15th Triennial World Congress, Barcelona, Spain

GENERALIZED RECEDING HORIZON CONTROL SCHEME FOR CONSTRAINED LINEAR DISCRETE-TIME SYSTEMS

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Abstract: In this paper, we propose a generalized stabilizing receding horizon control (RHC) scheme for input/state constrained linear discrete-time systems, which includes existing ones, and is much more flexible and profitable in terms of feasibility & computation. The control scheme is based on a time-varying horizon cost function with time-varying terminal weighting matrices, which can easily be implemented via linear matrix inequality (LMI) optimization. We discuss modified schemes of the proposed one, which are better than the proposed general scheme in terms of feasibility or computation. Through various simulation examples, we illustrate the proposed results.

Keywords: RHC, constraints, time-varying, feasibility, computation, LMI

1. INTRODUCTION

Receding horizon control (RHC) uses the current control law obtained by solving the optimization problem every sampling instant. Since the RHC can consider a finite horizon cost function for the closed-loop stability, it takes advantages of handling input/state constraints, time-varying systems, etc. Finite horizon formulations with finite terminal weighting matrices have been widely investigated as in (G. De Nicolao and L. Magni and R. Scattolini, 1998; Lee *et al.*, 1998; Park and Kwon, 1999; Kwon and Kim, 2000; Kim, n.d.), since they include the infinite horizon formulation as in (Kothare *et al.*, 1996) and have less computational burdens than that with the infinite terminal weighting matrix as in (Rawlings and Muske, 1993).

For constrained and/or time-varying systems, feasibility and computational burden are very important issues. For the stabilizing RHC, these are closely related to how to design the terminal weighting matrix and cost horizon. In literature, terminal weighting matrices have been often represented by inequality conditions, which guarantee the closed-loop stability.

For constrained systems, the terminal inequality conditions make the optimization problem infeasible in the presence of large initial states as shown in (Kothare *et al.*, 1996; G. De Nicolao and L. Magni and R. Scattolini, 1998; Lee *et al.*, 1998; Park and Kwon, 1999). The results in (Kothare *et al.*, 1996; Park and Kwon, 1999) have conservative feasible initial-state sets since they assume a linear feedback RHC and one-horizon cost function, respectively. In addition, they don't consider arbitrary time-varying systems. Although the results in (Lee *et al.*, 1998) and (G. De Nicolao and L. Magni and R. Scattolini, 1998) consider general constrained time-varying systems, the terminal inequality condition in (Lee *et al.*, 1998) is not practical as shown in (Kim, n.d.) and the fixed terminal weighting matrix in (G. De Nicolao and L. Magni and R. Scattolini, 1998) still has a small feasible reason.

For time-varying systems, we should solve many terminal inequality conditions for the closed-loop stability as shown in (Kim, n.d.). Too many conditions may make the optimization problem infeasible numerically and will cause a lot of computational burden. Here, if time-varying systems are constrained, then the optimization problem will be much more infeasible and have more computational burden. Thus, it will be interesting to investigate about how to implement a stabilizing RHC in terms of feasibility and computational burden for general constrained time-varying systems.

The horizon size is always fixed in literature to author's knowledge. For this reason, a large one causes a lot of computational burden and may make the optimization problem infeasible numerically for constrained and/or time-varying systems. The small one, however, has a small feasible initial-state set for constrained systems. Thus, it will also be interesting to investigate about how to deal with the horizon size in terms of feasibility and computational burden.

In this paper, we propose a generalized RHC scheme for input/state constrained discrete-time systems, which includes the whole existing ones in (Kothare *et al.*, 1996; G. De Nicolao and L. Magni and R. Scattolini, 1998; Lee *et al.*, 1998; Park and Kwon, 1999; Kim, n.d.) and is much more flexible and profitable in terms of feasibility & computation. The control scheme is based on a time-varying finite horizon cost function with time-varying finite terminal weighting matrices, which can easily be implemented via linear matrix inequality (LMI) optimization. We discuss modified schemes of the proposed one, which are better than the proposed general scheme in terms of feasibility or computation. Through simulation examples, we illustrate the proposed results.

2. GENERALIZED STABILIZING RECEDING HORIZON CONTROL SCHEME

Consider the linear discrete time-varying system:

$$x(i+1) = A(i)x(i) + B(i)u(i), \quad x(0) = x_0 \quad (1)$$

subject to the input and state constraints:

$$u_{\min}(i) \le G_u(i)u(i) \le u_{\max}(i), \; i \in [0, \; \infty) \quad (2)$$
$$x_{\min}(i) \le G_x(i)x(i) \le x_{\max}(i), \; i \in [0, \; \infty) \quad (3)$$

where $x(i) \in R^n$ is the state, $u(i) \in R^m$ the control, $G_u(i) \in R^{l\times m}$, and $G_x(i) \in R^{p\times n}$.

For the system (1), consider the following optimization problem:

$$\underset{u(i),u(i+1),\cdots,u(i+N_i-1),Q_f(i),H(i)}{\text{Minimize}} \quad \beta_1(i) + \beta_2(i) \quad (4)$$

subject to

$$\beta_1(i) \ge \sum_{\tau=i}^{i+N_i-1} x^T(\tau)Q(\tau)x(\tau) + u^T(\tau)R(\tau)u(\tau) \quad (5)$$
$$\beta_2(i) \ge x^T(i+N_i)Q_f(i)x(i+N_i) \quad (6)$$
$$u_{\min}(\tau) \le G_u(\tau)u(\tau) \le u_{\max}(\tau), \quad \tau \in [i, i+N_i-1] \quad (7)$$
$$x_{\min}(\tau) \le G_x(\tau)x(\tau) \le x_{\max}(\tau), \quad \tau \in [i, i+N_i] \quad (8)$$
$$-u_{\lim}(\tau) \le G_u(\tau)u(\tau)(= -H(i)x(\tau)) \le u_{\lim}(\tau), \; \tau \in [i+N_i, \infty) \quad (9)$$
$$-x_{\lim}(\tau) \le G_x(\tau+1)x(\tau+1)(= (A(\tau) - B(\tau)H(i))x(\tau)) \le x_{\lim}(\tau), \; \tau \in [i+N_i, \infty) \quad (10)$$

where $N_i \ge 1$, $Q(\tau) = C^T(\tau)C(\tau)$, $R(\tau) = R^T(\tau)$, $Q_f(i) = Q_f^T(i)$ are positive definite matrices, $u_{\lim}(\tau) = \min\{|u_{\min}(\tau)|, \; |u_{\max}(\tau)|\}$, and $x_{\lim}(\tau) = \min\{|x_{\min}(\tau)|, \; |x_{\max}(\tau)|\}$.

Here, we introduce the terminal inequality condition, which is proposed in (Kim, n.d.) for unconstrained time-varying systems with a fixed horizon $N_i = N$:

$$Q_f(i) \ge Q(\sigma) + H^T(i)R(\sigma)H(i) + (A(\sigma) - B(\sigma)H(i))^T Q_f(i)(A(\sigma) - B(\sigma)H(i)) \quad \text{for all } \; \sigma \ge i+N_i \; \text{ and some } \; H(i). \quad (11)$$

First, we investigate the feasibility of the optimization problem (4).

Lemma *1.* If the problem (4) subject to (5)-(11) is feasible at some time i and $N_{j+1} \ge N_j - 1$ for all $j \ge i$, then it is feasible for all time $j \ge i$ without any disturbances and noises.

Proof: It is clear from (9) and (10). □

Throughout the rest of this paper, assume that $N_{i+1} \ge N_i - 1$ for all i and there are no external disturbances and noises. For N_0 at the initial time $i = 0$, we select the smallest one, with which the problem (4) subject to (5)-(11) is feasible. Note that the larger one just increases the computational burden.

For handling of (9) and (10), we introduce a simple extension of the ellipsoid constraints in (Boyd *et al.*, 1994) as follows.

$$\begin{bmatrix} Z(\sigma) & G_u(\sigma)Y(i) \\ Y^T(i)G_u^T(\sigma) & S(i) \end{bmatrix} \geq 0 \text{ and}$$
$$Z_{j,j}(\sigma) \leq u^2_{\lim,j}(\sigma) \text{ for all } \sigma \geq i + N_i \quad (12)$$
$$G_x(\sigma)S(i)G_x^T(\sigma) \leq E(\sigma) \text{ and}$$
$$E_{j,j}(\sigma) \leq x^2_{\lim,j}(\sigma) \text{ for all } \sigma \geq i + N_i \quad (13)$$

where $S(i) = \beta_2(i)Q_f^{-1}(i)$, $Y(i) = H(i)S(i)$, $u_{\lim,j}(\sigma)$ and $x_{\lim,j}(\sigma)$ are jth elements of $u_{\lim}(\sigma)$ and $x_{\lim}(\sigma)$, respectively, and $Z_{j,j}(\sigma)$ and $E_{j,j}(\sigma)$ are the (j,j) elements of the matrices $Z(\sigma)$ and $E(\sigma)$, respectively.

Lemma *2.* If there exist solutions $u(\cdot)$ and $Q_f(i)$ for the problem (4) subject to (5)-(8) and (11)-(13) at the time i, then (9) and (10) are satisfied. Thus, if (4) subject to (5)-(8) and (11)-(13) is feasible at the initial time, then it is always feasible.

Proof: From the result in (Boyd *et al.*, 1994) for continuous time-invariant systems, we can easily know that (9) and (10) are satisfied if there exist solutions $u(\cdot)$ and $Q_f(i)$ for the problem (4) subject to (5)-(8) and (11)-(13) at the time i. Thus, the problem (4) subject to (5)-(8) and (11)-(13) can also be feasible at the next time $i+1$ with $\beta_2(i+1)$ and $Q_f(i+1)$ replaced by $\beta_2(i)$ and $Q_f(i)$, respectively. It holds for all times. □

From Lemma 2, we can define the feasible initial state-set as

$$\chi_0 = \{x(0)|\text{there exist } u(0), u(1), \cdots, u(N_0 - 1) \\ H(i), \text{ and } Q_f(i) \text{ for (4) subject to} \\ (5)-(8) \text{ and } (11)-(13) \text{ with } x(0)\} \quad (14)$$

Now, the first control $u^*(i)$ at each time i is called receding horizon control (RHC), which is obtained by solving the problem (4) subject to (5)-(8) and (11)-(13). Then, this proposed scheme is called RHC scheme. The resulting optimal cost at each time i is defined as $J^*(i, i + N_i)$.

Next, we investigate the closed-loop stability of the proposed RHC by using the result in (Kim, n.d.). To this end, we introduce the following lemma.

Lemma *3.* If the problem (4) subject to (5)-(8) and (11)-(13) is feasible, the resulting optimal cost is monotonically nonincreasing, i.e., $J^*(\tau_1, \tau_2) \geq J^*(\tau_1, \tau_2 + 1)$ for $\tau_2 \geq \tau_1 + 1$. Thus, $J^*(\tau_1, \tau_2) \geq J^*(\tau_1, N)$ for all $N \geq \tau_2$.

Proof: It can be proved in the similar way as that in (Kim, n.d.) for general unconstrained linear discrete time-varying systems. □

THEOREM *1.* If the optimization problem (4) subject to (5)-(8) and (11)-(13) is feasible at the initial time, then the closed-loop system with the resulting RHC $u^*(i)$ is uniformly asymptotically stable. If the pair $(A(i), C(i))$ is uniformly detectable, then it is uniformly attractive and bounded.

Proof: The optimization problem is always feasible from Lemma 2. By using the proof in (Kim, n.d.), we can easily know that $J^*(i, i + N_i) \geq x^T(i)Q(i)x(i) + u^{*T}(i)R(i)u^*(i) + J^*(i + 1, i + N_i + 1)|_{Q_f(i),\beta_2(i)}$ where $J^*(i + 1, i + N_i + 1)|_{Q_f(i),\beta_2(i)}$ is the optimal cost at $i+1$ with $Q_f(i+1)$ and $\beta_2(i+1)$ replaced by $Q_f(i)$ and $\beta_2(i)$, respectively. Here, $Q_f(i)$ and $\beta_2(i)$ are solutions for $J^*(i, i + N_i)$. When $N_{i+1} \geq N_i$, since $J^*(i+1, i+N_i+1)|_{Q_f(i),\beta_2(i)} \geq J^*(i+1, i+N_{i+1}+1)|_{Q_f(i),\beta_2(i)}$ from Lemma 3 and $J^*(i+1, i+N_{i+1}+1)|_{Q_f(i),\beta_2(i)} \geq J^*(i+1, i+N_{i+1}+1)$ by optimality, $J^*(i, i+N_i) \geq x^T(i)Q(i)x(i) + u^{*T}(i)R(i)u^*(i) + J^*(i+1, i+1+N_{i+1})$. This nonincreasing monotonicity holds when $N_{i+1} = N_i - 1$. Since $Q(\cdot)$ is positive definite, $J^*(i, i + N_i)$ is a Lyapunov function.

When the pair $(A(i), C(i))$ is uniformly detectable and $C(i)$ is not positive definite, then we can just guarantee that the resulting closed-loop system is uniformly attractive and bounded. If the system has no input/state constraints, it is clear that the closed-loop system is uniformly asymptotically stable under the uniform detectability. □

For the closed-loop stability, the result in (Lee *et al.*, 1998) also proposes a terminal inequality condition, which is different from (11):

$$Q_f(i) \geq Q(i + N) + H^T(i)R(i + N)H(i) \\ +(A(i + N) - B(i + N)H(i))^T Q_f(i + 1) \\ (A(i + N) - B(i + N)H(i)) \text{ for some } H(i) \quad (15)$$

However, as pointed out in (Kim, n.d.), it is impossible to solve (15) for all times since Q_{i+1} is a design parameter of the optimization problem at the next time $i + 1$. The results in (Kothare *et al.*, 1996; Park and Kwon, 1999) consider time-varying systems included in a convex hull. Note that the proof methods for the closed-loop stability in (Kothare *et al.*, 1996; Park and Kwon, 1999) cannot be applied directly to general time-varying systems since the cost monotonicity with time-varying terminal weighting matrices should be handled carefully as shown in (Kim, n.d.).

Next, we show that the proposed RHC scheme includes those in (Kothare *et al.*, 1996; G. De Nicolao and L. Magni and R. Scattolini, 1998; Lee *et al.*, 1998; Park and Kwon, 1999; Kim, n.d.).

Remark *1.* For convenience of comparison, we introduce

$$Q_f(i) \geq Q + H^T(i)RH(i) + (A_k - B_kH(i))^T Q_f(i)(A_k - B_kH(i)) \text{ for all } k \in [1, L] \text{ and} \quad (16)$$

$$\underset{u,Q_f(i),H(i)}{\text{Minimize}} \sum_{\tau=i}^{i+N-1} x^T(\tau)Q(\tau)x(\tau) + u^T(\tau)R(\tau) u(\tau) + x^T(i+N)Q_f(i)x(i+N). \quad (17)$$

The results in (Lee *et al.*, 1998), (Kothare *et al.*, 1996), (Park and Kwon, 1999) consider the following problems: (17) subject to (7), (8), (12), (13), and (15) with $\beta_2(i) = 1$ and $Q_f(i+1)$; (17) subject to (9) and (10) for $\tau \in [i, \infty)$, (12) and (13) with $\sigma = i + N$ and $\beta_2(i) = 1$, and (16) (without $Q_f(i)$); and (4) subject to (5)-(8), (12), (13), and (16) with $N_i = 1$, respectively. Since the result in (G. De Nicolao and L. Magni and R. Scattolini, 1998) is an extension of the result in (Lee *et al.*, 1998) for nonlinear systems and the result in (Kim, n.d.) can be handled as a special case of this paper, the proposed scheme in this paper includes the above existing results.

Based on Remark 1, we compare the proposed formulation with existing ones in (Kothare *et al.*, 1996; Lee *et al.*, 1998; Park and Kwon, 1999) in terms of feasibility & computation. Note that the horizon size is always fixed in literature.

Remark *2.* As N_0 increases, the feasible initial-state set in this paper gets larger than that in (Park and Kwon, 1999) with $N_i = 1$ for all i, although it holds after some finite value depending on system, constraints, and weighting matrices. For a small N_0, it is also larger than that in (Lee *et al.*, 1998) with $N_i = N_0$ and $\beta_2(i) = 1$ for all i. As N_0 increases, the feasible set in this paper becomes similar to that with $\beta_2(i)$ fixed as $\beta_2(i) = 1$. However, a large N_0 causes a lot of computational burden for constrained and/or time-varying systems. The linear feedback RHC in (Kothare *et al.*, 1996) has the smallest feasible state-set although it has a smaller computational burden than those in (Lee *et al.*, 1998; Park and Kwon, 1999), and the proposed RHC with the fixed horizon size N. Since N_i can be reduced as $N_i = N_{i-1} - 1$ in this paper, the proposed RHC with the reduced N_i has a much smaller computation time than that with the fixed one. Therefore, the proposed scheme in this paper is much more flexible and profitable than the existing ones in terms of feasibility & computation.

Note that N_i and $Q_f(i)$ don't need to be fixed even for time-invariant systems. As another method to reduce the computational burden for time-varying systems, we propose the following result based on Theorem 1.

Corollary *1.* Let $Q_f(0)$ be the solution for (11)-(13) at the initial time. If the optimization problem (4) with $Q_f(i)$ replaced by $Q_f(0)$ subject to (5)-(8) is feasible at the initial time, then the closed-loop system with the resulting RHC $u^*(i)$ is uniformly asymptotically stable. If the pair $(A(i), C(i))$ is uniformly detectable, then it is uniformly attractive and bounded.

The proof here comes from Theorem 1. Note that the RHC from Corollary 1 has a less computational burden than that from Theorem 1 since (11)-(13) is solved only once at the initial time, while its performance seems to be worse than that from Theorem 1.

In the next section, how to implement the proposed stabilizing RHCs is suggested.

3. IMPLEMENTATION OF THE RHC VIA LMI OPTIMIZATION

In this section, the proposed RHC scheme is converted to the equivalent linear matrix inequality (LMI) problem.

For simplicity, throughout the rest of this paper, define $\bar{F}(i)$ and $\hat{F}(i)$ as

$$\bar{F}(i) = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 \\ F(i) & \ddots & \ddots & \ddots & \vdots \\ 0 & F(i+1) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & F(i+N_i-2) & 0 \end{bmatrix}$$

$$\hat{F} = \begin{bmatrix} F(i) & 0 & \cdots & 0 \\ 0 & F(i+1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & F(i+N_i-1) \end{bmatrix},$$

respectively. $\bar{F}(i)$ and $\hat{F}(i)$ are used for presentation of $\bar{A}(i)$, $\bar{B}(i)$, $\hat{Q}(i)$, $\hat{R}(i)$, $\hat{G}_u(i)$, and $\hat{G}_x(i)$ which consist of $A(i)$, $B(i)$, $Q(i)$, $R(i)$, $G_u(i)$, and $G_x(i)$, respectively. Also define $\Phi_A(i+N_i, i)$ as $\Phi_A(i+N_i, i) = A(i+N_i-1)A(i+N_i-2)\cdots A(i)$. Then, the problem (4) subject to (5)-(8), and (11)-(13) can be converted to

$$\underset{U(i),\ Q_f(i)}{\text{Minimize}} \quad \beta_1(i) + \beta_2(i) \quad (18)$$

subject to (12)-(13),

$$\begin{bmatrix} M_1(i) & (W_2^{\frac{1}{2}}(i)U(i))^T \\ W_2^{\frac{1}{2}}(i)U(i) & I \end{bmatrix} \geq 0 \qquad (19)$$

$$\begin{bmatrix} 1 & M_2^T(i) \\ M_2(i) & S(i) \end{bmatrix} \geq 0 \qquad (20)$$

$$\begin{aligned} \left[u_{\min}^T(i),\ \cdots,\ u_{\min}^T(i+N_i-1)\right]^T &\leq \hat{G}_u(i) \\ U(i) \leq \left[u_{\max}^T(i),\ \cdots,\ u_{\max}^T(i+N_i-1)\right]^T & \quad (21) \end{aligned}$$

$$\begin{aligned} &\left[x_{\min}^T(i), \cdots, x_{\min}^T(i+N_i-1)\right]^T \leq \\ &\hat{G}_x(i)(I-\bar{A}(i))^{-1}(\bar{B}(i)U(i)+X_0(i)) \\ &\leq \left[x_{\max}^T(i),\ \cdots,\ x_{\max}^T(i+N_i-1)\right]^T \quad (22) \end{aligned}$$

$$\begin{bmatrix} S(i) & M_3^T(i) & M_4^T(i) & M_5^T \\ M_3(i) & S(i) & 0 & 0 \\ M_4(i) & 0 & \beta_2(i)I & 0 \\ M_5 & 0 & 0 & \beta_2(i)I \end{bmatrix} \geq 0 \qquad (23)$$

for all $\sigma \geq i+N_i$ where $M_1(i) = \beta_1(i) - W_1(i)U(i) - W_0(i)$, $W_1(i) = 2X_0^T(i)\hat{Q}_{\bar{A}}(i)\bar{B}_2$, $U(i) = [u^T(i),\ u^T(i+1),\ \cdots,\ u^T(i+N_i-1)]^T$, $W_0(i) = X_0^T(i)\hat{Q}_{\bar{A}}(i)X_0(i)$, $W_2(i) = \hat{R}(i) + \bar{B}^T(i)\hat{Q}_{\bar{A}}(i)\bar{B}(i)$, $\hat{Q}_{\bar{A}}(i) = (I-\bar{A}(i))^{-T}\hat{Q}(i)(I-\bar{A}(i))^{-1}$, $X_0(i) = [x^T(i),\ 0,\ \cdots,\ 0]^T$, $M_2(i) = (\Phi_A(i+N_i,i)x(i)+\bar{B}_2(i)U(i))$, $M_3(i) = A(\sigma)S(i) - B(\sigma)Y(i)$, $M_4(i) = C(\sigma)S(i)$, and $M_5 = R^{\frac{1}{2}}(\sigma)Y(i)$. When $N_i = 1$, (19)-(22) are changed into

$$\begin{bmatrix} \beta_1(i) - x^T(i)Q(i)x(i) & (R^{\frac{1}{2}}(i)u(i))^T \\ R^{\frac{1}{2}}(i)u(i) & I \end{bmatrix} \geq 0$$

$$\begin{bmatrix} 1 & (A(i)x(i)+B(i)u(i))^T \\ A(i)x(i)+B(i)u(i) & S(i) \end{bmatrix} \geq 0$$

$$u_{\min}(i) \leq G_u(i)u(i) \leq u_{\max}(i)$$

$$x_{\min}(i) \leq G_x(i)x(i) \leq x_{\max}(i).$$

The RHC is obtained by solving the above LMI optimization problem at each time i. Note that $x_{\min}(i+N_i) \leq G_x(i+N_i)x(i+N_i) \leq x_{\max}(i+N_i)$ is satisfied by (13) and (20). Next, we discuss how to deal with (12), (13), and (23) for all $\sigma \geq i+N_i$ in terms of computation & feasibility.

Remark *3.* When the set of all pairs $(A(i), B(i))$ can be expressed using a finite number of pairs, we have only to solve (12), (13), and (23) for the finite number of pairs. However, even if the set of all pairs $(A(i), B(i))$ can be expressed using a finite number of pairs, the optimization problem (18) can be infeasible numerically when the maximum and minimum eigenvalues of $W_2(i)$ have a big difference and the computer recognizes the minimum eigenvalue as zero, although all eigenvalues of $W_2(i)$ are always positive from the problem formulation. When information of the pairs $(A(i), B(i))$ is not given for all times or the set of all pairs $(A(i), B(i))$ is infinite, we cannot also solve the optimization problem. In these cases, we consider (12), (13), and (23) with $\sigma = i+N_i$ instead of all $\sigma \geq i+N_i$. Then, the optimization problem becomes easier to handle in terms of computation & numerical feasibility; but the cost monotonicity is no longer guaranteed theoretically as we can expect easily from Lemma 3. However, as in the simulation example in the next section, the cost is seen to be monotonically nonincreasing in many cases.

If (12) and (13) for all $\sigma \geq i+N_i$ cannot be expressed with finite number of conditions, then we simply set all $u_{\lim}(\tau)$ and $x_{\lim}(\tau)$ for $\tau \geq i+N_i+T$ as $u_{\lim}(\tau) = \min\{u_{\lim}(t),\ t \in [i+N_i+T, \infty)\}$, and $x_{\lim}(\tau) = \min\{x_{\lim}(t),\ t \in [i+N_i+T, \infty)|\}$ where T is a user-designed nonnegative integer.

4. SIMULATION EXAMPLES

For illustration of the proposed RHCs, we consider two types of constrained time-varying systems.

For all examples, assume that $i = 0, 1, 2, \cdots, 18$, $x_{\lim}(i) = 10$, $G_u(i) = 1$, $G_x(i) = [1\ \ 1]$, $Q(i) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $R(i) = 1$, and $x_0 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$. In this section, the RHCs from (18) subject to (12), (13), (19)-(23) for all $\sigma \geq i+N_i$ with $N_i = N_{i-1}-1$, for all $\sigma \geq i+N_i$ with $N_i = N$, for $\sigma = i+N_i$ with $N_i = N_{i-1}-1$, and for $\sigma = i+N_i$ with $N_i = N$ are called RH1, RH2, RH3, and RH4, respectively.

4.1 Constrained Time-Varying System I

Consider a time-varying system where the pair $(A(i), B(i))$ belongs to a polytope.

$$\begin{aligned} A(i) &= \frac{1}{3}sin(\frac{i}{\gamma_1}\pi)\begin{bmatrix} 1.2 & 0.2 \\ 0.3 & 0.5 \end{bmatrix} + \frac{1}{3}cos(\frac{i}{\gamma_2}\pi) \\ &\quad \begin{bmatrix} 0.9 & 0.5 \\ 0.3 & 1.2 \end{bmatrix} + \frac{1}{3}\alpha(i)\begin{bmatrix} 1 & 0.2 \\ 0.9 & 0.7 \end{bmatrix} \\ B(i) &= \frac{1}{3}sin(\frac{i}{\gamma_1}\pi)\begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} + \frac{1}{3}cos(\frac{i}{\gamma_2}\pi)\begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix} \\ &\quad + \frac{1}{3}\alpha(i)\begin{bmatrix} 1 \\ 0.7 \end{bmatrix} \end{aligned}$$

where $u_{\lim}(i) = 1$, $\gamma_1 = 5$, $\gamma_2 = 8$, and $\alpha(i)$ is an uniformly distributed random variable in $(0, 1)$. Note that this system has three pairs of vertices.

Table 1 shows that RH1 has the smallest norm of state and RH4 has the widest feasible initial state-set. The problems for RH1 and RH2, and RH3 and RH4 are feasible when $N_0 = N \geq 4$ and $N_0 = N \geq 1$, respectively. Although the RHC with the fixed Q_f is not shown in this paper due to the page limit, it has the smallest computation time.

The optimal costs from RH1 and RH2 are monotonically decreasing. Note that the optimal costs from RH3 and RH4 also seem to decrease monotonically for many values of γ_1 and γ_2 while the cost monotonicity is not guaranteed theoretically.

Table 1. $\sqrt{\|x\|^2}$ & Total Online Simulation Time (sec.)

	$\sqrt{\|x\|^2}$	Time
$\text{RH1}_{N_0=4}$	14.51	6.0
$\text{RH2}_{N=4}$	14.92	7.4
$\text{RH3}_{N_0=2}$	14.91	3.3
$\text{RH4}_{N=2}$	14.94	3.9

4.2 Constrained Time-Varying System II

Consider a periodic time-varying system:

$$A(i) = \begin{bmatrix} 1 + \alpha_1 sin(\frac{i}{\gamma_3}\pi) & 0.2 \\ 0.3 & 0.5 + \alpha_2 cos(\frac{i}{\gamma_4}\pi) \end{bmatrix}$$

$$B(i) = \begin{bmatrix} 0.8 + \alpha_3 cos(\frac{i}{\gamma_5}\pi) \\ 0.2 + \alpha_4 sin(\frac{i}{\gamma_6}\pi) \end{bmatrix} \tag{24}$$

where $u_{\text{lim}}(i) = 2$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = \frac{1}{2}$, $\alpha_4 = 1$, $\gamma_3 = 2$, $\gamma_4 = 3$, $\gamma_5 = 5$, and $\gamma_6 = 7$.

As mentioned in Remark 3, the problems for RH1 and RH2 are infeasible numerically for many different values of $\alpha_1 - \gamma_6$ including this example. However, RH3 and RH4 can be obtained when $N_0 = N \geq 2$ for many different values of $\alpha_1 - \gamma_6$.

Table 2 shows the results of RH3 and RH4. Note that the optimal cost from RH3 and RH4 seems to decrease monotonically for many different values of $\alpha_1 - \gamma_6$.

Table 2. $\sqrt{\|x\|^2}$ & Total Online Simulation Time (sec.)

	$\sqrt{\|x\|^2}$	Time
$\text{RH3}_{N_0=4}$	24.96	4.2
$\text{RH3}_{N_0=2}$	24.96	4.0
$\text{RH4}_{N=4}$	24.90	5.6
$\text{RH4}_{N=2}$	24.74	4.3

5. CONCLUSION

In this paper, we propose a generalized stabilizing receding horizon control (RHC) scheme for input/state constrained linear discrete-time systems, which can easily be implemented by the linear matrix inequality (LMI) optimization. We discuss modified schemes of the proposed one, which make the optimization problem more feasible numerically and the computation time smaller than the proposed one for constrained time-varying systems.

The proposed scheme, which includes all of the existing ones in (Kothare *et al.*, 1996; G. De Nicolao and L. Magni and R. Scattolini, 1998; Lee *et al.*, 1998; Park and Kwon, 1999; Kim, n.d.), is more flexible and profitable in terms of feasibility & computation. Modified schemes have some practical benefits in terms of feasibility & computation. Thus, the results in this paper are expected to be very useful for other regulation problems, which consider constrained and/or time-varying systems.

Acknowledgement: This work was supported by the Post-doctoral Fellowship Program of Korea Science & Engineering Foundation (KOSEF).

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