2. Consider the following strategy for each voter on either side: cast only the regular vote over the first $T-2$ decisions; at $T-1$, cast all bonus votes if $v_i > \alpha$ ($i \in \{m, M\}$), for a fixed $\alpha > 0$, and none otherwise; cast all remaining votes in the last election. We show in step (i) that if $m > 2$ then there exists a B_0 for which such strategies are equilibrium strategies. We then show in (ii) that in such an equilibrium $EV_0 > EW_0$ if $m > M/2$.

(i). Suppose all other voters are following such a strategy. In the first $T-2$ periods, $m + B_0 < M$ (or $B_0 < M - m$) is sufficient to rule out deviation by a minority voter, because he can cast at most all his bonus votes. In period $T-1$, $B_0 < M-m$ is again sufficient to rule out deviation by a minority voter if $v_m < \alpha$, because the voter can hope to overturn the decision in minority's favor only if the majority is not using its bonus votes. But note that the condition is also sufficient to rule out deviation when $v_m > \alpha$ because in such a case a minority voter can be tempted to withdraw some or all of his bonus votes only if by doing so he can overturn a T-period decision against the minority, or, again, only if $m + B_0 < M$. Majority voters always win the first $T-2$ decisions. At $T-1$, if $v_M < \alpha$, a majority member can be tempted to cast some or all of his bonus votes only if by doing so he can turn in majority's favor a decision that would otherwise be won by the minority. Thus a sufficient condition ruling out such a deviation is: $M + B_0 < m(1+B_0)$, or $B_0 > (M-m)/(m-1)$. As in the case of the minority, the condition is also sufficient to rule out deviation when $v_M > \alpha$. Thus for all $m > 2$, there exists $B_0 \in ((M - m)/(m - 1), M - m)$ such that the strategies are equilibrium strategies for all voters. Note that $M + B_0 < m(1 + B_0)$ implies $M < m(1 + B_0)$: the minority wins at $T - 1$ if $(v_{mT-1} > \alpha, v_{MT-1} < \alpha)$, and wins at T if $(v_{mT-1} < \alpha, v_{MT-1} > \alpha)$. The majority wins at all other times.

(ii). When all voters follow these strategies, $EV_0 > EW_0$ iff:

$$F(\alpha)\left[M\int_0^\alpha vdF(v)+F(\alpha)M\int_0^1 vdF(v)\right]+$$
$$+[1-F(\alpha)]\left[M\int_\alpha^1 vdF(v)+[1-F(\alpha)]M\int_0^1 vdF(v)\right]+$$
$$+F(\alpha)\left[M\int_\alpha^1 vdF(v)+[1-F(\alpha)]m\int_0^1 vdF(v)\right]+$$
$$+F(\alpha)\left[m\int_\alpha^1 vdF(v)+[1-F(\alpha)]M\int_0^1 vdF(v)\right]>2M\int_0^1 vdF(v)$$

Simplifying:

$$F(\alpha)\left[MF(\alpha)+m(1-F(\alpha))\right]\int_0^1 vdF(v)+ \tag{A1}$$
$$+[mF(\alpha)+M(1-F(\alpha))]\int_\alpha^1 vdF(v)>M\int_0^1 vdF(v)$$

Note that the left-hand side simplifies to $M\int_0^1 vdF(v)$ when evaluated at either $\alpha=0$ or $\alpha=1$, since in both cases the majority always wins (and thus $EV_0 = EW_0$). Taking the derivative of (A1) with respect to α and evaluating it at $\alpha=0$, we obtain:

$$\left.\frac{\partial(EV_0-EW_0)}{\partial\alpha}\right|_{\alpha=0}=f(0)\int_0^1 vdF(v)(2m-M)>0\Leftrightarrow m>M/2$$

Thus if $m > M/2$ there exists a threshold $\alpha > 0$ such that the strategies described above lead to higher ex ante welfare than simple majority voting. □

Example. *Model B.*

(A) Equilibrium. To verify that the strategy described is an equilibrium, consider the best response for voter i. If i casts x_{i1} votes in the vote over the first proposal, his expected utility over the whole game is: $EU_i|x_{i1} = v_{i1}prob(W_1|x_{i1}) + E(v)prob(W_2|4-x_{i1})$ where $prob(W_t|x_{it})$ is i's probability of obtaining the desired outcome in period t conditional on casting x_{it} votes, and $E(v)=0.5$. Since $(n-1)$ is an even number, and every other voter is casting either 1 or 3 votes, the difference in votes between the two sides, excluding i, must be even for both proposals. Thus, when i considers the choice between casting 3, 2 or 1 votes, the only case in which the choice matters is a difference of 2 votes in his side disfavor, either over proposal 1 or proposal 2:

$$EU_i|3 \quad > \quad EU_i|2 \Leftrightarrow v_{i1}[prob(\Delta x_{1-i}=2)] > 0.5[prob(\Delta x_{2-i}=2)]$$

$$EU_i|2 \quad > \quad EU_i|1 \Leftrightarrow v_{i1}[prob(\Delta x_{1-i}=2)] > 0.5[prob(\Delta x_{2-i}=2)]$$

(where Δx_{1-i} indicates the number of votes by which i's side is losing, absent i's vote). Given the symmetry of $F(v)$, in the candidate equilibrium the probability of any other voter casting 1 or 3 votes is identical, implying: $prob(\Delta x_{1-i} = 2) = prob(\Delta x_{2-i} = 2)$. Thus i's best response is to cast 1 vote if $v_{i1} < 0.5$ and 3 votes if $v_{i1} > 0.5$; the conclusion holds for all i, and the strategy is indeed an equilibrium. If $M > 3m$, $prob(\Delta x_{1-i} = 2) = prob(\Delta x_{2-i} = 2) = 0$, and the number of votes cast is irrelevant.

(B) Frequency of minority victories. Write the majority size as $M = m + 2k - 1$, with $k \geq 1$ (recall than n is odd). The minority wins the first vote if there are at least k more valuations above 0.5 among the minority than the majority. Given the symmetry of the Uniform, the probability of this event is given by the formula in the text. The minority wins the second vote if there are at least k more valuations below 0.5 over the first proposal among the minority than the majority, an event that again, given the symmetry of the Uniform distribution, has the probability given in the text. Note that k must be smaller than m, implying that the majority always wins if $M \geq 3m$.

(C) Efficient frequency of minority victories. According to our efficiency criterion, the minority should win whenever the sum of its valuations is larger than the sum of the majority's valuations. Call y (z) the sum of m (M) independent random variables, each distributed Uniformly over $[0, 1]$. The efficient frequency of minority victories is then given by $\int_0^m \left(\int_z^m P_m(y)dy\right) P_M(z)dz$ where:

$$P_m(y) = \frac{1}{2(m-1)!}\sum_{s=0}^{m}(-1)^s\binom{m}{s}(y-s)^{m-1}sign(y-s) \qquad \text{(A2)}$$

(and correspondingly for $P_M(z)$).

(D) *Expected payoff. (i) Equilibrium.* With n odd and the equilibrium strategies described above, the difference in votes cast by the two groups is always an even number. In addition, the symmetry of the Uniform distribution guarantees that the probability of any given difference in votes is equal over the two proposals. If we call $prob(W_M|x)$ the probability of obtaining the desired outcome for $i \in M$, conditional on casting x votes, we can write the ex ante expected payoff of a majority member as:

$$EV_{Bi} = (3/8)prob(W_M|1) + (5/8)prob(W_M|3) \qquad \forall i \in M$$

where $prob(W_M|1) = prob(x_{M-i} \geq x_m)$ and $prob(W_M|3) = prob(x_{M-i} \geq x_m - 2)$. Recall that $M = m+2k-1$. Given the equilibrium strategies, the symmetry of the Uniform distribution, and the independence of the valuation draws, if we call "high" a valuation above 0.5, $prob(x_{M-i} \geq x_m)$ equals the probability that the number of high draws in the minority group is at most $k-1$ higher than for the majority group, excluding voter i:

$$prob(W_M|1) = 1 - \sum_{s=k}^{m}\left[\sum_{r=0}^{m-s}\binom{M-1}{r}\binom{m}{r+s}\right]2^{-(M-1+m)}$$

Similarly, $prob(x_{M-i} \geq x_m - 2)$ equals the probability that the number of high draws in the minority group is at most k higher than for the majority group, excluding voter i:

$$prob(W_M|3) = 1 - \sum_{s=k+1}^{m}\left[\sum_{r=0}^{m-s}\binom{M-1}{r}\binom{m}{r+s}\right]2^{-(M-1+m)}$$

Analogous calculations yield the ex ante expected payoff of a minority member:

$$EV_{Bj} = (3/8)prob(W_m|1) + (5/8)prob(W_m|3) \qquad \forall j \in m$$

where:

$$prob(W_m|1) = \sum_{s=k}^{m-1}\left[\sum_{r=0}^{m-s-1}\binom{M}{r}\binom{m-1}{r+s}\right]2^{-(M+m-1)}$$

and

$$prob(W_m|3) = \sum_{s=k-1}^{m-1}\left[\sum_{r=0}^{m-s-1}\binom{M}{r}\binom{m-1}{r+s}\right]2^{-(M+m-1)}$$

Having derived the ex ante expected payoff of a majority and a minority member, respectively - payoffs that are reported in Figure 2 - we can write the ex ante aggregate expected payoff in equilibrium as $EV_B = M(EV_{Bi}) + m(EV_{Bj})$, $i \in M$, $j \in m$.

(ii) First best efficiency. For each proposal, the ex ante efficient aggregate payoff EU_B^* is easily derived, given (A2):

$$EU_B^* = \int_0^m \left(\int_z^m yP_m(y)dy\right)P_M(z)dz + \int_0^m \left(\int_y^M zP_M(z)dz\right)P_m(y)dy \tag{A3}$$

Over the two proposals, the ex ante efficient payoff is $2EU_B^*$. The first term in (A3) corresponds to the efficient expected payoff for the minority group, and the second for the majority group. The corresponding per capita values (multiplied by 2) are plotted in Figure 1b. *(iii) Simple majority voting.* With simple majority voting, the majority always wins. Its expected payoff equals the aggregate expected payoff and is given by: $\int_0^M zP_M(z)dz = M/2$ or M over the 2 proposals. *(iv) Random choice.* If each group has a fifty percent chance of winning any vote, the aggregate expected payoff is $1/2(M/2) + 1/2(m/2)$ over each proposal, or $(M+m)/2$ for the 2-proposal game.

Example. *Model C.*

(A) Equilibrium. The majority can ensure itself victory over all proposals if $2M > 3m$. Suppose then $2M \leq 3m$. When $x_m = m$, the minority always loses ($m < \max\{M, m+3\} < \min\{3M, 4M-(m+3)\}$). The only possible deviation for a minority member is to cast 2 or 3 votes when $x_{m-i} = m-1$, but $m+2 < \max\{M, m+3\} < \min\{3M, 4M-(m+3)\}$: the deviation cannot be profitable. The majority always wins when casting $\min\{3M, 4M-(m+3)\}$ votes, but loses when $x_M = \max\{M, m+3\}$ if $x_m = 3m$. A majority member could deviate and use his bonus votes when $x_{M-i} = \max\{M-1, m+2\}$. But

casting 2 votes cannot be profitable: with $2M \leq 3m$, $\max\{M+1, m+4\} < 3m$. And neither can casting 3: with $2M \leq 3m$, either $\max\{M+2, m+5\} < 3m$ and $\min\{3M-2, 4M-(m+5)\} > 3m$, in which case the outcomes are unchanged; or $\max\{M+2, m+5\} > 3m$ and $\min\{3M-2, 4M-(m+5)\} < 3m$, in which case the certainty of winning at $v_M > 0.5$ is traded for the certainty of winning in the future, with $E(v) = 0.5$ - a net loss in expected utility.

(B) Frequency of minority victories. If $2M \leq 3m$ the minority wins the first vote if $(v_{m1} > 0.5 \cap v_{M1} < 0.5)$ and the second if $(v_{m1} < 0.5 \cap v_{M1} > 0.5)$ - given the symmetry of the Uniform distribution, it wins each vote with probability 0.25.

(C) Efficient frequency of minority victories. Given the perfect correlation of valuations within each group, the efficient frequency of minority victories is given by $prob(Mv_M < mv_m) = \int_0^1 \int_0^{(m/M)v_m} dv_M dv_m = m/(2M)$.

(D) *Expected payoff. (i) Equilibrium.* If $2M > 3m$, the majority always wins and the expected aggregate payoff over the two proposals equals M. If $2M \leq 3m$, the expected aggregate payoff equals: $(1/4)(M/4 + M/2) + (1/4)(3M/4 + M/2) + (1/4)(3M/4 + m/2) + (1/4)(3m/4 + M/2) = (13M + 5m)/16$ (where the first term is the expected payoff over the two proposals when $(v_{m1} < 0.5 \cap v_{M1} < 0.5)$, the second when $(v_{m1} > 0.5 \cap v_{M1} > 0.5)$, the third when $(v_{M1} > 0.5 \cap v_{m1} < 0.5)$, and the fourth when $(v_{m1} > 0.5 \cap v_{M1} < 0.5)$ - all events with probability 1/4). *(ii) First best efficiency.* In model C we can represent the total valuation of the minority (majority) group by a random variable y (z), Uniformly distributed over $[0, m]$ ($[0, M]$). The efficient aggregate expected payoff, per proposal, is given by:

$$EU_C^* = \int_0^m \left(\int_z^m \frac{y}{m} dy \right) \frac{1}{M} dx + \int_0^m \left(\int_y^M \frac{z}{M} dz \right) \frac{1}{m} dy = \frac{m^2 + 3M^2}{6M} \quad \text{(A4)}$$

Over the two proposals, the ex ante efficient payoff is $2EU_C^*$. The first term in (A4) corresponds to the efficient expected payoff for the minority group $(m^2/(3M))$, and the second for the majority group $((3M^2 - m^2)/6M)$. The corresponding per capita values (multiplied by 2) are plotted in Figure 1b. *(iii) Simple majority voting.* With simple majority voting, the majority always wins, and its expected payoff, which equals the aggregate expected payoff, is given by: $\int_0^M \frac{z}{M} dz = M/2$ or M over the 2 proposals. *(iv) Random choice.* If each group has a fifty percent chance of winning any vote, the aggregate expected payoff is $1/2(M/2) + 1/2(m/2)$ over each proposal, or $(M+m)/2$ for the 2-proposal game.

SAMPLE INSTRUCTIONS (CChat1)

Thank you for agreeing to participate in this decision making experiment, and for arriving on time. During the experiment we require your complete, undistracted attention, and ask that you follow instructions carefully. You may not open other applications on your computer, chat with other students, or engage in other distracting activities, such as using your phone, reading books, etc.

You will be paid for your participation in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance.

The entire experiment will take place through computer terminals, and all interaction between you will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiments.

We will start with a brief instruction period. During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. If you have any questions during the instruction period, raise your hand and your question will be answered out loud so everyone can hear. If you have any questions after the experiment has begun, raise your hand, and an experimenter will come and assist you.

The experiment you are participating in is a voting experiment, where you will be asked to allocate a budget of several votes over two different proposals. We will begin with a practice session. The practice session will be followed by the paid session, which will consist of 20 matches. Each match will have elections for two different proposals, and you will receive a new budget of votes at the beginning of each match.

At the end of the paid session, you will be paid the sum of what you have earned, plus a show-up fee of $10.00. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Your earnings during the experiment are denominated in FRANCS. Your DOLLAR earnings are determined by multiplying your earnings in FRANCS by a conversion rate. For this experiment the conversion rate is 0.01, meaning that 100 FRANCS equal 1 DOLLAR.

DESCRIPTION

At the beginning of the first match, you will be randomly assigned with 4 other persons in the room to form a 5-voter committee, which votes over two different proposals, in sequence. Of the 5 voters of this committee, 2 voters belong to the FOR group; the remaining 3 voters belong to the AGAINST group. Whether you belong to the FOR or to the AGAINST group is decided randomly by the computer and will be displayed on your computer monitor.

The groups not only differ in size, but also differ in their preference over proposals. Specifically, all voters in the FOR group are always in favor of all proposals; all voters in the AGAINST group are always against all proposals.

Each voter is given one "regular" vote to cast in each of the two proposal elections. You must always use this vote in each proposal election. In addition, each voter is given a total of 2 “bonus votes” at the beginning of each match that you will use in addition to the regular votes.

The first proposal your committee votes on is called Proposal A. You may cast up to 3 votes in the A election (your regular A vote plus either 0, 1, or 2 of your bonus votes.) Before proceeding to the vote, you are assigned your personal Proposal A value. If your value is positive, you are in favor of Proposal A; if your value is negative, you are against Proposal A. Each voter of the FOR group is in favor of Proposal A and has a positive value for Proposal A which is equally likely to be any amount between 1 and 100 francs. Every member of the FOR group is assigned the SAME Proposal A value by the computer. Each voter of the AGAINST group is against Proposal A and has a negative value for Proposal A which is equally likely to be any amount between -1 and -100 francs. Every member of the AGAINST group is assigned the same Proposal A value by the computer.

If you are in the FOR group, you earn your value if A passes. If you are in the AGAINST group, you earn the absolute value of your value if A does not pass. For example, if your are in the AGAINST group and your proposal A value is –55, then you earn 55 francs if A does not pass, and 0 francs if A passes. A passes if there are more YES votes than NO votes in the A election. A does not pass if there are more NO votes than YES votes. Ties are broken randomly. In this example, you also know that the other two members of the AGAINST group also have Proposal A values of –55.

After being told your proposal A value, you will be allowed two minutes to exchange messages with the other members of your group. The messages you send and receive are not seen by members of the other group. They are private messages within your group. The messages must conform to the following rules. 1. Your messages must be relevant to the experiment. Do not engage in social chat. 2. You are not permitted to send messages that are intended to reveal your identity or participant ID number. 3. The use of threatening or offensive language, including profanity, is not permitted.

At any time during this 2 minute period, you can make your individual voting decision. You must decide whether to cast 1 vote, 2 votes, or 3 votes in the proposal A election. If you are in the FOR group, any votes you cast will be automatically counted as YES votes for A. If you are in the AGAINST group, any votes you cast will be automatically counted as NO votes.

The experimenter will announce when the two minute period is finished. If you haven’t yet voted, please vote when the announcement is made, so we can all proceed to the next proposal. You are not told how the other people have voted until after you cast your vote, although you are free to say whatever you wish about your voting decision to the other members of your group during the two minute message stage.

Whatever bonus votes you do not use in the A election, will be saved for you to use in the proposal B election. For example, if you cast 1 vote in the A election, all your bonus votes will be saved for the B election. If you cast 2 votes in the A election, only 1 of your bonus votes will be saved, and if you cast 3 votes in the A election, none of your bonus votes are saved.

After you and the other voters in your committee have made voting decisions, you are told the outcome of the proposal A election, and the total number of votes FOR and AGAINST. You then proceed to the proposal B election. You are in the same committee for the proposal B election as you were for the proposal A election. In addition, if you were in the FOR group in the A election, you remain in the FOR group in the B election (and if you were in the AGAINST group, you remain in the AGAINST group).

There is no message stage for Proposal B. When you and the voters in your committee are ready to proceed, you will each be assigned proposal B values in the same manner that your proposal A values were assigned. Each voter's assigned value for proposal B will typically be different than their proposal A values. All voters in the FOR group still receive positive values, and these values are the same for all members of the FOR group. All voters in the AGAINST group receive negative values, and these values are the same for all members of the AGAINST group. All your remaining votes will automatically be cast as YES votes for proposal B if you are a FOR voter, and as NO votes if you are an AGAINST voter. The outcome of the B election is then reported to you.

When everyone has finished this completes the first match, and we will then go to the next match. You will be rematched with 4 other people to form a new 5-person committee, and repeat the procedure described above. The voters in your new committee will be selected randomly by the computer, but if you were a FOR voter in the first committee, you will still be a FOR voter for the rest of the experiment. And if you were an AGAINST voter in the first match, you will still be in the AGAINST group for the rest of the experiment. As in the first match, your new committee has 2 FOR voters and 3 AGAINST voters.

After your new committee has finished voting on both proposals in the second match, you will again be rematched into a new committee in a similar way, and this will continue for 30 matches. Remember that each match consists of 2 proposals, every committee has 2 FOR voters and 3 AGAINST voters. Also remember, if you are a FOR voter, you will always be a FOR voter, and if you are an AGAINST voter, you will always be an AGAINST voter.

PRACTICE SESSION

We will now give you a chance to get used to the computers with a brief practice session. Are there any questions before we begin the practice match?

[ANSWER QUESTIONS]

You will not be paid for this practice session; it is just to allow you to get familiar with the experiment and your computers. During the practice session, do not press any keys or click with your mouse, unless instructed to. When we instruct you, please do exactly as we ask. We will now hand out record sheets for you to record important information during the experiment. Please raise your hand if you need a pen or pencil.

HAND OUT RECORD SHEETS AND PENS AND COLLECT YELLOW CARDS

Please pull out your dividers so we can begin the practice session.

[START GAME on SERVER]

FIRST PPT SLIDE

This is the decision screen for Proposal A in match 1. Your ID# is printed at the very top left of your screen. Please record this on your record sheet.

The screen tells you your proposal A value, whether you are in the FOR group or in the AGAINST group, and the number of people in each group (always 2 for the FOR group and 3 for the AGAINST group in this experiment). Then the screen tells you the number of votes you have available. The bottom window of your screen is the history table, which is blank now because nothing has happened yet.

Please record your proposal A value on your record sheet in the row labeled "Practice 1 A". Rememer that everyone in your group has the same value as you do. That is, everyone in the FOR group of your committee has the same positive proposal A value in this round, and everyone in the AGAINST group of your committee has the same negative proposal A value this round. For example, if you are in the FOR group and your proposal A value is 41, then this tells you that both of the other members of your committee's FOR group in your committee also have a proposal A value equal to 41. The AGAINST group members of your committee also share a proposal A value, but all you would know is that it is some negative number between -1 and -100.

It is important that you understand how these values are assigned. Are there any questions before we proceed with the practice round?

After recording this information, we begin the 2 minute message stage. Messages are entered by typing on the line at the very bottom of the screen and then clicking the send button. Everyone please practice this once by sending the message "Hello" now. Notice that this is echoed in the message display box, and your message is also displayed on your screen. Also notice that each of your have been assigned a temporary number that identifies you anonymously to the other members of your group. For example, the two members in a FOR group, are assigned temporary id numbers 1 and 2. The three members in the AGAINST group are assigned temporary id numbers 1, 2, and 3.

At any time during the 2 minute message stage, you may choose how many votes to cast in the A election, by clicking on the arrow key. You may cast either 1, 2, or 3 votes in this election. Any unused votes in this election will be saved for you to use in the B election of this match.

If your proposal A value is positive, then all votes you cast will count as YES votes for A, and if your proposal A value is negative, then all votes you cast will count as NO votes. When you have selected the number of votes you wish to cast in this election, please click on the "vote" button. Please record the number of votes you cast on your record sheet. Then wait for all other voters in the room to finish casting their Proposal A votes. The proposal passes if there are more YES votes than NO votes. Tie votes are broken randomly by the computer.

SECOND PPT SLIDE

The experimenter will announce when the 2 minute message stage is over. Please make your voting decision at this time, if you have not done so already. Once everyone has made their vote decision for the A election, the votes are tallied and the results for your match are displayed in the results window. The window displays your Proposal A value, the number of votes you cast, the total number of YES votes cast in the election, the total number of NO votes, the outcome, and your payoff from the A election. Please record all of this information on your record sheet.

Then click OK when you are ready to proceed to the proposal B election.

THIRD PPT SLIDE

We are now in the B election. Notice that the history screen has been updated and includes a summary of the previous proposal A election. There is no message stage for Proposal B, and your voting decision is determined completely by how many votes you cast in the A election. But you will need to read the information on the screen and record it. Please record your proposal B value on your record sheet in the row labeled "Practice 1 B". This screen reminds you how many votes you have remaining. This number equals the number of bonus votes you did not use for proposal A plus your regular proposal B vote. Please record this number on your record sheet in the column labeled "your vote". Then click on the "Vote" button. All these votes are now automatically cast by the computer. They are recorded as YES votes for proposal B if you are in the FOR group, and as NO votes if you are in the AGAINST group.

FOURTH PPT SLIDE

Once everyone has made their vote decision for the B election, the votes are tallied and the results for the people in your committee are displayed in the results window. The screen displays the number of YES votes and the number of NO votes, the outcome, and your payoff in francs. Please record this information on your record sheet.

Please press OK when you are ready to proceed.

FIFTH PPT SLIDE

Once everybody has pressed OK, a new window appears and displays what your dollar payoff would have been if this were a paid match instead of a practice match. It also displays your total dollar payoff from all previous matches, which so far is zero. You do not need to record your cumulative payoff after each match. But you will need to record it at the very end of the experiment. Please press OK when you are ready to proceed.

We have now completed the first practice match. We will now proceed to the second practice match. Remember that you are assigned to a new committee in this match, although you will continue to be a FOR voter if you were a FOR voter in the first committee; you will continue to be an AGAINST voter if you were an AGAINST voter in the first committee. Everyone is randomly assigned to a new committee after every match in the experiment. Notice that the full-view history contains the information about what you did in the first match. Please raise your hand if your history screen does not show this information.

Please complete the second practice match on your own, by following the same directions as in the first practice match. Don't forget to record the information as it appears on your screen. Remember, you are not paid for these practice matches. Feel free to raise your hand if you have any questions.

When everyone has made their vote decisions for proposal A and proposal B in this practice match, and the screen with the proposal B results has appeared at the end of the match, please wait for further instructions. Do NOT click OK on that screen.

[WAIT FOR SUBJECTS TO COMPLETE PRACTICE MATCH 2]

Practice match 2 is now over. Please press OK to go to the final screen of the practice session, displaying your payoff from the current match, and your total payoff in the experiment so far. Do not press OK yet. You do not need to record your total payoff because this was a practice session. You will have to record it at the end of the paid session. Any questions?

Please press OK when you are ready to proceed.

If you have any questions from now on, raise your hand, and an experimenter will come and assist you.

Please pull out the dividers to ensure your privacy and the privacy of others.

Please click OK and begin the first paid match.

(Play matches 1 – 20)

This completes the experiment. Please make sure to record your total payoffs on your record sheet, including your $10 show-up fee. Please remain in your seat and we will come by to check your total. Do not use the computers or talk with each other. We will pay each of you in private in the next room in the order of your seat numbers. Please sign and turn in your record sheet when you receive payment. You are under no obligation to reveal your earnings to the other participants. Thank you for your participation.