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FINITE PERFECT INFORMATION EXTENSIVE GAMES WITH GENERIC PAYOFFS

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Abstract

In finite perfect information extensive (FPIE) games, backward induction (BI) gives rise to all pure-strategy subgame perfect Nash equilibria, and iterative elimination of weakly dominated strategies (IEWDS) may give different outcomes for different orders of elimination. Duggan recently posed several conjectures in an effort to better understand the relationship between BI and IEWDS in FPIE games. One conjecture states that the unique BI strategy profile in FPIE games with generic payoffs is guaranteed to survive IEWDS when all weakly dominated strategies are eliminated at every round. This paper exhibits a counterexample to this conjecture.

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Key words: perfect information games, extensive games, backward induction, weakly dominated strategies, iterative elimination of weakly dominated strategies, generic payoffs

Finite Perfect Information Extensive Games with Generic Payoffs *

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Finite perfect information extensive (FPIE) games are well-understood. Backward induction (BI) gives rise to all pure-strategy subgame perfect Nash equilibria of the game, and iterative elimination of weakly dominated strategies (IEWDS) may give different outcomes for different orders of elimination. Duggan ([1]) recently posed several conjectures in an effort to better understand the relationship between BI and IEWDS. I address these conjectures here and in [3]. This paper deals with the question: Does the unique BI strategy profile in FPIE games with generic payoffs always survive IEWDS when all weakly dominated strategies are eliminated at every round?

FPIE games with generic payoffs are a class of FPIE games where no player receives the same payoff at two distinct terminal nodes. Such games satisfy $u_i(x) = u_i(y) \Leftrightarrow u_j(x) = u_j(y)$, for any two terminal nodes x and y, where $u_i(x)$ denotes player *i's* payoff at x. As shown by Moulin, any games satisfying this condition are dominance solvable ([2], [4]). In other words, using Moulin's order of iterative elimination of weakly dominated strategies (IEWDS), if S denotes the set of strategies in the game, Z^k denotes all weakly dominated strategies in $S \backslash Z^1, \ldots, Z^{k-1}$, and $S^\infty = S \backslash Z^1, \ldots, Z^m$ denotes the set of strategies when no further eliminations are possible, each player's payoff function is constant on S^∞.

The unique backward induction (BI) strategy profile need not survive IEWDS for all orders of elimination ([5]). Consider, for example, the following FPIE game with generic payoffs:

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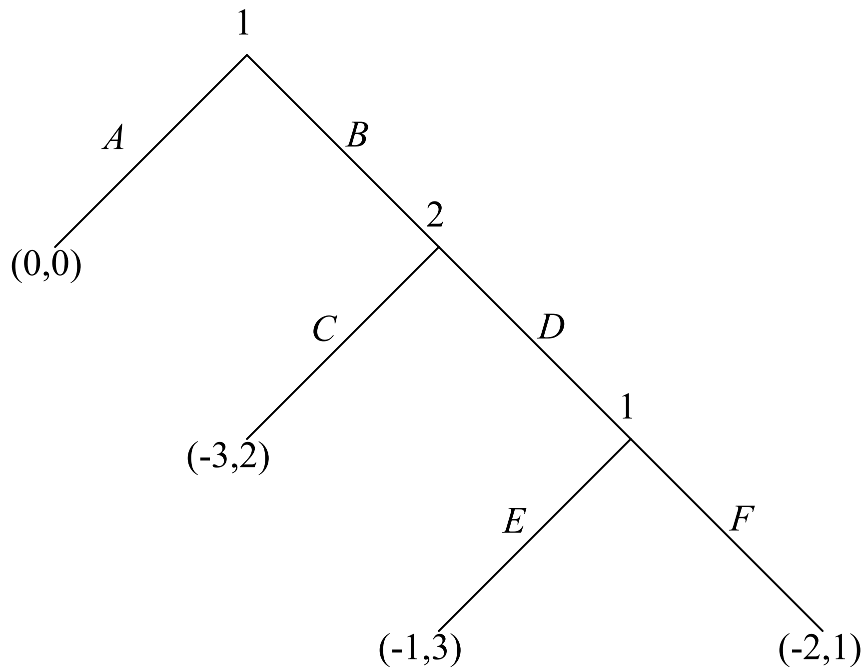
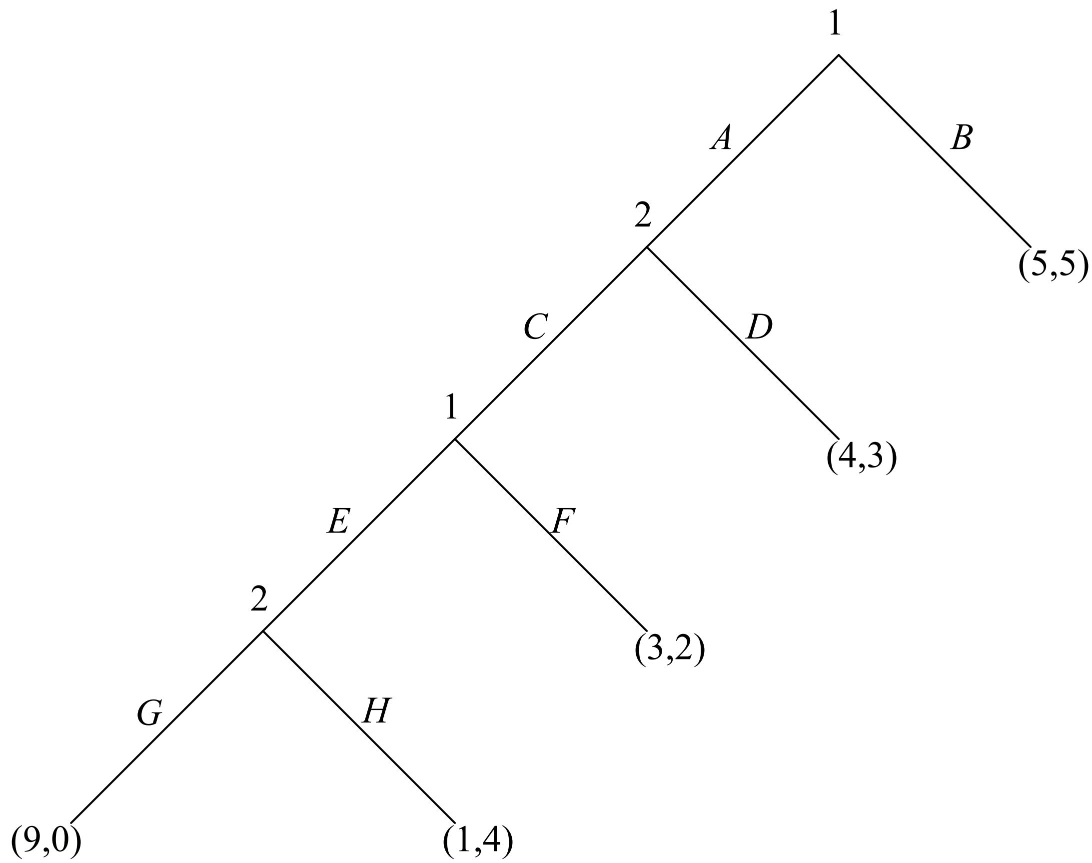


Here one valid order of IEWDS is to first eliminate strategy *BE* and then eliminate strategy *D*. In this case, the unique BI strategy profile, (*AE*, *D*), is eliminated. However, Osborne and Rubinstein note that the BI strategy profile in FPIE games with generic payoffs always survives IEWDS for some order of elimination ([5]). In fact, in this example, the BI strategy profile is not eliminated in the order of elimination considered by Moulin. Duggan ([1]) notes that if the BI strategy profile survives IEWDS for this order of elimination, then the unique path of play realized by IEWDS is equivalent to the unique path of play realized by BI. With these results in mind, Duggan proposes an open problem: Is the BI strategy profile guaranteed to survive IEWDS under this order of elimination?

Duggan conjectured that this was true, but I demonstrate the following counterexample:



The unique BI strategy profile is (*BF*, *DH*). But in Moulin's order of elimination, *AF* and *CG* are eliminated in the first round of IEWDS, and *AE*, *DG*, and *DH* are eliminated in the final round of IEWDS. This leaves just the strategy profiles (*BE*, *CH*) and (*BF*, *CH*), so the BI strategy profile need not survive IEWDS using Moulin's order of elimination.

References

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