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# ON THE PROBABILITY OF THE COMPETITIVE EQUILIBRIUM BEING GLOBALLY STABLE: THE C.E.S. EXAMPLE

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## Abstract

This paper extends an anaylsis proposed by Hirota (1981) to a class of economies with C.E.S. utility functions that include Scarf (1960)'s second example as a special case and shows by the use of *numerical methods*that (i) a Walrasian price adjustment mechanism converges to an equilibrium with very high probability and (ii) the weak axiom in revealed preference for market excess demands is satisfied with high probability, but the gross substitutability is rarely satisfied. Also, this paper suggests a possible interpretation of a Walrasian price adjustment that is based on the observations of experiments done by Anderson, Plott, Shimomura and Granat (2000).

JEL classification numbers:

Key words: probability, numerical method, global stability, Walrasian price adjustment, weak axiom, gross substitutes

# On the Probability of the Competitive Equilibrium Being Globally Stable: The C.E.S. Example

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## 1 Introduction

The problem of stability of the competitive equilibrium was intensively discussed in the 1950's by Arrow and Hurwicz (1958), Arrow, Block and Hurwicz (1959), and many others: the most important result was that global stability occurs if all of the goods are gross substitutes, though the plausibility of such condition has been left still unclear. It was finally settled by Scarf (1960)'s demonstration of the counter examples to the conjecture that the Walrasian price adjustment mechanism was always stable: Scarf provided two examples, the first with Leontief type utility function, the second with C.E.S. utility function. From those instability examples of Scarf and subsequent analyses of market excess demands studied by Sonnenschein, Mantel, Debreu, and others, it is widely held that the phenomenon of instability of the competitive equilibrium will be relatively common. This recognition naturally led to the study of advanced mathematical techniques (see Scarf (1973)) for computing an equilibrium instead of a simple price adjustment that seems to have become an old-fashion method.

Twenty years ago, the author (1981) asked whether or not the instability of Scarf's first example is robust against changes in the assignment of initial holdings and clarified that global stability returns for about 80 percent of all assignment patterns. Additionally Hirota (1985) extended this analysis to a class of economies with arbitrary utility functions which include the perfect complementarity of Scarf's example as a limiting case. However, this dominancy of global stability is asserted only in a very restricted class of economies, and nothing was mentioned about a more general class of economies such as Scarf's second example with the C.E.S. utility functions that represent more reality than types of perfect complementarity.

Recently Anderson, Plott, Shimomura and Granat (2000) have applied a model of Scarf's first example to the experimental study of double auction markets[2] and have observed very remarkable facts: the stream of the average prices

[1] I would like to thank Peter Bossaerts, Charles Plott, and other faculties at Caltech for a number of stimulating discussions. In particular, I am grateful to Charles Plott for his suggestions in this version.

[2] There is an experiment of multiple tatonnement market system (different from double auction market) proposed by Plott (1988). He demonstrated that due to the lack of incentive compatibility, such system never converged in the sense that the system iterated without ever receiving quantities demanded and supplied that would allow the system to stop at the price.

of the actual transaction prices roughly follows the solution paths of a tatonnement process. That is, when the solutions oscillate without tending to converge, the data show the same motion, and when the solutions converge to the equilibrium, the data also show such convergence. Their results may suggest a possible interpretation of a Walrasian price adjustment which is quite different from the common one (for example, see Hicks (1989; pp.7-11)) and might have been partially close to the final intention of Walras (1873; pp.83-91, 153-172): Walras states that the actual, well organized markets close to his theoretical, frictionless, competitive markets are those of the stock exchange, the grain, etc. in which purchases and sales are made by auction (as buyers, the traders make their demands by outbidding each other, and as sellers, the traders make their offers by underbidding each other). Although Walras does not succeed to precisely describe the dynamic complexity of disequilibrium transaction prices in such markets, he seems to consider or at least to intend that as time proceeds, the stream of complex transaction prices, through the mechanism of free competition, will gravitate to a general equilibrium, and its motion of prices may to some extent be represented by his successive tatonnement process. (as to an experimental study of a traditional adjustment process that is different from the Walrasian one, see Plott and George (1992).)

Motivated by the above, in this paper we extend an analysis proposed by Hirota (1981) to a class of economies with the C.E.S. utility functions that include Scarf's second example as a special case and show by the use of *numerical method* that the differential equations representing a simple price adjustment process converge to an equilibrium point with very high probability. In addition, we explore to what extent the above class of economies will satisfy the important stability conditions such as the gross substitutability and the weak axiom in revealed preference for market excess demands and show that the weak axiom is satisfied with high probability, but the gross substitutability is rarely satisfied. This explore seems to be important because there is nothing in the literature so far which enables us to see how those conditions will be plausible not only in theory but in practice.

The meaning of this study developed here is as follows:

(1) As long as the C.E.S. utility functions often used in empirical studies are supposed, the phenomenon of stability will be considered to be common, contrary to the conventional understanding (see Negishi (1962)) that is widely held. As Scarf (1981) states in comment on Hirota (1981), a simple tatonnement process may be still useful in practice as an algorithm for computing an equilibrium because it may converge with high probability.

(2) The gross substitutability has very few possibility to be satisfied, so that many assertions based on this assumption have to be reconsidered. On the other hand, the probability that the weak axiom is satisfied will be very high and considerably close to that of stability. This fact means that the market excess demands may be highly probable to be well behaved functions which bring about fruitful results in theory.

(3) If, as Anderson, Plott, Shimomura and Granat show and Walras might have considered, the Walrasian price adjustment could describe, as a general

tendency, the direction of the motion of the transaction prices in well organized markets, our study here would give an indication of the extent to which the competitive equilibrium in theory may be prevalent in some reality.

## 2 Model and Specific Space of Parameters

Let us consider an exchange economy with three commodities and three individuals each of whose utility functions is assumed to be of the form

$$U(x_1, x_2, x_3) = [b_1^{\rho+1} \cdot x_1^{-\rho} + b_2^{\rho+1} \cdot x_2^{-\rho} + b_3^{\rho+1} \cdot x_3^{-\rho}]^{-\frac{1}{\rho}} \tag{1}$$

The parameters $(b_1, b_2, b_3) \geq 0$ and $\rho \geq 0$ respectively denote the relative importance among goods and the degree of substitutability among goods, and $1/(1+\rho) = c$ is called "elasticity of substitution." It should be noted that as $c \to 0$, (1) becomes $\mathrm{Min}[x_1/b_1, x_2/b_2, x_3/b_3]$, and as $c = 1$, (1) becomes the Cobb-Douglas function.

Since, in our analysis, the parameters $(b_1, b_2, b_3)$ for all individuals are randomly chosen from a parameter space we are about to give, we shall represent them as $b^j = (b_{j1}, b_{j2}, b_{j3})$'s, where the first subscript refers to the person and the second to the commodity. In order to avoid the complexity of description let us assume that a value of $c$ is common for all the individuals (even if different values are supposed, the results studied below do not change). Also let the initial endowments of all individuals be represented by $a^j = (a_{j1}, a_{j2}, a_{j3})$'s that will be randomly chosen from a certain space.

When given all the parameters, for any prices $p = (p_1, p_2, p_3) > 0$ the excess demands of each individual are derived from utility maximization subject to the budget constraint, and summing these up, we obtain the following market excess demand functions that are defined parametrically:

$$f_i(p; a, b, c) = \sum_{j=1}^{3} \left[ \frac{b_{ji} \cdot p_i^{-c} \cdot M_j}{\sum_{k=1}^{3} b_{jk} \cdot p_k^{1-c}} - a_{ji} \right], \quad i = 1, 2, 3, \tag{2}$$

where $M_j \equiv \sum p_i a_{ji}$ is the nominal income of the $j^{th}$ individual, both $a = (a^1, a^2, a^3)$ and $b = (b^1, b^2, b^3)$ are points in nine-dimensional real space. (2) is homogeneous of degree zero with respect to $p$ and $b$, and is homogeneous of degree one with respect to $a$.

Now consider a simple price adjustment process in which the rate of change of price of each commodity is assumed to be simply equal to the excess demand of that commodity (for simplicity we are neglecting the problem of adjustment speed). Mathematically this is given by the following parametric differential equations:

$$\frac{dp_i}{dt} = f_i(p; a, b, c), \quad i = 1, 2, 3. \tag{3}$$

If we take the initial prices $p(0) > 0$ on the sphere $\sum_{i=1}^{3} p_i^2 \equiv \lambda > 0$, then the solution $p[t; p(0)]$ to (3) will always be on that sphere as long as the existence is assured, since $\sum p_i \frac{dp_i}{dt} = \sum p_i f_i \equiv 0$ by the Walras Law.

In this section a restriction is put on the parameters $a^j$'s and $b^j$'s that enables a study of the concern at hand in close relation to the past works of Scarf (1960) and Hirota (1981).

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$,

where the former is called the endowment matrix and the latter the evaluation matrix.

**Assumption 1** *A and B are doubly stochastic, i.e.,*

$$\sum_{i=1}^{3} a_{ji} = 1, (j = 1, 2, 3), \quad \sum_{j=1}^{3} a_{ji} = 1, (i = 1, 2, 3), \tag{4}$$

$$\sum_{i=1}^{3} b_{ji} = 1, (j = 1, 2, 3), \quad \sum_{j=1}^{3} b_{ji} = 1, (i = 1, 2, 3). \tag{5}$$

When $p^* = (1, 1, 1)$ or its multiples prevail, the supply of each commodity and the income of each trader do not change for any $a = \left(a^1, a^2, a^3\right)$ satisfying (4), and the total demand for each commodity is unity for any $b = (b^1, b^2, b^3)$ satisfying (5), so that $p^* = (1, 1, 1)$ and its multiples are always an equilibrium. Of course some other equilibrium prices may emerge depending on the parameters chosen. Since the multiples of (5) do not change the demand functions, throughout this paper let us consider they also belong to a class of Assumption 1.

Before proceeding further, a short review of the past works seems useful for understanding the intention here. Taken here is a special case of parameters that are given by cyclic permutations of goods:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & \alpha & 1 \\ 1 & 0 & \alpha \\ \alpha & 1 & 0 \end{bmatrix}. \tag{6}$$

The famous instability examples of Scarf may now be described as the following cases: In addition to (6), $\alpha$ and $c$ satisfy either

$$\alpha = 1 \text{ and } c = 0 \tag{7}$$

or

$$\alpha > \frac{1}{1-2c} \text{ and } \frac{1}{2} > c > 0. \tag{8}$$

Case (7) corresponds to Scarf's first example in which each trader always demands the same quantity of only two goods and initially has a single unit of either good. On the other hand, case (8) corresponds to his second example in which a C.E.S. utility function having more reality is adopted; the indifference curves of both examples are of the forms in figure 1.

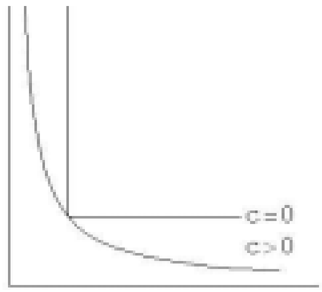


Figure 1:

Scarf (1960) showed the following: For the initial price vector $p(0)$ such that $\sum p_i^2 \equiv 3$, case (7) yields that the solutions of system (3) are subject to a limit cycle that is explicitly determined as the intersection of $p_1 \cdot p_2 \cdot p_3 = p_1(0) \cdot p_2(0) \cdot p_3(0)$ and the sphere $\sum p_i^2 \equiv 3$, and case (8) yields that the solutions move away from the equilibrium $p^* = (1, 1, 1)$ and approach some limit cycle that does not contain $p^*$. As Scarf states, the property of instability in the second example does not depend on a delicate change of parameters as long as (8) is satisfied. However, when given different assignments of initial holdings such as $a^1 = (1, 0, 0)$, $a^2 = (0, 1, 0)$, $a^3 = (0, 0, 1)$, global stability returns for both cases (7) and (8).

Let us consider a four-dimensional real set $Y$ (soon defined) satisfying (4), and define the stable set $G \subset Y$, each point of which leads to global stability of system (3) with case (7). Hirota (1981) showed that the ratio of measure $G$ to measure $Y$ is equal to $3/2 - \log 2$, approximately $0.8$. Since the stable set is this much, such complementarity as Scarf's alone cannot stem the stable function of the tatonnement processes, permitting the assertion that special assignments of initial holdings may be responsible for the instability of Scarf's first example.

The purpose of this paper is to extend the above results to a more general class of parameters $a, b$, and $c$ that includes Scarf's second example, but to do this following the mathematical method of Hirota (1981) seems extremely difficult. So in this paper the purpose is achieved by the use of a numerical method based on computer programs.

Now turning to describe the study here, Assumption 1 naturally leads to a four-dimensional real space as follows:

$$Y = \{y = (y_{11}, y_{12}, y_{21}, y_{22}) \in R_+^4 \mid \begin{array}{rcl} 1 & = & y_{i1} + y_{i2} \ \ (i = 1, 2) \\ 1 & = & y_{1i} + y_{2i} \ \ (i = 1, 2) \\ & & y_{11} + y_{12} + y_{21} + y_{22} = 1\}. \end{array} \tag{9}$$

Next are considered a space of "lattice points" included in $Y$,

$$\widetilde{Y}(n) = \left\{ \left( \frac{i}{n}, \frac{j}{n}, \frac{k}{n}, \frac{r}{n} \right) \in Y \right\}, \tag{10}$$

where $n$ is a positive integer, and integers $(i, j, k, r)$ are nonnegative integers less than or equal to $n$. The region of points in $\widetilde{Y}(n)$ may be demonstrated by Figure 2, which may help in making a program of system (3).

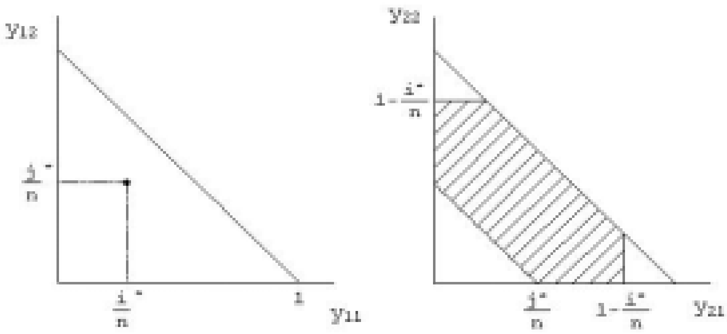


Figure 2:

When $\widetilde{Y}(n)$ is concerned with the initial endowments, it will be written as $C(n)$, and on the other hand when concerned with the evaluation parameters in utility functions, it will be written as $Q(m)$. Of course, $n$ and $m$ are positive integers.

**Definition 1** *(stable set of initial endowments)*

*For a given* $\widetilde{b} = (b_{11}, b_{12}, b_{21}, b_{22}) \in Q(m)$ *and a given* $c$*, a set* $G(b, c) \subset C(n)$ *is defined, each point of which leads to global stability of system (3) in the* *following sense: The solutions to (3) with the initial price vector* $p(0)$ *satisfying* $\sum p_i^2 \equiv 3$ *converge close enough to the equilibrium prices* $p^* = (1,1,1)$*, i.e.,* $|\, \mathrm{p}(t; p(0)) - \mathrm{p}^* | < \varepsilon$*, where* $\varepsilon$ *is a sufficiently small positive number.*

Definition 2 *(probability of global stability)*
*Let us assume that points of* $C(n)$ *and* $Q(m)$ *independently occur with equal frequency. In other words, they are randomly chosen from* $C(n)$ *and* $Q(m)$*. (I) For a given* $\widetilde{b} \in Q(m)$ *and a given* $c$*, the ratio of the number of points in* $G(b,c)$ *to the number of points in* $C(n)$ *implies a "conditional probability" of the system (3) being globally stable, written as*

$$\mathrm{P}(b,c) = \frac{\# G(b,c)}{\# C(n)}, \tag{11}$$

*where* $\# C$ *denotes the number of points in* $C$*. (II) A conditional probability may be defined for all points in* $Q(m)$*, so that its average implies the "total" probability of system (3) being globally stable. Formally,*

$$\mathrm{P}(c) = \frac{\sum_{\widetilde{b} \in Q} P(b,c)}{\# Q(m)}. \tag{12}$$

The computer simulation based on the programs mainly written with Fortran tells what values the above $P(b,c)$ and $P(c)$ actually take. (In this program the Runge-Kutta method and the Runge-Kutta-Fehlberg method have been adopted. Since the programs with Fortran are too long to demonstrate, simple examples of program written with Mathematica will be instead given in Appendix.) It is conceivable that these values will depend on the size of the lattice space considered, i.e. actually on the given values of $n$ and $m$. However, numerical studies show that for $n$ and $m$ larger than some value the above probability does not change very much. Thus the aim here will be achieved only if suitable values of $n$ and $m$ are tried.

Throughout the computer operations in this section a convergence measure $\varepsilon = 0.001$ and an initial price vector (for example, $p(0) = \left(\frac{1}{2}, \frac{1}{2}, \sqrt{\frac{5}{2}}\right)$ ) that is far from the equilibrium $p^* = (1,1,1)$ were given. Below lies a demonstration of the typical results obtained. In order to determine the sizes of two lattice spaces $C(n)$, $Q(m)$, in (I),(II) and (III) $n = 30$ was set, so that $\# C(30) = 123256$, and in (IV) $n = 20$ and $m = 6$ were set, so that $\# C \cdot \# Q = 10879176$.

(I) When $b^1 = (0,1,1)$, $b^2 = (1,0,1)$, $b^3 = (1,1,0)$, the conditional probability is:

$$
\begin{aligned}
P(b,c) &= \frac{123256}{\#C} = 1 \qquad \text{for } c = 0.4 \\
&= \frac{123232}{\#C} = 0.999 \qquad \text{for } c = 0.3 \\
&= \frac{116000}{\#C} = 0.941 \qquad \text{for } c = 0.1 \\
&= \frac{103660}{\#C} = 0.841 \qquad \text{for } c = 0
\end{aligned}
$$

(This data reconfirm the theorem in Hirota (1985).)
(II) When $b^1 = (0,6,1)$, $b^2 = (1,0,6)$, $b^3 = (6,1,0)$, the conditional probability is seen in:

$$
\begin{aligned}
P(b,c) &= \frac{123193}{\#C} = 0.999 \qquad \text{for } c = 0.4 \\
&= \frac{122382}{\#C} = 0.993 \qquad \text{for } c = 0.2 \\
&= \frac{121383}{\#C} = 0.983 \qquad \text{for } c = 0.1 \\
&= \frac{119749}{\#C} = 0.972 \qquad \text{for } c = 0
\end{aligned}
$$

(It should be noted that condition (8) is satisfied for $c \leq 0.4$. Therefore this case corresponds to the utility functions in Scarf's second example.)
(III) When $b^1 = (6,3,1)$, $b^2 = (3,1,6)$, $b^3 = (1,6,3)$, the conditional probability is seen in:

$$
\begin{aligned}
P(b,c) &= \frac{123256}{\#C} = 1 \qquad \text{for } c = 0.4 \\
&= \frac{121541}{\#C} = 0.990 \qquad \text{for } c = 0.2 \\
&= \frac{110767}{\#C} = 0.900 \qquad \text{for } c = 0.1 \\
&= \frac{86856}{\#C} = 0.704 \qquad \text{for } c = 0
\end{aligned}
$$

(IV) Finally, the total probability is seen in :

$$\begin{aligned} P(c) &= \frac{10875657}{\#C \cdot \#Q} = 0.999 \qquad \text{for } c = 0.4 \\ &= \frac{10800291}{\#C \cdot \#Q} = 0.993 \qquad \text{for } c = 0.2 \\ &= \frac{10320058}{\#C \cdot \#Q} = 0.949 \qquad \text{for } c = 0.1 \\ &= \frac{6409691}{\#C \cdot \#Q} = 0.589 \qquad \text{for } c = 0 \end{aligned}$$

The data above show that it will be highly probable, except for $c = 0$, that system (3) with a class of parameters satisfying Assumption 1 converges to the equilibrium $p^* = (1, 1, 1)$, and that in effect the equations of $f_i(p; a, b, c) = 0$ $(i = 1, 2, 3)$ have a unique equilibrium. It is also noted that for $c$ larger than 0.5, these probabilities become almost unity.

# 3 Gross Substitutes and Weak Axiom

It was seen in the previous section that the phenomenon of stability dominates system (3). In this section, by asking to what extent the well known stability conditions such as gross substitutability and weak axiom would be satisfied within the framework of system (2) with Assumption 1, we want to compare the results of both the previous and the present sections. This study clarifies how ad hoc presumptions imposed on excess demands will be applicable not only in theory but in practice.

The condition of gross substitutes is that the Jacobian of the excess demands $f_1$, $f_2$, $f_3$ has all of the off-diagonal elements being positive for all $p \geq 0$, i.e.,

$$\frac{\partial f_i}{\partial p_j} > 0 \qquad \text{if } \ i \neq j \tag{13}$$

("weak" gross substitutes in which some inequalities are permitted equal to zero is neglected in this paper.)

(13) yields the weak axiom in revealed preference for market excess demand functions ( see Arrow, Block, and Hurwicz (1959)), that is, $p^* \cdot f(p;\, a, b, c) > 0$ for all $p \neq p^*$ (an equilibrium price vector), which , under Assumption 1, is reduced to

$$\sum_{i=1}^{3} f_i(p;\ a, b, c) > 0 \ \text{ for } \ p \neq p^* \tag{14}$$

since all goods are able to have an identical prices at equilibrium. (It is to be noted that (13) is sufficient for (14), but not necessary.)

Let prices be taken on the interior of unit simplex, which leads to use a set

$$S = \{0 < p_1 + p_2 < 1, \ \ p_i > 0\}.$$

(No case with zero price is dealt with because our excess demand functions are defined only on the positive orthant.)

In order to try numerical study let us define a lattice set of $S$,

$$\widetilde{S}(h) = \left\{ \left( \frac{i}{h}, \frac{j}{h} \right) \in S \right\},$$

where $h$ is a positive integer giving the size of a space of lattice points, and integers$(i,\ j)$ are positive integers less than $h$.

When parameters $\widetilde{b} \in Q(m)$ and $c$ are given, by the computer simulation based on the programs written with Fortran, we can define two subsets of $C(n)$ for one set of which the excess demands $f_i(p;\, a, b, c)$'s satisfy (13) for all points of $\widetilde{S}(h)$ with a sufficiently large $h$ and for another set of which the excess demands satisfy (14), and then similarly as in the previous section we can also define these probabilities. It should be noticed that since only finite points of $S$ are examined in numerical studies, the "true"probability (should be defined for all points of $S$) must be somewhat less than the one in the consideration here. However, if a suitable size of $\widetilde{S}(h)$ is taken into consideration, its differences will be expected a very little.

(probability of gross substitutes) At first let us demonstrate typical results on gross substitutability that are obtained by the program with Fortran. (Simple example of the program with Mathematica will be given in Appendix.) To determine the sizes of three lattice spaces $C(n), Q(m), \widetilde{S}(h), n = 30$ and $h = 10$ were set in (I),(II),(III), so that $\#C = 123256$,and $n = 20, m = 10, h = 10$ were set in (IV), so that $\#C \cdot \#Q = 59245956$.

(I) When $b^1 = (0,1,1), b^2 = (1,0,1), b^3 = (1,1,0)$, the conditional probability is seen in:

$$\begin{aligned} P(b,c) &= \frac{98200}{\#C} = 0.796 \qquad \text{for } c = 0.9 \\ &= \frac{29162}{\#C} = 0.236 \qquad \text{for } c = 0.8 \\ &= \frac{121}{\#C} = 0.001 \qquad \text{for } c = 0.6 \end{aligned}$$

(II) When $b^1 = (0,6,1), b^2 = (1,0,6), b^3 = (6,1,0)$, the conditional probability is seen in:

$$\begin{aligned} P(b,c) &= \frac{105386}{\#C} = 0.855 \qquad \text{for } c = 0.9 \\ &= \frac{36590}{\#C} = 0.296 \qquad \text{for } c = 0.7 \\ &= \frac{69}{\#C} = 0.001 \qquad \text{for } c = 0.4 \end{aligned}$$

(III) When $b^1 = (6,3,1)$ , $b^2 = (3,1,6)$ , $b^3 = (1,6,3)$ , the conditional probability is seen in:

$$\begin{aligned} P(b,c) &= \frac{121397}{\#C} = 0.984 && \text{for } c = 0.9 \\ &= \frac{13907}{\#C} = 0.113 && \text{for } c = 0.7 \\ &= \frac{746}{\#C} = 0.006 && \text{for } c = 0.6 \end{aligned}$$

(IV) Finally, the total probability is as follows:

$$\begin{aligned} P(c) &= \frac{53906161}{\#C \cdot \#Q} = 0.909 && \text{for } c = 0.9 \\ &= \frac{9704706}{\#C \cdot \#Q} = 0.164 && \text{for } c = 0.7 \\ &= \frac{366649}{\#C \cdot \#Q} = 0.006 && \text{for } c = 0.5 \end{aligned}$$

The numerical study tells : in addition to the above data, for any $c$ less than $0.5$ the probabilty is almost equal to zero, so that the gross substitutability, except when the elasticity of substitution is close enough to unity, cannot be expected to dominate system (2) with Assumption 1. It seems, therefore, that this condition is too stringent to impose on the market excess demands.

(probability of weak axiom) What happens to the weak axiom? An answer lies in a demonstration below. In (I),(II),(III), $n = 30, h = 20$ or $h = 40$ or $h = 80$ were set, so that $\#C = 123256$,and in (IV) $n = 10, m = 10$ and $h = 30$ were set, so that $\#C \cdot \#Q = 59245956$.

(I) When $b^1 = (0,1,1)$ , $b^2 = (1,0,1)$ , $b^3 = (1,1,0)$ , the conditional probability is as follows:

$$\begin{aligned} P(b,c) &= \frac{123256}{\#C} = 1 && \text{for } c = 0.4 \\ &= \frac{122072}{\#C} = 0.990 && \text{for } c = 0.2 \\ &= \frac{113808}{\#C} = 0.923 && \text{for } c = 0.1 \\ &= \frac{80935}{\#C} = 0.656 && \text{for } c = 0 \end{aligned}$$

(II) When $b^1 = (0,6,1)$ , $b^2 = (1,0,6)$ , $b^3 = (6,1,0)$ , the conditional probability is seen in:

$$
\begin{aligned}
P(b,c) &= \frac{122818}{\#C} = 0.996 \qquad \text{for } c = 0.3 \\
&= \frac{120954}{\#C} = 0.981 \qquad \text{for } c = 0.1 \\
&= \frac{118106}{\#C} = 0.958 \qquad \text{for } c = 0
\end{aligned}
$$

(III) When $b^1 = (6,3,1)$ , $b^2 = (3,1,6)$ , $b^3 = (1,6,3)$ , the conditional probability is seen in:

$$
\begin{aligned}
P(b,c) &= \frac{123238}{\#C} = 0.999 \qquad \text{for } c = 0.3 \\
&= \frac{120775}{\#C} = 0.980 \qquad \text{for } c = 0.2 \\
&= \frac{107231}{\#C} = 0.870 \qquad \text{for } c = 0.1 \\
&= \frac{74102}{\#C} = 0.601 \qquad \text{for } c = 0
\end{aligned}
$$

(IV) Finally the total probability is as follows:

$$
\begin{aligned}
P(c) &= \frac{59171907}{\#C \cdot \#Q} = 0.998 \qquad \text{for } c = 0.3 \\
&= \frac{58491774}{\#C \cdot \#Q} = 0.988 \qquad \text{for } c = 0.2 \\
&= \frac{53488875}{\#C \cdot \#Q} = 0.902 \qquad \text{for } c = 0.1 \\
&= \frac{29079035}{\#C \cdot \#Q} = 0.491 \qquad \text{for } c = 0
\end{aligned}
$$

The data say that the weak axiom will be satisfied with high probability, except for zero value of elasticity of substitution (put differently, the substitution terms in the Slusky equation vanish). Suprisingly, the above result is very close to the one of the previous section. This suggests that the weak axiom may be not only a sufficient condition for global stability, but considerably near the necessary condition. Of course it will be uncertain whether this may be asserted under more general framework beyond C.E.S. preferences.

# 4 General Space of Parameters and Remarks

In the previous section it was shown that the phenomenon of stability dominates system (3) when the parameters satisfy Assumption 1. Although the change on the measure of quantity of goods covers for some more possible cases of parameters, it is not sufficient. Therefore in this section it is desirable to study what will happen to the completely general class of parameters.

Put forth are the following assumptions:

Assumption 2 *The sum of all elements in the endowment matrix $A$ is unity, and the sum of each row and column in $A$ is positive, i.e.,*

$$\sum_{j=1}^{3}\sum_{i=1}^{3} a_{ji} = 1, \tag{15}$$

$$\sum_{i=1}^{3} a_{ji} > 0 \quad (j = 1, 2, 3), \tag{16}$$

$$\sum_{j=1}^{3} a_{ji} > 0 \quad (i = 1, 2, 3). \tag{17}$$

(16) implies that each person initially possesses at least one commodity, and (17) implies that each commodity is supplied by at least one person.

Assumption 3 *The sum of each row in the evaluation matrix $B$ is unity, and at least two elements of each row are positive, and the sum of each column is positive, i.e.,*

$$\sum_{i=1}^{3} b_{ji} = 1 \quad (j = 1, 2, 3), \tag{18}$$

$$b_{ji} < 1 \quad (i, j = 1, 2, 3), \tag{19}$$

$$\sum_{j=1}^{3} b_{ji} > 0. \tag{20}$$

Now let the implication of these assumptions be explained. Since the market excess demand functions $f = (f_1, f_2, f_3)$ are homogeneous of degree one with respect to $a = \left(a^1, a^2, a^3\right) \in R^9$, for any positive scalar $\lambda > 0$, $f(p; \lambda a, b, c) = \lambda \cdot f(p; a, b, c)$. Suppose system (3) with $(a, b, c)$ is globally stable. Then there is a Liapunov function $V(p)$, that is again a Liapunov function for system (3) with $(\lambda a, b, c)$, since

$$\begin{aligned} \dot{V}(p) &= \sum \frac{\partial V}{\partial p_i} \cdot \dot{p}_i = \sum \frac{\partial V}{\partial p_i} \cdot f_i(p; \lambda a, b, c) \\ &= \lambda \sum \frac{\partial V}{\partial p_i} \cdot f_i(p; a, b, c). \end{aligned}$$

Because the demand functions do not change for multiples of $b^j$, (16) is completely general.

Though $a = \left(a^1, a^2, a^3\right)$ is a point of the nine-dimensional real space, a restriction of Assumption 2 leads to the consideration of an eight-dimensional space by elimination one variable, $a_{33}$. Let this real space be denoted by $Y$ and a lattice space of $Y$ be defined as follows:

$$\widehat{C}(n) = \{\widehat{a} = (a^1, a^2, a_{31}, a_{32}) = \left(\tfrac{i}{n}, \tfrac{j}{n}, \tfrac{k}{n}\right) \in Y\},$$

where $i = (i_1, i_2, i_3)$, $j = (j_1, j_2, j_3)$, $k = (k_1, k_2)$ are all nonnegative integers less than or equal to a given $n$.

On the other hand, Assumption 3 leads to a six-dimensional real space $Z$ that is expressed as three time products of $\{z_1 + z_2 \leqq 1,\ z_i \geq 0\}$. Let us define a lattice space of $Z$ as follows :

$$\widehat{Q}(m) = \{\widehat{b} = (b_{11}, b_{12}, b_{21}, b_{22}, b_{31}, b_{32}) = \left(\tfrac{i}{m}, \tfrac{j}{m}, \tfrac{k}{m}\right) \in Z\},$$

where $i = (i_1, i_2)$, $j = (j_1, j_2)$, $k = (k_1, k_2)$ are all nonnegative integers less than or equal to a given integer $m$.

If $\widehat{a}$ and $\widehat{b}$ are independently taken from the above sets, then an identical equilibrium price cannot be supposed as was done in the previous sections. However it is possible, without loss of generality, to change the measurement of each good so as to obtain an identical equilibrium price. Let $f_i(p)$ be an excess demand function of the $j^{th}$ good, and $p^* = (p_1^*,\ p_2^*,\ p_3^*)$ an equilibrium price vector on the unit simplex. Measuring one unit of each good as $p_i^*$ unit provides the new excess demand funcitons that are defined as

$$g_i(q_1,\ q_2,\ q_3) = p_i^* \cdot f_i(q_1 p_1^*,\ q_2 p_2^*,\ q_3 p_3^*),\ i = 1, 2, 3,$$

where $q$ is price vector in new measurment and again on the unit simplex.

It is quite clear that $q_1 = q_2 = q_3$ gives an equilibrium prices for the new excess demands, and $g_i(q)$'s satisfy the weak axiom if and only if $f_i(p)$'s satisfy. It is also clear that if one system is globally stable, so is another system : a Liapunov function $V(p)$ for the former system yields $V^*(q) = V(q_1/p_1^*,\ q_2/p_2^*,\ q_3/p_3^*)$ that is a Liapunov function for the latter system.

These facts enable us to concentrate on some specifie distributions of pair $(a, b)$'s for which the equilibrium prices of all goods are always the same ; both $p_1 = p_2 = p_3$ and $f(p; a, b, c) = 0$ yield

$$\sum_{j=1}^{3} b_{ji} \sum_{k=1}^{3} a_{jk} - \sum_{j=1}^{3} a_{ji} = 0, \quad i = 1, 2, 3, \tag{21}$$

only two of which are independent by the Walras Law.

For a given $\widehat{b} \in \widehat{Q}(m)$, a set of initial endowments $\widetilde{C}(n, \widehat{b}) \subset \widehat{C}(n)$ can be defined so as to satisfy (21), and two subsets of $\widetilde{C}(n, \widehat{b})$ can be defined each point of whose one set leads to stability, and each point of whose another set leads to the weak axiom, and furthermore these probabilities can be defined similarly as in sections 2 and 3.

Below are demonstrated typical results under this completely general setting of parameters, which are given by the computer simulation based on the program with Fortran.

(probability of weak axiom) In order to determine the sizes of three lattice spaces, in (I) $n = 39$ and $h = 30$ were, set so that $\#\widetilde{C}\left(39, \widehat{b}\right) = 260517$,in (II) and (III) $n = 60$ and $h = 30$ were set, so that $\#\widetilde{C}\left(60, \widehat{b}\right) = 199196$ and $111090$ respectively, and in (IV) $n = 15, m = 9$ and $h = 20$ were set, so that $\#\widetilde{C} \cdot \#\widehat{Q} = 17782875$.

(I) When $b^1 = (0,1,1)$, $b^2 = (1,0,1)$, $b^3 = (1,1,0)$, the conditional probability is:

$$\begin{aligned} P(b,c) &= \frac{258776}{\#\widetilde{C}} = 0.993 \qquad \text{for } c = 0.2 \\ &= \frac{237411}{\#\widetilde{C}} = 0.911 \qquad \text{for } c = 0.1 \\ &= \frac{136899}{\#\widetilde{C}} = 0.525 \qquad \text{for } c = 0 \end{aligned}$$

(II) When $b^1 = (0,6,1)$, $b^2 = (1,0,6)$, $b^3 = (6,1,0)$,the probability is seen in:

$$\begin{aligned} P(b,c) &= \frac{199061}{\#\widetilde{C}} = 0.999 \qquad \text{for } c = 0.4 \\ &= \frac{196281}{\#\widetilde{C}} = 0.985 \qquad \text{for } c = 0.2 \\ &= \frac{181785}{\#\widetilde{C}} = 0.913 \qquad \text{for } c = 0 \end{aligned}$$

(III) When $b^1 = (6,3,1)$, $b^2 = (3,1,6)$, $b^3 = (1,6,3)$, the probability is seen in:

$$\begin{aligned} P(b,c) &= \frac{109005}{\#\widetilde{C}} = 0.981 \qquad \text{for } c = 0.2 \\ &= \frac{94584}{\#\widetilde{C}} = 0.851 \qquad \text{for } c = 0.1 \\ &= \frac{58356}{\#\widetilde{C}} = 0.525 \qquad \text{for } c = 0 \end{aligned}$$

(IV) Finally, the total probability is as follows:

$$
\begin{aligned}
P(c) &= \frac{17730771}{\#\widetilde{C}\cdot\#\widehat{Q}} = 0.997 \qquad \text{for } c = 0.4\\
&= \frac{16840818}{\#\widetilde{C}\cdot\#\widehat{Q}} = 0.947 \qquad \text{for } c = 0.2\\
&= \frac{13831998}{\#\widetilde{C}\cdot\#\widehat{Q}} = 0.778 \qquad \text{for } c = 0.1\\
&= \frac{5408884}{\#\widetilde{C}\cdot\#\widehat{Q}} = 0.304 \qquad \text{for } c = 0
\end{aligned}
$$

The above data show that the probability of weak axiom is again very high even in this completely general parameters, except for zero value of elasticity of substitution. The weak axiom is not necessarily required to obtain stability (it is just a sufficient condition), so that the probability of stability in this general setting will be expected to be somewhat larger than the above data. The numerical data about stability in this case (which are not demonstrated here) tell us: when the elasticity of substitution is larger than 0.2, the difference of both probabilities is very small, but when it is 0.1 and ,in particular, 0, there appears some difference of about 10 percent.

It seems that the data observed in sections 2 and 3 and the present section may permit this concluding assertion:

**Proposition** *When the C.E.S. utility functions with the "positivity" of elasticity of substitution are assumed, system (3) converges to an equilibrium with very high probability, and system (2) satisfies the weak axiom with high probability and rarely satisfies the gross substitutability if the elasticity of substitution is not close enough to unity.*

It is noted from the data that as the value of elasticity of substitution becomes larger (in other words, as the substitution effect in the Slusky equation becomes larger), the degree of stability increases. This seems to support the traditional view (see Hicks (1939; pp.315-317)) that the source of instability is due to asymmetrical income effects in markets and the substitution effects have the effect of stabilizing the economic system.

Finally, some remarks on the characteristics of market excess demand functions and on an interpretation of a Walrasian price adjustment process.

(I) It is widely held that since an arbitrary market excess demands satisfying the homogeneity and the Walras Law can be decomposed into a number of individual excess demand functions that are supported by utility maximization behavior, it will be disagreeable to impose specific conditions on market excess demands. However our research here may suggest that as long as C.E.S. utility functions are assumed, a class of excess demands that defy a nice property will be rarely met in practice and will have relatively a only small measure.

(II) Consider an exchange economy that is characterized by $U^j(x_{j1}, x_{j2}, x_{j3})$, $a^j = (a_{j1}, a_{j2}, a_{j3})$, $j = 1, 2, 3$, where $U^j(\cdot)$ and $a^j$ denote the $j^{th}$ type trader's utility function and initial endowment vector. Further suppose:
(i) There are $N$ people in each type.
(ii) This economy is stationary for certain periods from period 1 to period $\widetilde{T}$, that is, in all periods every person has the same utility function and initial endowments as in the first period.
(iii) All the transactions are performed subject to some rules of double auction markets (for instance, see Davis and Holt (1993)), and are performed through *numeraire* (consider the third commodity to be *numeraire*). As buyers, the traders make their demands (either exact quantities or some range of quantities) by outbidding each other, and as sellers, the traders make their offers by underbidding each other.

Let $p_i^b(\tau, T)$ and $p_i^0(\tau, T)$ denote the bid price and offer price of commodity $i$ at a time $\tau$ in period $T$, i.e., $G_i(T) = \{p_i^b(\tau, T),\ p_i^0(\tau, T)\}$, $i = 1, 2$.

The set of the actual transaction prices $S_i(T) = \{p_i^a(\tau, T)\}$ will be a subset of $G_i(T)$. $G_i$ and $S_i$ will be regulated by utility functions, initial holdings, history of the past data, and various expectations. However, it seems extremely difficult to represent the motion of all the prices in each period by some differential equations since it involves a sort of complexity.

In their several experiments, Anderson, Plott, Shimomura and Granat (2000) have observed the following: When such a perfect complementarity as Scarf's is assumed, and types of assignment patterns of initial holdings are given, the stream of the average transaction prices in each period $\left\{\overline{p}_i(1),\ \overline{p}_i(2), ..., \overline{p}_i\left(\widetilde{T}\right)\right\}$ roughly follows the prediction of the solution paths of Walrasian tatonnement process. This observation may yield some conjectures that seem to be very stimulating for future research:
(i) The average trader of each type may be considered the price taker.
(ii) The average transaction prices in each period may be approximately determined as the average transaction prices in the previous period plus some estimated value of the excess demand in the previous period, i.e.,

$$p_i(T+1) = p_i(T) + \varphi(\text{excess demand of commodity } i) + \epsilon, \ \ i = 1, 2.$$

This price movements (which, at a glance, look like a tatonnement) may approximately represent the dynamics of the average prices of many complex disequilibrium prices at which the various transactions are actually performed.

# Appendix

The numerical study in this paper has been performed by computer simulation based on the programs that are written mainly with Fortran. For the interested readers the use of Fortran will be highly recommended in various reasons, but the Fortran programs in this research are too long to demonstrate in this appendix, so that for the reference of the readers very simple examples of programs for Sections 2 and 3 written with Mathematica are provided. Since the program for Section 4 can be similarly made, it will be omitted.

### (A-1) Program for Stability

The program for demonstrating whether or not system (3) with parameters $a$ and $b$ and $c$ converges to $p^* = (1, 1, 1)$ is as follows:

```
process[a_, b_, c_] := Module[{sol}, sol = NDSolve[Join[Table[p[k]'[t] ==
Sum[(b[[j, k]] Sum[p[i][t] a[[j, i]], {i, 3}] p[k][t] ^-c)/(Sum[b[[j, i]]
p[i][t]^(1-c), {i, 3}])- a[[j, k]], {k, 3}], {p[1][0] == p[2][0] == 1/2, p[3][0]
== √5/2}], {p[1], p[2], p[3]},{t, 0, 100}, MaxSteps -> 20000] ;
If[Sqrt[Sum[(First[p[i][100]/.sol] - 1.0)^2, {i, 3}]] < 0.001, s1 =s1 + 1,
s2 =s2 +1 ]]
```

When both the evaluation matrix $b$ and elasticity of substitution $c$ are given and the size of the lattice space $C(n)$ is also given the program for conditional probability is as follows:

```
proba[b0_, c0_, n_] := ( b = b0; c = c0; Do[process[{{i/n, j/n, 1 - (i+j)/n},
{k/n, r/n,1 - (k+r)/n}, {1 - (i+k)/n, 1 -(j+r)/n, (i+j+k+r)/n - 1}}, b, c],
{i, 0, n}, {j, 0, n-1},{k, 0, n-i}, {r, Max[0, n-k-i-j], Min[n-k, n-j]} ;
{s1, s2})
```

Finally, the program for the total probability is seen in:

```
toproba[c00_, n0_, m_] := (n = n0; c0 = c00;
Do[proba[{{i/m, j/m, 1-(i+j)/m},{k/m, r/m, 1-(k+r)/m},
{1-(i+k)/m, 1-(j+r)/m, (i+j+k+r)/m - 1}}, c0, n], {i, 0, m},
{j, 0, m-i}, {k, 0, m-i}, {r, Max[0, m-i-j-k], Min[m-k, m-j]}];
Print[{s1, s2}];Print["probability of stability"," : ", N[s1/(s1+s2)]])
```

It is noted: the above program does not take the boundary problem (on the differential equations) into consideration, so that there will appear small differences between the results of Section 2 by Fortran program. In the above program s1 and s2 represent the numbers of stability and instability respectively. When this program is performed, s1 and s2 should be first set zero.

### (A-2) Program for Gross Substitutability

The Jacobian of market excess demands with parameters $a$ and $b$ and $c$ is programmed as follows:

```
g[a_, b_, c_] := (t = Outer[D, Table[Sum[(b[[j, k]] Sum[p[i] a[[j, i]], {i, 3}]
p[k]^-c)/(Sum[b[[j, i]] p[i]^(1-c), {j, 3}], {k, 3}], Table[p[i], {i, 3}]];
Flatten[t /. Table[t[[i, i]] ->- t[[i, i]], {i, 3}]])
```

The program for examining whether or not excess demands (2) with parameters $a$ and $b$ and $c$ satisfy the condition of gross substitutes is as follows:

```
gross[a0_, b0_, c0_, h_] := (a = a0; b = b0; c = c0; Module[{s}, s = 0;
Catch[Do[If [Apply[Or, Table[Less[g[a, b, c] [[k]] /.
{p[1] -> i, p[2] -> j, p[3] -> h - i - j}, 0], {k, 9}]],
Throw[s = s + 1]], {i, 1, h -2}, {j, 1, h - i -1}]];
If[s == 1, s1 = s1 +1, s2 = s2 +1]])
```

When both the evaluation matrix $b$ and the elasticity of substitution $c$ are given and the size of the lattice space $C(n)$ is also given the program for conditional probability is seen in:

```
grossproba[b0_, c0_, h_, n_]:= (Do[gross[{{i, j, n-i-j},{k, r, n-k-r},
{n-i-k, n-j-r, (i+j+k+r) -n}}, b0, c0, h], {i, 0, n}, {j, 0, n-i}, {k, 0, n-i},
{r, Max[0, n-k-i-j], Min[n-k, n-j]}]; {s1, s2})
```

Finally the program for total probability is seen in:

```
grosstoproba[c00_, h0_,n0_, m_]:=(c0=c00; h=h0; n=n0;
Do[grossproba[{{i, j, m-i-j},{k, r, m-k-r},{m-i-k, m-j-r, (i+j+k+r)-m}},
c0, h, n], {i, 0, m},{j, 0, m-i},{k, 0, m-i},{r, Max[m-k, m-j],
Min[m-k, m-j]}]; Print["total probability",":", N[s1/(s1+s2)]])
```

In the above s1 and s2 respectively represent the numbers of gross substitutability being satisfied and not satisfied

### (A-3) Program for Weak Axiom

The summation of the market demands with parameters $a$ and $b$ and $c$ is programmed as follows:

```
w[a_, b_, c_] := Sum[Sum[(b[[j, k]] Sum[p[i] a[[j, i]], {i, 3}] p[k]^-c)/
(Sum[b[[j, i]] p[i]^(1-c), {i, 3}]), {j, 3}], {k, 3}]
```

The program for examining whether or not the market excess demands with parameters $a$ and $b$ and $c$ satisfy the condition of weak axiom is made as follows:

```
weak[a0_, b0_, c0_, h_] := (a = a0; b = b0; c = c0; Module[{s}, s = 0;
Catch[Do[If [And[LessEqual[ w[a, b, c]/.
```

```
{p[1]-> i, p[2]-> j, p[3]-> h - i -j}, 3], Unequal[3 i, h]],
Throw[s = s +1]], {i, 1, h -2}, {j, 1, h - i -1}]];
If[s ==1, s2 = s2 + 1, s1 = s1 + 1]])
```

The program for conditional probability is as follows:

```
weakproba[b0_, c0_, h_, n_]:=(Do[weak[{{i, j, n-i-j},{k, r, n-k-r},
{n-i-k, n-j-r, (i+j+k+r)-n}}, b0, c0, h], {i, 0, n}, {j, 0, n-i}, {k, 0, n-i},
{r, Max[0, n-k-i-j], Min[n-k, n-j]}]; {s1,s2})
```

Finally the total probability is seen in:

```
weaktoproba[c00_, h0_, n0_, m_]:=(c0=c00; h=h0; n=n0;
Do[weakproba[{{i, j, m-i-j},{k, r, m-k-r},{m-i-k, m-j-r, (i+j+k+r)-m}},
c0, h, n], {i, 0, m}, {j, 0, m-i}, {k, 0, m-i}, {r, Max[0, m-k-i-j],
Min[m-k, m-j]}]; Print[”total probability”,”:”, N[s1/(s1+s2)]])
```

In the above s1 and s2 respectively represent the numbers of weak axiom being satisfied and not satisfied.

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