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POST-STRATIFICATION WITHOUT POPULATION LEVEL INFORMATION ON THE POST-STRATIFYING VARIABLE, WITH APPLICATION TO POLITICAL POLLING

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POST-STRATIFICATION WITHOUT POPULATION LEVEL INFORMATION ON THE POST-STRATIFYING VARIABLE, WITH APPLICATION TO POLITICAL POLLING*

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Abstract

We investigate the construction of more precise estimates of a collection of population means using information about a related variable in the context of repeated sample surveys. The method is illustrated using poll results concerning presidential approval rating (our related variable is political party identification). We use post-stratification to construct these improved estimates, but since we don't have population level information on the post-stratifying variable, we construct a model for the manner in which the post-stratifier develops over time. In this manner, we obtain more precise estimates without making possibly untenable assumptions about the dynamics of our variable of interest, the presidential approval rating.

Keywords: Bayesian Inference; Post-stratification; Sample surveys; State-space models.

1 Introduction

Post-stratification is widely recognized as an effective method for obtaining more accurate estimates of population quantities in the context of survey sampling. Not only does it correct for non-sampling error, but it can lead to less variable estimates. The basic idea is, if we know that our population is composed of distinct groups (strata) that differ with regard to the quantity which we are interested in estimating, and we know the sizes of these strata in our population, then we can obtain a more accurate estimate of the

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quantity of interest by correcting our estimate for any imbalance in the representation of the strata in our sample. This correction is obtained by using a weighted average (using the known weights from the population) of the averages within strata as our estimate of the population mean. If we calculate the variance of this estimate conditional on the observed number of respondents falling into each of the strata (as is generally recommended, see Holt and Smith (1979)), the variance of this estimate will be a linear combination of the variance of the strata means, hence the estimate could have zero variance (if group membership exactly determines the quantity of interest), but in practice our gains will depend on how strongly our quantity of interest is related to the variable(s) we use to post-stratify. Although post-stratification is not always used in academic studies, it is a commonplace tool in commercial public opinion polls (Voss, Gelman and King (1995)).

One of the greatest practical limitations to the use of post-stratification is the need to know the proportion of the population in each strata. We only have population level information for certain variables, and so it would appear that post-stratification is only useful if our quantity of interest is related to one of a handful of characteristics for which we have population level information. Here, we overcome this difficulty by constructing a dynamic model for the variable by which we post-stratify, thereby estimating the strata weights from our sample. The dynamic model for the post-stratifier allows for more efficient estimation of the weights for each time period than would be possible if we analyzed each sample separately. Clearly, if the method of obtaining the samples does not change over time, we can not hope to correct for sampling bias if we estimate our weights, hence we here use post-stratification solely to obtain more efficient estimates. Note that we are not required to propose any dynamic model for the quantity of interest, only for the post-stratifier. Since we are free to select the post-stratifier, we try to choose a variable which is related to the quantity of interest and has dynamic behavior which is relatively well understood (for example, the variable is basically constant over time).

1.1 Structure of the Data and Preliminary Considerations

We analyze data from a (self-weighted) sample survey of U.S. adults, the "WISCON" project, from the University of Wisconsin at Madison's Letters and Science Survey Center. For each respondent, we have his or her rating of the president on a scale of 1 to 10, the party with which he or she most closely identifies (which we group into one of three categories, Democrat, Republican or Independent, based on the respondent's answer to two questions about party identification), and the date of the interview. We group each respondent by the week in which he or she was interviewed so as to have a sequence of samples of these quantities (i.e. the approval rating within each party and the size of each party in our sample) from the week starting 1/19/93 until the week starting 8/13/96 (which constitutes most of Clinton's first term). We are ultimately interested in estimating the mean approval rating of the president for each week, μ_t for $t = 1, \ldots, T$, given all of the data up to time T. The weekly samples collect information from about 40 to 60 respondents (we do not try to estimate the mean approval rating for weeks with too few interviews, hence we exclude several weeks, leaving a total of 171 weeks of data), and so a natural estimate of μ_t (and a basis for comparison for any other method) is the sample mean with standard error given by the sample standard deviation divided by the square root of the sample size at time t (moreover, since the sample sizes are large, we can appeal to the central limit theorem to conclude that the distribution of the sample mean is approximately normal).

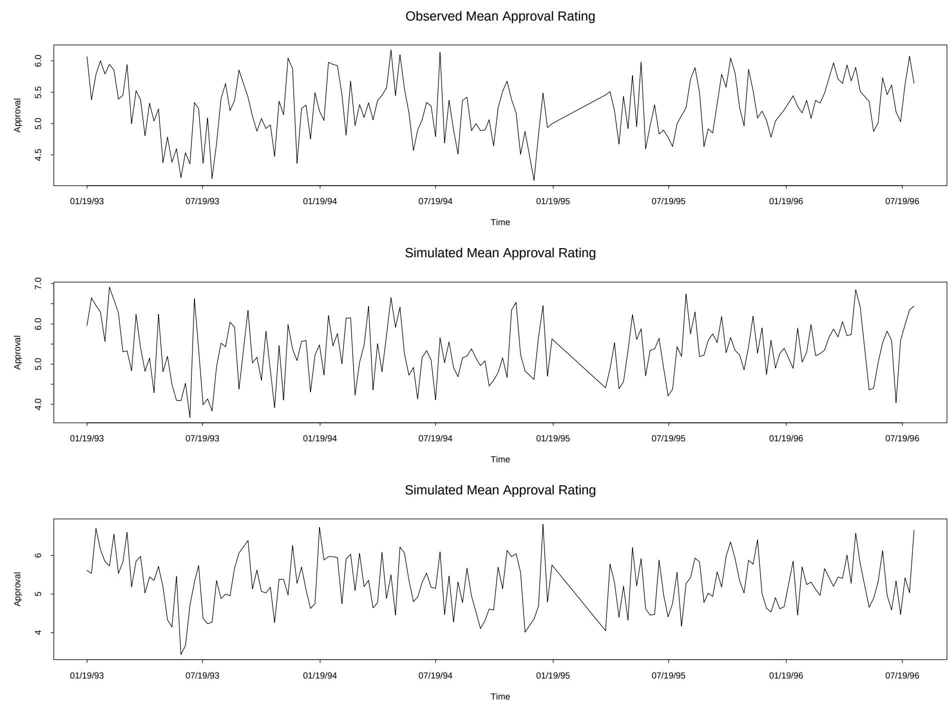

Figure 1: *Observed mean approval rating, and 2 simulations of the mean approval rating under the model.*

The top plot in Figure 1 displays the mean approval rating for all of the weeks. We suppose that these sample means are independent over time since they are based on independent random samples. Since our dynamic model for the weights is a Bayesian model (as we shall see below), it is useful to note that using the sample mean (based on samples large enough for the central limit theorem to take effect) with the aforementioned standard error as an estimate of μ_t is equivalent in Bayesian terms to assuming a normal distribution for the sample mean given μ_t and σ_t (where σ_t is the standard deviation of the approval ratings at time t), using a normal prior for μ_t with arbitrarily large variance and using the sample standard deviation as an estimate for the unknown quantity σ_t.

That is, if we let n_{1t} = the number of Democrats in our sample at time t, n_{2t} = the number of Republicans in our sample at time t, n_{3t} = the number of Independents in our sample at time t, and we set $N_t = \sum_j n_{jt}$ for $t = 1, \ldots, T$, then we find the posterior distribution of μ_t by supposing that for $t = 1, \ldots, T$ we have $\overline{y}_t | \mu_t, \sigma_t, N_t \sim \mathrm{N}(\mu_t, \sigma_t^2/N_t)$ and $\mu_t | \sigma_t \sim \mathrm{N}(\mu_0, \sigma_0^2)$ where $\overline{y}_t$ is the mean approval rating for our sample at time t and we take σ_0 to be arbitrarily large. This implies $\mu_t | \overline{y}_t, \sigma_t, N_t \sim \mathrm{N}(\overline{y}_t, \sigma_t^2/N_t)$. We estimate

σ_t^2 with the usual unbiased estimate, s_t^2, and so we obtain draws from the posterior distribution of μ_t using the normal distribution in a completely straightforward manner (so we are ignoring the fact that the sample variance is subject to variability). Later, we will treat the mean approval rating within each party μ_{jt} for $j = 1, 2, 3$ in the same manner, and we shall assume that the approval rating is independent across parties. In this sense we have no dynamic model for the approval rating within party. Our intention is to post-stratify presidential approval by political party identification, and show that by correcting our estimate for imbalances in political party representation, we can obtain a more efficient estimate. While it is difficult to propose a dynamic model for approval, it is reasonable to suppose that the proportion of a population which holds a given political attitude is almost constant from week to week.

1.2 A Simple Model and Method

As a simple investigation into the efficacy of this method, we use the average over all time periods of the proportion of our sample in each party for the strata weights, and treat these weights as known. This is equivalent to the dynamic model which supposes that the proportion in each party is constant over time (and we ignore the uncertainty in the estimation of the weights, an entirely reasonable practice since these averages of sample proportions are sample averages based on $\sum_t N_t = 8{,}462$ observations). If we use these averages for the weights for all the weeks, and treat these as known, then we can estimate the efficiency of our post-stratification estimate relative to the sample mean for each week by the ratio of the variance of the estimated sample mean s_t^2/N_t to the estimated variance of the post-stratification estimate at time t. We find that these estimated efficiencies range from 0.48 to 2.8 with an average of 1.23. The correlation between approval and party identification is about 0.35 (treating party identification as continuous), so we see that even a weak correlation can be useful. These results are in accord with the findings of others (e.g. Holt and Smith (1979)), in particular $\overline{y}_{PS}$ often has lower variance (and here, on average, has lower variance), but sometimes the sample mean is preferable. While this simple method indicates that the post-stratification estimator can outperform the sample mean (on average here), a model that assumes the strata proportions are constant over time (3.5 years) is not very reasonable (see, for example, MacKuen et al. (1983)). A more plausible model is provided in the next section, but we note that in some settings this analysis may be satisfactory.

1.3 Political Polling and the Presidency

Presidential approval has been a central concept in the study of both presidential power and public opinion in political science. With the advent of the "new presidency" in the age of mass media politics, having high levels of approval is seen as an important political resource for presidents (Kernell 1986). Having high levels of approval is thus a central component of presidential power (Neustadt 1990), and influences electoral outcomes and legislative success (Brody 1991; Rivers and Rose 1985; Ostrom and Simon 1989).

Since the early 1970's a long list of studies have examined various presidential approval series, although the series from the Gallup Organization is most common because it is available starting with the Truman administration. In general, these studies have been interested in examining how the percentage of the population approving of the job of the current president varies with economic conditions and "rally events"–such as armed conflict or political scandal (see, inter alia, Brace and Hincklely 1991; Beck 1991, 1992; Brody 1991; Clarke and Stewart 1994; Kernell 1978; Kiewiet and Rivers 1985; MacKuen 1983; Norpoth and Yantek 1983; Ostrom and Simon 1989). More recent work on presidential approval has paid particular attention to the dynamics of presidential approval. The consensus has been that the approval within the population is highly persistent from month to month, but there has been some debate on how best to model this persistence (see Box-Steffensmeier and Smith 1998; Smith 1992; Williams 1992).

2 Models and Posterior Simulation

2.1 Parameterization of a Categorical Post-Stratification Variable as a Multivariate Outcome

Since we don't have population level information on party identification, in order to effectively post-stratify we first posit a model for the temporal evolution of the party identification series. Rather than directly modeling the two series n_{1t} and n_{2t} (the number of respondents in each party), we first transform our data so that we model a vector with components which are approximately independent. The approximate independence thereby induced should make our inference less sensitive to our model for the covariance structure utilized in our dynamic model of the proportions. For the political party identification series, we model the proportion of respondents who identify with one of the two major parties, and the proportion of those who identify with the Democrats amongst those who identify with one of the major parties. So, if we let $n_t = n_{1t} + n_{2t}$ and define the 2-vector $y_t = (n_t/N_t, n_{1t}/n_t)$, then, since N_t and n_t are large, it is reasonable to suppose that y_t has a bivariate normal distribution (for the derivations which follow we adopt the convention that $y_t = (0, 0)$ if $n_t = 0$). If we let θ_{1t} = the proportion of the population which is in one of the major parties (i.e. Democrat or Republican), and θ_{2t} = the proportion of Democrats amongst those in a major party, then the measurement covariance (i.e. sampling error) of y_t given θ_{1t},θ_{2t} and N_t under simple random sampling (ignoring finite population correction factors) can be expressed as

$$V_t^* = \begin{pmatrix} \theta_{1t}(1-\theta_{1t})/N_t & \theta_{1t}\theta_{2t}(1-\theta_{1t})^{N_t} \\ \theta_{1t}\theta_{2t}(1-\theta_{1t})^{N_t} & \frac{\theta_{2t}(1-\theta_{2t})}{N_t}\sum_{j=0}^{\infty}\frac{\mu_j(\theta_{1t},N_t)}{N_t^j} + \theta_{2t}^2(1-\theta_{1t})^{N_t}(1-(1-\theta_{1t})^{N_t}) \end{pmatrix},$$

where $\mu_j(\theta, N) = \sum_{k=i0}^{N-1}\binom{N}{k}k^j(1-\theta)^k\theta^{N-k}$. We obtain this expression by noting that, conditional on N_t, θ_{1t}, and θ_{2t}, if we use 1_A to represent the indicator function of the set

A, then (if we use the convention that $1_{\{n_{3t}<N_t\}}/(N_t - n_{3t})$ is zero when $N_t = n_{3t}$ in the second line) if y_{jt}, $j = 1, 2$, is the j^{th} element of y_t:

$$\begin{aligned}
\mathrm{Var}(y_{2t}) &= \mathrm{E}(\mathrm{Var}[y_{2t}|n_{1t} + n_{2t}]) + \mathrm{Var}(\mathrm{E}[y_{2t}|n_{1t} + n_{2t}]) \\
&= \mathrm{E}\big[1_{\{n_{1t}+n_{2t}>0\}} \frac{\theta_{2t}(1-\theta_{2t})}{N_t - n_{3t}}\big] + \mathrm{Var}(1_{\{n_{1t}+n_{2t}>0\}}\theta_{2t}) \\
&= \frac{\theta_{2t}(1-\theta_{2t})}{N_t} \sum_{j=0}^{\infty} N_t^{-j}\, \mathrm{E}(1_{\{n_{1t}+n_{2t}>0\}} n_{3t}^j) + \theta_{2t}^2 (1-\theta_{1t})^{N_t}(1-(1-\theta_{1t})^{N_t}),
\end{aligned}$$

from which we obtain the element on the second diagonal of V_t^*, the other elements being straightforward. Although we could substitute our sample proportions, y_{jt}, for the unknown population proportions, θ_{jt}, in this expression and thereby obtain an estimate of the measurement covariance matrix (using 20 terms in the infinite sums is more than sufficient to obtain 7 digit accuracy, and 1 or 2 terms is probably adequate for most practical purposes), we instead use the simple approximation to the desired estimate (which is good to within 1% of the desired estimate of the standard error of y_{2t}, and is obviously good for the off-diagonal element since N_t is large and θ_{1t} is at least 0.7),

$$V_t = \begin{pmatrix} y_{1t}(1-y_{1t})/N_t & 0 \\ 0 & y_{2t}(1-y_{2t})/n_t \end{pmatrix}.$$

We will treat these measurement variances as known in our analysis.

2.2 Dynamic Model for the Post-Stratifying Variable

Given V_t and N_t for $t = 1, \ldots, T$, and the initial conditions m_0 and C_0, we propose the following state-space model for $t = 1, \ldots, T$:

$$\begin{gathered}
y_t = \theta_t + \nu_t \quad \text{where} \quad \nu_t \sim \mathrm{N}(0, V_t) \\
\theta_t = \theta_{t-1} + \omega_t \quad \text{where} \quad \omega_t \sim \mathrm{N}(0, W) \\
\theta_0 \sim \mathrm{N}(m_0, C_0)
\end{gathered}$$

where $\{\nu_t\}$ and $\{\omega_t\}$ are mutually orthogonal sequences of independent disturbances. We treat the matrix W as a random variable and estimate it from the data. This model is motivated by the fact that political attitudes in the contemporary United States do not change much over the course of a single week. For known W, this is a special case of a model for which one can use the Kalman filter to obtain the posterior moments of the state vectors, θ_t for $t = 0, \ldots, T$ (see e.g. West and Harrison (1997)).

2.3 Analytic Expressions for Posterior Inference

In order to obtain samples from the posterior distribution of the weights for our post-stratification estimate, we first obtain samples from the posterior distribution of the state process in our dynamic model given all of the data up to time T, but since we do not know W, we suppose this is a (matrix valued) random variable and conduct Bayesian inference for this matrix. Our goal is to first simulate W from its marginal posterior distribution, and then simulate the state vectors, θ_t, given W, i.e. we will use the fact $p(\theta, W|y) = p(\theta|W, y)p(W|y)$ where $\theta = (\theta_0, \theta_1, \ldots, \theta_T)$ and $y = (y_1, \ldots, y_T)$. These results can be given a non-Bayesian interpretation as predictive inference for θ conditional on a marginal likelihood estimate of W.

We find the posterior distribution of the state vectors given the state covariance matrix W by using standard formulae from the Kalman filter. Now, under our model, we have (by the Kalman filter)

$$\theta_t|y_1, \ldots, y_t, W \sim \mathrm{N}(m_t, C_t)$$

with

$$m_t = V_t(C_{t-1} + W + V_t)^{-1}m_{t-1} + (C_{t-1} + W)(C_{t-1} + W + V_t)^{-1}y_t$$

and

$$C_t = C_{t-1} + W - (C_{t-1} + W)(C_{t-1} + W + V_t)^{-1}(C_{t-1} + W)$$

for $t = 1, \ldots, T$, hence it is elementary to show

$$p(\theta|W, y) = \mathrm{N}(\theta_T|m_T, C_T)\prod_{t=1}^{T}\mathrm{N}(\theta_{t-1}|h_{t-1}, H_{t-1})$$

where

$$h_t = W(C_t + W)^{-1}m_t + C_t(C_t + W)^{-1}\theta_{t+1}$$

and

$$H_t = C_t - C_t(C_t + W)^{-1}C_t',$$

for $t = 0, \ldots, T-1$.

We can obtain the marginal posterior density of the state covariance matrix by writing down the likelihood for y as a function of W. That is $y_t = \sum_{s=1}^{t}\omega_s + \theta_0 + \nu_t$, and so

$$p(W|y) = p(W)\prod_{t=1}^{T}\mathrm{N}(y_t|m_0, tW + C_0 + V_t).$$

In this manner we obtain the posterior distribution of the state covariance matrix once we determine an appropriate prior. We take $p(W) \propto 1$ (so that our posterior mode coincides with the MLE of W treating θ as a nuisance).

2.4 Other Modeling Issues

In light of the previous development, simulation is relatively straightforward, but we must attend to some details. For example, a minor complication is the fact that for some weeks we have no (or insufficient) data, and so our time series has unequal time increments (so in the previous development W should have been a function of t). The simple remedy is to realize that since we have assumed that θ_t follows a random walk, if it has been k weeks since we last obtained survey results, and the state covariance matrix is W (i.e. the covariance matrix of an increment of the state process based on one week of data is W), then the covariance matrix of the state process over an increment of k weeks is kW. For our dataset, and the way in which we use the Kalman filter, this correction has no discernable impact on our results. We also must specify initial values for the Kalman filter, m_0 and C_0. Based on rough guesses we set $m_0 = (0.8, 0.5)$, and to convey our lack of accurate information on these quantities we made C_0 a diagonal matrix with elements 0.2^2. With 171 weeks of data the specification of the initial values has little impact on our estimation of the state process θ_t for $t = 1, \ldots, T$ and has no practical impact on our post-stratification estimator (this was verified experimentally by altering m_0 and C_0).

2.5 Computation

2.5.1 Posterior Simulation of the Post-stratification Proportions

We use the Metropolis algorithm to obtain draws from $p(W|y)$, then we draw θ from the appropriate sequence of normal distributions. Our methodology is as outlined in Gelman et al. (1995): our candidate distribution is a multivariate normal with variance based on the curvature of the posterior at the mode (and we scale this matrix so that the proportion of jumps which were accepted was in the 40% range), and we used multiple sequences started from overdispersed starting points (which were selected by drawing deviates from a properly centered and scaled Student-t distribution with four degrees of freedom). We used 4 sequences of 10,000 iterations, and the resulting values of the convergence diagnostic statistic, $\sqrt{\hat{R}}$, were all less than 1.1.

Given W it is completely straightforward to simulate θ. Note that we do not require iterative simulation for simulating θ, we simply use draws from the bivariate normal distribution with mean and covariance matrix given by h_t and H_t since the joint distribution of the state vectors was found above. That is, we use the forward filtering, backward sampling algorithm of Carter and Kohn (1994) and Frühwirth-Scnhnatter (1994).

2.5.2 Simulation of the Mean within each Post-stratification Category and the Post-Stratified Estimate of the Population Mean

We estimate the mean within each party in the same manner that we estimated the mean approval without regard to party identification, hence, it is trivial to obtain simulations of μ_{jt}. To obtain draws from the posterior distribution of the post-stratification estimate we assume that the approval rating within each party is conditionally independent of the proportion of the population in each of the parties given the sample means within parties and the number of respondents in each party. Therefore we simulate a draw from the posterior distribution of μ_t^{PS} by simply combining simulations from both parts of the above model in the obvious fashion, namely, if we let $\pi_{1t} = \theta_{1t}\theta_{2t}$, $\pi_{2t} = \theta_{1t}(1 - \theta_{2t})$, and $\pi_{3t} = 1 - \theta_{1t}$, then we obtain μ_t^{PS} by $\mu_t^{PS} = \sum_{j=1}^{3} \mu_{jt}\pi_{jt}$.

2.5.3 Comments on Computations

This method of obtaining draws from the posterior distribution of θ, averaging over our uncertainty in the estimation of the state covariance matrix, can be generalized to deal with any unknown parameters in the usual Gaussian linear Kalman filter, such as unknown autoregressive coefficients in state-space autoregressions or unknown variance components in dynamic regression models.

For example, we also tried fitting first order state-space autoregressions with unknown state variances and unknown autoregressive coefficients to the two series y_{1t} and y_{2t} separately using this methodology (with only 2 parameters we were able to obtain simulations for the autoregressive coefficient and the state space variance by discretizing the bivariate posterior distribution and using the inverse cdf method, see for example Gelman et al. (1995)). Since the autoregressive coefficients were definitely very close to 1 (as we would expect with such low values of the state variances), we ignored the complication that the autoregressive coefficient matrix might be different from the identity matrix in our model for y_t (since this would augment the dimension of the state space of our Markov chain by 3 in the implementation of the Metropolis algorithm). In any event, we see how simple our approach to unknown model parameters can be, indeed, no iterative simulation is required at all for these low dimensional problems. The advantage of this technique for averaging over our uncertainty in the model parameters compared to simply using the Gibbs sampler to simulate the state process given the model parameters and then simulate the model parameters given the state process (as is frequently done, see e.g. West and Harrison (1997)) is that in our method, no iterative simulation is required for the state vectors. This is a great simplification since adjacent state vectors are highly correlated in their joint posterior distribution, hence obtaining convergence of the chain can be difficult if we must use an iterative simulation method to simulate the state vectors. This posterior correlation is especially troubling for typical filtering applications since one can have hundreds (or even thousands) of state vectors. For our application, this means that we just need to obtain draws from the equilibrium distribution of a 3 dimensional Markov Chain rather than a 345 dimensional Markov Chain.

In the sample survey literature, researchers have reported difficulty with using the MLE of the state variance when the series is short (see e.g. Pfeffermann (1991)). In such cases, averaging over the uncertainty in the estimation of the state variance in the above manner should eliminate these problems. In particular, with short series the MLE of the state variance will occasionally be zero (even if the data was produced by a mechanism with a nonzero state variance), but since this point estimate is subject to uncertainty, if we average over the uncertainty of the estimated state variance we will find that the Kalman filter can still lead to more accurate inference without implying that the level of the process is constant. Moreover, if we have information about the state covariance matrix (or any parameters in the more general linear Gaussian model) we can incorporate this information through a prior on W (rather than taking the flat prior $p(W) \propto 1$ as we have here). With short series, the judicious use of such prior information can lead to more reliable inference since the posteriors of the model parameters may be quite diffuse if we use flat priors.

2.6 An Alternative Model for the Time Series of Post-Stratification Proportions

The model described in the previous sections was not the first model we fit to this data. The first model we fit follows the approach to multinomial time series developed in Cargnoni, Müller and West (1997). We did not end up using this model because we found that it did not fit our data (see Section 3.2.2); however, we present it here for completeness and because it might be useful in other settings. Using the same notation as before, if we let $\pi_t = (\pi_{1t}, \pi_{2t}, \pi_{3t})$, then we first assume that for $t = 1, \ldots, T$

$$n_{1t}, n_{2t}, n_{3t} | N_t, \pi_t \sim \text{Mult}(N_t, \pi_t).$$

Next, let $\eta_{jt} = \text{logit}(\theta_{jt})$ for $j = 1, 2$. These transformations separate partisan changes from changes in affiliation within the two largest parties, and change scale in such a way that additive models are more reasonable (they also yield diagonal measurement covariance matrices, as we saw above). Now we define the vector $\eta_t = (\eta_{1t}, \eta_{2t})$, and we suppose that for $t = 1, \ldots, T$,

$$\eta_t = \xi_t + \epsilon_t \quad \text{where} \quad \epsilon_t \sim \text{N}(0, V)$$

$$\xi_t = \xi_{t-1} + \delta_t \quad \text{where} \quad \delta_t \sim \text{N}(0, W^*),$$

where $\{\epsilon_t\}$ and $\{\delta_t\}$ are mutually orthogonal sequences of independent disturbances. We finish our specification of the dynamics of π_t by supposing $\xi_0 | m_0^*, C_0^* \sim \text{N}(m_0^*, C_0^*)$. In addition we suppose that V and W^* are random variables (matrices), and we specify inverse Wishart priors with scale equal to the identity matrix and 2 degrees of freedom in the hope of obtaining a prior which has little impact on our inference (a hope which is realized, as we see by experimentation). This model implies that the dynamics of the vector η_t are basically equivalent to a vector process which follows an ARIMA(0,1,1) model. The values of the moving average parameters in the equivalent ARIMA(0,1,1) model are determined by V and W^* (for more on this equivalence see West and Harrison (1997)). Although one may be tempted to set $V = 0$ in the hope of obtaining an efficient algorithm for simulating draws from a model which specifies that the transformed proportions follow a vector random walk, this will not work because, if we use the sampling algorithm of Cargnoni, Müller and West (1997), we will iteratively sample from two conditional distributions which degenerate into point masses as V approaches zero (thus no mixing takes place for the parameters of interest). We then can draw samples from the posterior distribution of all parameters in our model for the party identification series using the

Metropolis-Hastings algorithm as explained in Cargnoni, Müller and West (1997). In order to assess convergence we used 4 independent sequences started from overdispersed starting points (and the general methodology presented in Gelman et al. (1995)). To obtain overdispersed starting points for our example, we conducted a preliminary run of 1,000 iterations, and then we used 2 times the medians for the variance parameters as our starting values for these parameters, while for the η_{jt}'s we used the medians of the values obtained from this trial run as our starting values. By specifying unrealistically large values for the variance parameters we got the sampler to spread out the values of ξ_t and η_t in the first iteration in a way which would be very difficult to do "by hand" since there are over 370 initial values that we must supply. The convergence of the chains was rapid, after a burn in of 2,000 iterations the next 1,000 were saved, and all of the values of the $\sqrt{\hat{R}}$ statistic were less than 1.02.

2.7 Model Criticism

Since our models do not attempt to represent every conceivable facet of the phenomenon under investigation, it is essential to understand the shortcomings of our models. A simple, yet sensitive, method for detecting model weaknesses is to use the model to simulate another dataset, then to compare the simulated data to the observed data (posterior predictive checks, see e.g. Gelman et al. (1995)). The first step is to examine several of the simulated datasets graphically. After this, one can design test statistics and compare the distribution of these test statistics under the posterior predictive distribution to their distribution under the posterior distribution (if a test statistic doesn't depend on any of the model parameters it is constant under the posterior distribution). In the time series modeling context, several natural test statistics can be proposed on general grounds. First, if our series is x_t for $t = 1, \ldots, T$, then the average absolute value of the change in the level of the series $T_1(x_1, \ldots, x_T) = \frac{1}{T-1}\sum_{t=2}^{T}|x_t - x_{t-1}|$ is a simple measure of the volatility of the series (if our fitting method smoothes the data too much then T_1 will be too large under the posterior predictive distribution). If ϕ_t is the forecast of x_t conditional on the observed data, another natural diagnostic is the average of the absolute value of the prediction error, $T_2(x_1, \ldots, x_T, \phi_1, \ldots, \phi_T) = \frac{1}{T}\sum_{t=1}^{T}|x_t - \phi_t|$. If the fitting method smoothes too much, the prediction errors will be too large on average. Although obtaining analytic expressions for these quantities is a daunting task, it is simple to draw simulations of these quantities from the appropriate distributions.

3 Results for Our Example

3.1 Fitting the Normal Theory Model

Figure 2 shows the marginal posterior distribution of the components of W and the correlation between the elements of the state vectors, based on 40,000 simulation draws from the Metropolis algorithm. Figure 3 displays 95% probability intervals for the proportion in each party obtained by the model (these intervals are laid over the sample proportions), while Figure 4 shows posterior predictive draws of the sample proportions. In Figure 5 we find the 95% confidence intervals for the average approval rating within each party, while in Figure 6 we find the 95% probability intervals given by our post-stratification estimate, and 95% confidence intervals based on the sample mean (whose construction was given in the introduction, but, of course, no simulation was used here). From the last graph we see that our post-stratification estimator is more precise than the sample mean.

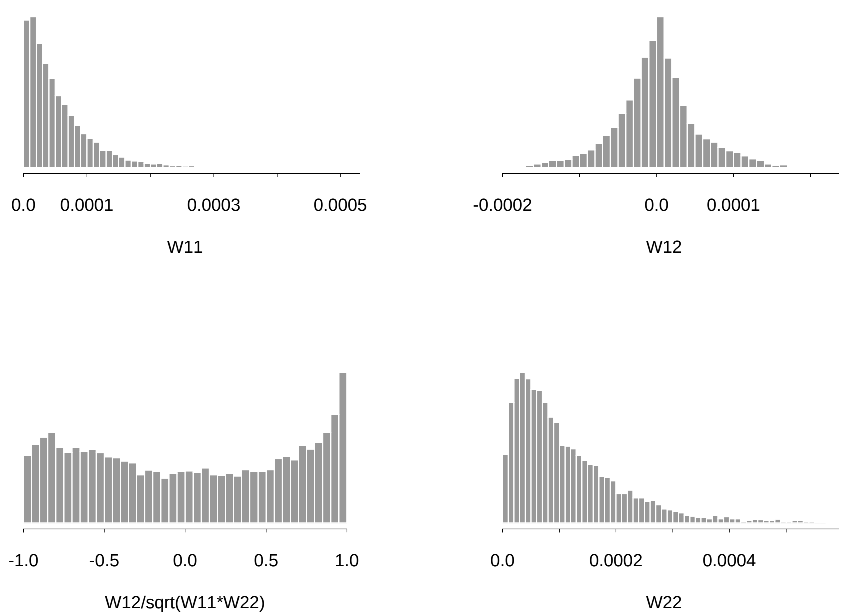

Figure 2: *The marginal posterior distribution of each element of the state covariance matrix assuming a flat prior. In the lower left hand corner we find the marginal posterior distribution of the correlation of the states.*

In order to more fully understand how the post-stratification estimator is working, it is instructive to see if our estimator really does respond to imbalances in the representation of the parties within our samples. To examine this we should consider Figure 7. From these graphs we easily see that if the proportion of Democrats relative to the proportion of Republicans in our sample is too large (relative to the estimate based on our dynamic model), then our post-stratification estimator will have a tendency to make the estimated approval rating smaller than the raw estimate (based on the sample mean). The same correction is made if there are too many Democrats in our sample (but the relative proportion of Democrats to Republicans is seen to be more important in determining the correction), and the opposite correction is made if there are too many Republicans. This is exactly the sort of behavior we expect since Clinton is a Democrat. From Figure 8 we see that the post-stratification estimate performs best for moderate sized samples (again, each dot represents one week of data in all of the plots). We also see that the

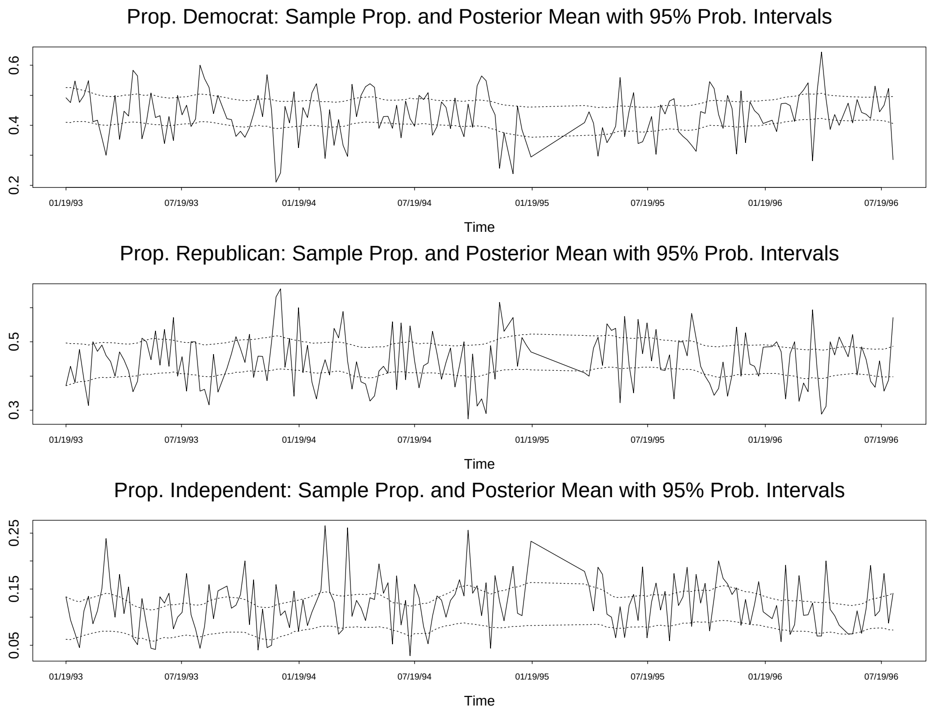

Figure 3: *The proportion in each party for all weeks with 95% probability intervals given by the model.*

largest corrections are for the smaller samples (as we would expect), and that the size of the correction does not have much to do with the estimated efficiency. Lastly, the fact that our state-space model for the party identification series is actually a hierarchical model for the increments of the state-space process is manifested in the shrinkage of the increments of our post-stratification estimate (as witnessed in the lower right hand corner of Figure 8).

3.2 Model Checking

3.2.1 Checking the Fit of Our Basic Model

The normal theory Kalman filter model presented above seems acceptable for our purposes. In Figure 4 we find a draw from the posterior predictive distribution for the number of respondents falling into each of the parties, while in Figure 1 we see two draws from the posterior predictive distribution for the average approval rating for each week. We obtain a draw from the posterior predictive distribution of the average approval rating by using a weighted mean of draws from approval within party, with weights given by the simulated sample proportions in each party under the posterior predictive distribution

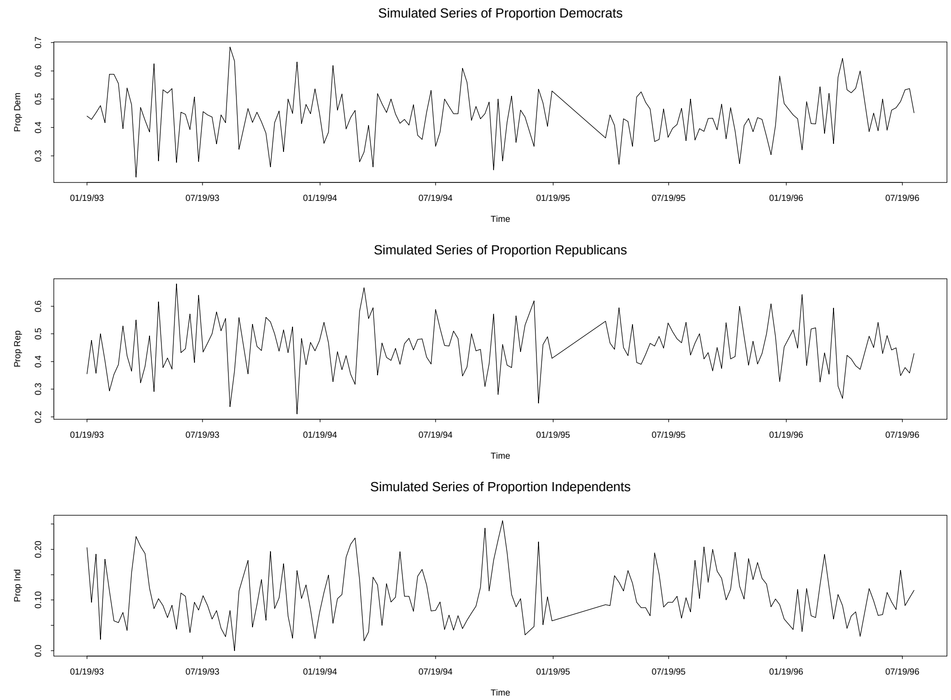

Figure 4: Simulated sample proportions for each week under the model. Compare to Figure 3.

for these proportions. We find the observed value of T_1, where

$$T_1(n_{1,1}, \ldots , n_{1,T}) = \frac{1}{T-1} \sum_{t=2}^{T} \left| \frac{n_{1,t}}{n_t} - \frac{n_{1,t-1}}{n_{t-1}} \right|,$$

is 0.089 and the 95% probability interval for T_2, where

$$T_2(n_{1,2}, \ldots , n_{1,T}, \theta_{2,1}, \ldots , \theta_{2,T-1}) = \frac{1}{T-1} \sum_{t=2}^{T} \left| \frac{n_{1,t}}{n_t} - \theta_{2,t-1} \right|,$$

under the posterior distribution is (0.065, 0.072). We find that 95% probability intervals for these two quantities based on 1,000 simulation draws from their posterior predictive distributions under the normal theory model are (0.076, 0.099) and (0.056, 0.069), and we find a 95% probability interval for the difference, $T_2(n^k_{1,2}, \ldots , n^k_{1,T}, \theta^k_{2,1}, \ldots , \theta^k_{2,T-1}) - T_2(n_{1,2}, \ldots , n_{1,T}, \theta^k_{2,1}, \ldots , \theta^k_{2,T-1})$ (where $n^k_{1,t}$ is the draw from the posterior predictive distribution corresponding to θ^k_t from the posterior distribution for $t = 1, \ldots , T$ and $k = 1, \ldots , 1000$), is (-0.014, 0.002). These posterior predictive checks indicate our normal theory model fits these aspects of the data.

3.2.2 Checking the Fit of Our Alternative Model

Once we examine our simulations for the proportion in a major party and the proportion of those in a major party who are Democrats based on the multinomial model, it appears

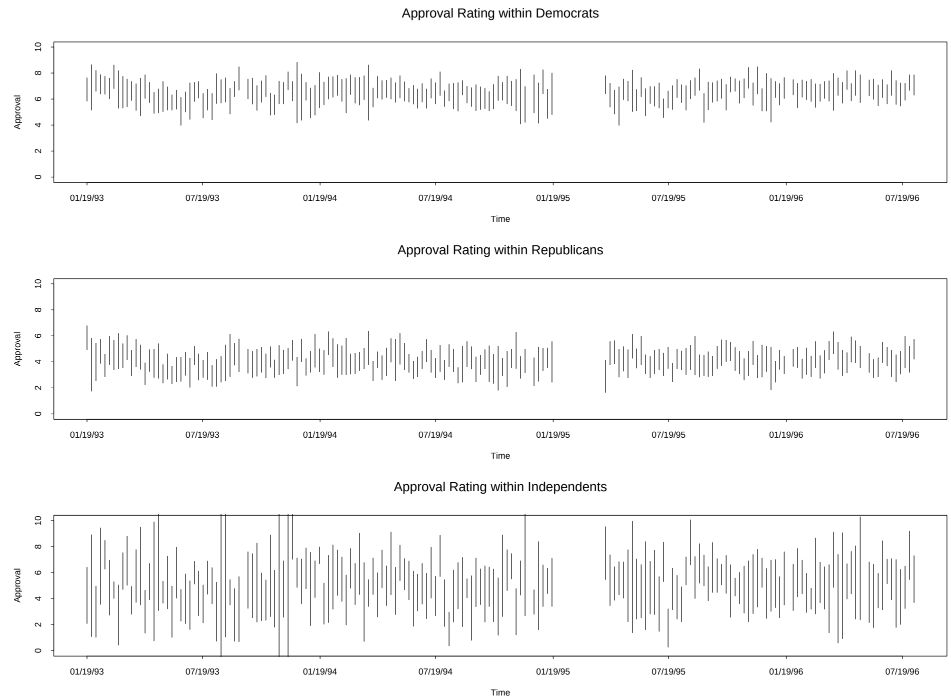

Figure 5: 95% confidence intervals for the mean approval rating within party.

that the posterior medians of these variables are too variable. Since the normal theory model for the party identification series is actually only based on a subset of the data we used to fit the multinomial model (our multinomial model was fit to data which included a portion of Bush's presidency), our observed value of T_1 is not the same as above (and we don't expect T_2 under the posterior distribution to be the same as above). Based on 1,000 posterior predictive samples we found that a 95% probability interval for T_1 under the multinomial model is (0.115, 0.147), while our observed value is 0.097. For our other test statistic, T_2, we find a 95% probability interval based on the posterior predictive distribution is (0.102, 0.129), while a 95% probability interval for T_2 based on the posterior distribution is (0.089, 0.107). We also find that a 95% probability interval for the difference, $T_2(n_{1,2}^k, \ldots, n_{1,T}^k, \theta_{2,1}^k, \ldots, \theta_{2,T-1}^k) - T_2(n_{1,2}, \ldots, n_{1,T}, \theta_{2,1}^k, \ldots, \theta_{2,T-1}^k)$, is (0.005, 0.028). These shortcomings indicate that the model is overfitting (i.e. our model doesn't smooth the series of proportions enough). It is difficult to construct a simpler model for the party identification series within the context of the model proposed by Cargnoni, Müller and West (1997), and so we chose to use the model based on the normal theory Kalman filter for the sample proportions.

4 Conclusions

The resulting estimates are more precise than the weekly sample means (the estimated efficiencies ranging from 0.66 to 2.3 with a mean of 1.19). If one considers the cost of

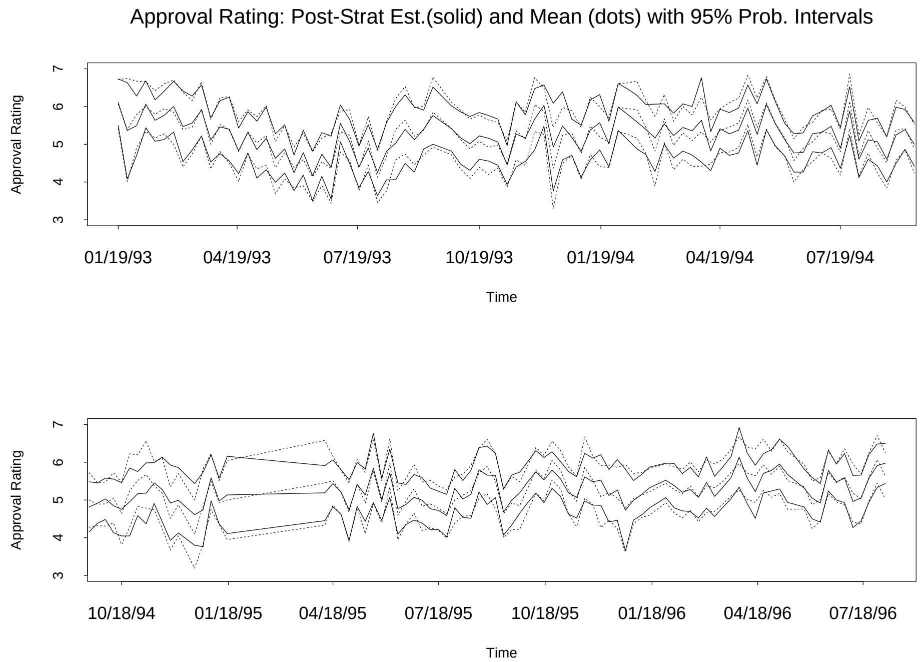

Figure 6: 95% probability intervals for the mean approval rating for each week based on the post-stratification estimate (solid line), and based on the sample mean (dotted line). The series is broken up to fit on one page.

obtaining survey data (since many questions are asked to each respondent, it can take 30 minutes to complete an interview), this is a great savings (with 8,462 observations, it is like we get over 1,600 more observations for free). If one has a long series for the quantity of interest, it may be feasible to identify an appropriate time series model for the quantity of interest. In such a case, one could base estimates on this model and obtain substantially more precise estimates (for example, one may be able to conclude that a random walk plus error model describes the movement of the series of interest over time). One advantage of this post-stratification estimate is that we are not required to propose a dynamic model for the quantity of interest, we only need a dynamic model for some quantity which is related to our quantity of interest. This is a great help here since specification of a dynamic model for a volatile variable (like approval rating) is controversial, while the slow changing nature of political attitudes implies models which allow for almost constant levels are suitable for separating measurement error from shifts in attitudes. Also, the results from our model for the party identification series can be used to construct post-stratified estimates for other variables. In this manner one can post-stratify a short series using simulations based on a more extensive dataset, and thereby obtain more precise estimates.

The failure of the multinomial model led us to consider other sorts of state-space models for discrete variables. The fact that state-space models are hierarchical models for the increments of the state process suggests that one can treat discrete variable

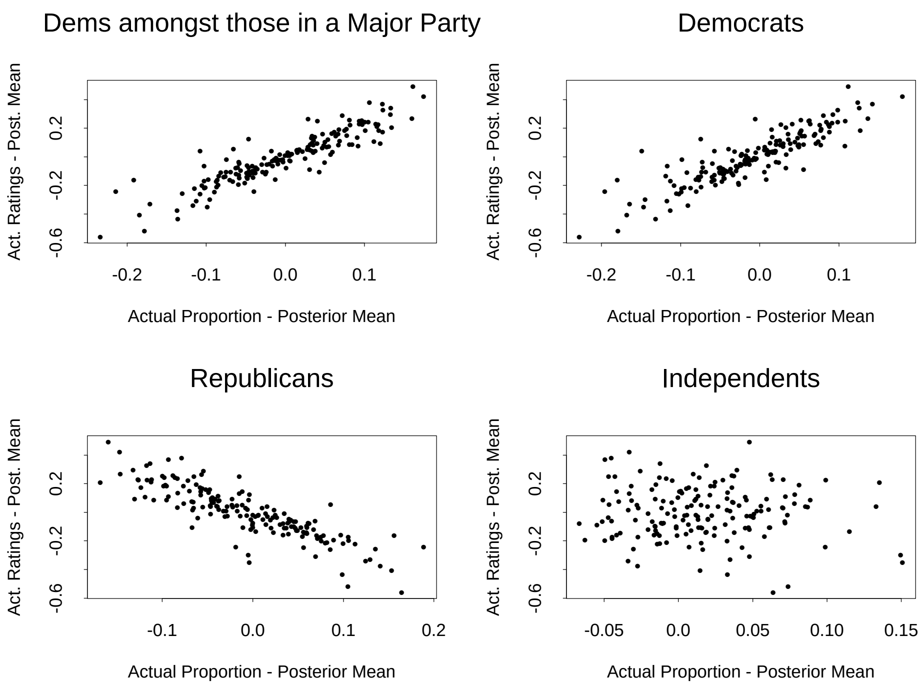

Figure 7: The difference in the observed proportions and the posterior means by the difference in the observed ratings and the posterior means for subsets of the samples. The post-stratification estimate corrects for unequal representation of the parties in our samples. Each dot represents one week.

filtering problems (by filtering we also refer to the associated problems of smoothing and prediction) exactly like random effects generalized linear models (on which there is an extensive literature, ranging from analytic approximations to several methods of posterior simulation), see the comments by Meyer in West, Harrison and Mignon (1985). Since adjacent states will have high posterior correlation, it seems sensible to parameterize the state process in terms of the increments of the state process rather than the levels of the process (this should yield a sampling algorithm which converges faster than one which samples the levels of the state process). This reparameterization is quite natural when one treats the filtering problem as a random effects generalized linear model.

There are also many approximations for filtering and smoothing in the time series literature (see for example, West, Harrison and Migon (1985)). These approximations provide reasonable initial values for iterative methods, or of course can be used as estimates themselves. If we are going to use approximate smoothing methods, a convenient way to obtain an approximation to the marginal likelihood of any model parameters, ϕ (e.g. state variances or autoregressive coefficients), is to use a method common in the random effects literature (see, for example, Rubin (1981) or Besag (1989)), namely

$$p(\phi|y) \propto \frac{p(y|\theta,\phi)p(\theta,\phi)}{p(\theta|\phi,y)}.$$

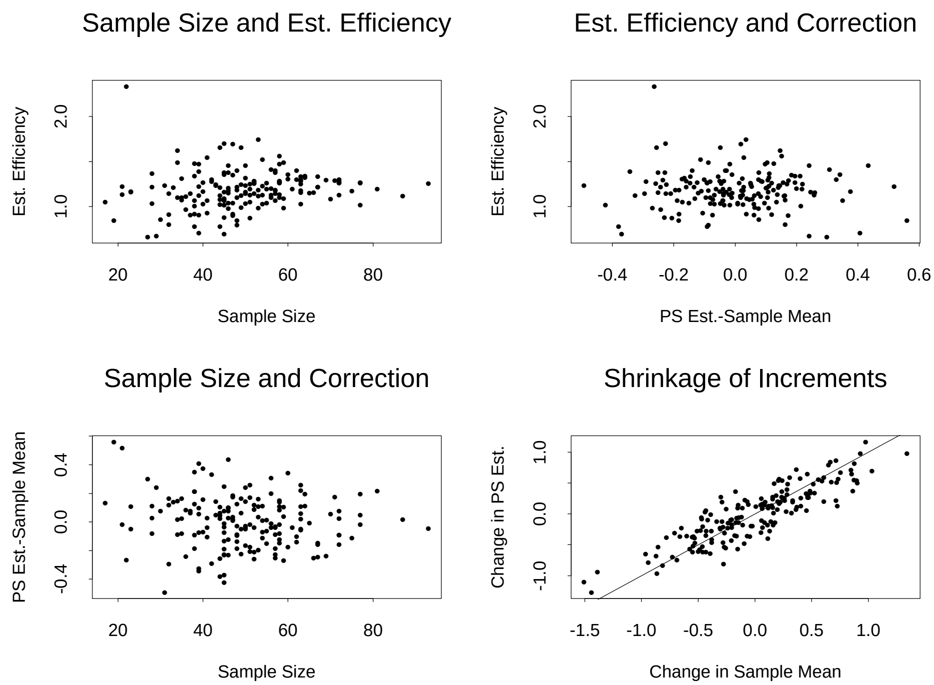

Figure 8: The post-stratification estimate performs best for moderate sized samples. The line in the plot which illustrates the shrinkage of the increments is a $y = x$ line. Each dot represents one week.

But if the state space is Markovian,

$$p(\theta, \phi) = p(\phi)p(\theta_0) \prod_{t=1}^{T} p(\theta_t | \theta_{t-1}, \dots, \theta_0, \phi),$$

thus it is typically straightforward to write down the numerator in the marginal likelihood. For the denominator we can use a multivariate normal with moments given by our approximate method. We also note that this expression is the easiest way to obtain the posterior distribution of the model parameters in the context of the extended Kalman filter.

In conclusion, we find that the post-stratification estimator gives more precise results than the sample mean, and it does this by correcting our estimate for imbalances in the representation of the political parties in our sample. Moreover, these gains are achieved without recourse to an explicit dynamic model for the quantity of interest.

References

Beck, N. (1991), "Comparing Dynamic Specifications: The Case of Presidential Approval," *Political Analysis*, 3, 51-88.

Besag, J. (1989), "A Candidate's Formula: A Curious Reult in Bayesian Prediction," *Biometrika*, 76, 183.

Brace, P., and Hinckley, B. (1991), "The Structure of Presidential Approval: Constraints within and across Presidencies," *The Journal of Politics*, 53(4), 993-1017.

Brody, R. (1991), *Assessing the President*, Stanford: Stanford University Press.

Cargnoni, C., Müller, P., and West, M. (1997), "Bayesian Forecasting of Multinomial Time Series Through Conditionally Gaussian Dynamic Models," *Journal of the American Statistical Association*, 92, 640-647.

Carter, C., and Kohn, R. (1994), "On Gibbs Sampling for State Space Models," *Biometrika*, 81, 541-553.

Clarke, H., and Stewart, M. (1994), "Prospections, Retrospections, and Rationality: The "Bankers" Model of Presidential Approval Reconsidered." *American Journal of Political Science*, 38(4), 1104-1123.

Frühwirth-Shnatter, S. (1994), "Data Augmentation and Dynamic Linear Models," *Journal of Time Series Analysis*, 15, 183-202.

Gelman, A, Carlin, J., Stern, H., and Rubin, D. (1995), *Bayesian Data Analysis*, London: Chapman & Hall.

Holt, D., and Smith, T. M. F. (1979), "Post Stratification," *Journal of the Royal Statistical Society A*, 142, 33-46.

Kernell, S. (1978), "Explaining Presidential Popularity," *American Political Science Review*, 72, 506-22.

Kernell, S. (1986), *Going Public*, Washington, D.C.: CQ Press.

Kiewiet, D., and Rivers, D. (1985), "The Economic Basis of Reagan's Approval," In *Contemporary Political Economy*, ed. D. A. Hibbs and J. Fasbender, pp. 49-71, Elseview: North-Holland.

Little, R. J. A. (1993), "Post-Stratification: A Modeler's Perspective," *Journal of the American Statistical Association*, 88, 1001-1012.

Little, T. (1996), "Models for Non-response Adjustment in Sample Surveys," Ph.D. Thesis, University of California, Berkeley.

MacKuen, M. (1983), "Political Drama, Economic Conditions, and the Dynamics of Presidential Popularity," *American Journal of Political Science*, 27, 165-192.

Neustadt, R. (1990), *Presidential Power*, New York: Free Press.

Ostrom, C., Jr., and Simon, D. (1989), "The Man in The Teflon Suit," *Public Opinion Quarterly*, 53, 353-387.

Ostrom, C., Jr., and Smith, R. M. (1993), "Error Correction, Attitude Persistence, and Executive Rewards and Punishments: A Behavioral Theory of Presidential Approval," *Political Analysis*, 4, 127-184.

Pfeffermann, D. (1991), "Estimation of Seasonal Adjustments of Population Means using Data from Repeated Surveys," *Journal of Business and Economic Statistics*, 9, 163-177.

Rivers, D., and Rose, N. (1985), "Passing the President's Program: Public Opinion and Presidential Influence in Congress," *American Journal of Political Science*, 29, 183-196.

Rubin, D. B. (1981), "Estimation in Parallel Randomized Experiments," *Journal of Educational Statistics*, 6, 377-401.

Smith, R. (1992), "Error Correction, Attractors, and Cointegration: Substantive and Methodological Issues," *Political Analysis*, 4, 249-254.

Voss, S., Gelman, A., and King, G. (1995), "Pre-Election Survey Methodology: Details From Nine Polling Organizations, 1988 and 1992," *Public Opinion Quarterly*, 59, 98-132.

West, M., and Harrison, J. (1997), *Bayesian Forecasting and Dynamic Models*, New York: Springer-Verlag.

West, M., Harrison, P.J., and Migon, H. (1985), "Dynamic Generalized Linear Models and Bayesian Forecasting (with discussion)," *Journal of the American Statistical Association*, 80, 73-97.

Williams, J. (1992), "What Goes Around Comes Around: Unit Root Tests and Cointegration," *Political Analysis*, 4, 229-236.