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MARKET STABILITY: BACKWARD BENDING SUPPLY IN A LABORATORY EXPERIMENTAL MARKET

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Abstract

The paper investigates the stability properties of markets with backward bending supply curves. Parameters are chosen so that the two classic models of price dynamics, the Walrasian model and the Marshallian model, give opposite predictions. The results are: (1) market instability can be observed; (2) in the backward bending case stability is captured by the Walrasian model and the Marshallian model of dynamics is rejected. Previous experiments have demonstrated that the Marshallian model works in the forward falling case. Thus, which theory of dynamics is appropriate for a market depends upon the underlying reasons for demand and supply shapes.

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I. Introduction

Two classical concepts of market stability exist in the literature. One is based on analysis introduced by Alfred Marshall, traditionally called Marshallian stability, and the second was introduced by Leon Walras, traditionally called Walrasian stability.[1] The Walrasian notion seems to have survived in a popular sense and is the one typically found in the literature and textbooks.[2] However, in spite of a lack of popularity, it is the Marshallian concept that has received support in experiments. This study continues an investigation of the Marshallian and Walrasian concepts by exploring experimental environments in which income effects play an important role. Previous experiments, in which the Marshallian concept of stability characterized the nature of the market adjustment process, were based on the existence of an externality, much like a "fad" or a "cascade." The question posed is whether or not markets can exhibit instability when income effects are pronounced and, if so, which concept most accurately represents market behavior in such cases.

Interest in stability concepts stems from complicated questions about the nature of market dynamics, disequilibrium market behavior, and the general process of market adjustments. Interest also stems from a natural academic curiosity about the predictive accuracy of classical theoretical ideas. Since the classical models of stability are relatively simple and have not been fully tested,[3] they are natural models to explore in the hope that they will lead to insights about the price discovery process in markets.

The class of economic environments with downward sloping market supply provides an opportunity for an experimental investigation of market stability. That is, when price decreases the quantity supplied increases or equivalently, when price increases the quantity supplied decreases. Within this class of environments, where both the demand curve and the supply curve have negative slope, the Marshallian and Walrasian models give substantially different predictions about the conditions under which market instability will be observed. Thus, appropriately defined experiments can test the two concepts against each other.

Two different types of downward sloping market supply are consistent with the existence of multiple, competitive suppliers. First is the case of the "forward falling" supply which is created by the existence of an externality. In the forward falling case the supply curve decreases because the costs of suppliers decrease as the market volume, which includes the sales of competitors, increase. Marshall called this phenomenon "external economies" and referred to industry economies of scale as opposed to economies of scale at the firm level. Experiments in the forward falling, "externality" environment, have lead to a clear rejection of the Walrasian model and acceptance of the Marshallian model (Plott and George, 1992; Plott and Smith, 1999).[4] The second is the case of the "backward bending" supply which is created by the existence of negative income effects. For example, as income goes up due to wage increases, people might be less inclined to work so the supply of labor goes down. The research reported here is focused on this income effect or a "backward bending" supply curve.

II. Two Classical Models of Market Dynamics and Stability

Both of the classical models assume that all information relevant for market adjustments is captured by aggregates of individual behavior represented by market demand and market supply. These aggregates are of the form:

(1) $D(X_d , P_d) = 0$

(2) $S(X_s, P_s) = 0.$

The two equations capture the relationship between a price variable and a market quantity variable for each of the demand side of the market and the supply side of the market. The variables P_d and P_s are respectively the demand price and the supply price and the X_d and demanded, 18 units, so the price falls. Notice that the movement is toward the equilibrium point A, so the equilibrium is said to be stable according to the Walrasian model.

Three observations are useful. First, the two models give different conditions under which market instability will be observed. In cases like point A, under condition D_1, the predictions of the two models are exactly opposite. If supply is negatively sloped and if demand cuts supply from below, then the equilibria is Walrasian stable and Marshallian unstable. If demand hits supply from above, then the point is Walrasian unstable and Marshallian stable.

Second, the two models give different (essentially opposite) predictions about the direction of movements of prices and quantities. These differences provide a partial basis for the tests that will be reported here.

Third, the two models employ different definitions of equilibrium. When the parameters are sufficiently smooth, the definitions are equivalent. Ambiguities begin to appear when variables can take only discrete values. Since discreteness is a property of almost any experiment, conventions must be developed to deal with the problem. This observation will be discussed in greater detail later.

III. The Research Questions

The logic of the experimental design can be seen in Figure 1 and is closely related to the research questions that are posed. The strategy was for the experiments to begin with one demand condition and then after equilibrium occurred, if it did occur, the demand would be shifted to the other demand condition. If the data had converged to one the equilibrium represented by the points A, B, or C in Figure 1, under the first demand condition, the presumption would be that the equilibrium was stable. After the change in demand the equilibrium the equilibrium would be unstable according to the same theory that predicted it to be stable. Regardless of which theory applied, any equilibrium that

X_s are the demand and supply quantities. The literature has recognized different theories about how the aggregates might be formed from individual decisions, but the analysis here will follow general equilibrium models (Arrow and Hahn, 1971).

Figure 1 contains a graphical representation of the parameters that were implemented in the experiments to be discussed later. The market supply curve is downward sloping. Exactly how this is achieved will be discussed in Section III. Two different market demand situations were studied. These are represented by the curves D_1 and D_2 .

The Marshallian model assumes that $X_s = X_d = X$ and that the direction and speed of price adjustments are of the form:

(1) $dX/dt = G(P_d(X) - P_s(X))$.

The equation reflects the fact that the Marshallian model represents dynamic market behavior as a change in volume that results from a discrepancy between the demand price at the existing volume and the supply price at that volume. Specifically, in the left panel of Figure 1, at a volume of 22 units and under demand condition D_1, the demand price of 152 is less than the supply price of 165. According to the Marshallian model, volume should decrease because the demand price is less than the supply price. Notice that the movement is away from the equilibrium at point A, so the equilibrium is unstable according to the Marshallian model of market dynamics.

The Walrasian model assumes that $P_d = P_s = P$ and that the direction and speed of price adjustments follow a law of the form:

(2) $dP/dt = F(X_d(P) - X_s(P))$.

The implication of the equation is that the Walrasian model represents the dynamics of market movements as resulting from a difference in the quantity demanded at a given price and the quantity supplied at that price. Suppose the price is 156 and that the demand condition is D_1. At that price the quantity supplied, 25 units, is above quantity was stable under the first condition would be unstable under the second (with the exception of the extreme points which might disappear as equilibria after the shift).

Three basic research questions are posed. First, does the ordinary demand and supply model work in the sense that equilibria of that model predict the ultimate "resting place" of the market data? The answer to this first question is not obvious because in the backward bending case the theoretical market supply curve is derived from the hypothesis that individual agent suppliers solve a constrained maximization problem with a constant market price. In the continuous double auction there is no constant market price so the hypothesis of constrained maximization on which demand curves are developed can be questioned. Furthermore, rules of thumb or heuristics, which behaviorists suggest are generally used might lead to decisions in these markets that is much different from the competitive model. The answer to the question is further obscured by the presence of nonconvexities in the experiment, which will be described in the next section. Thus, failure of the equilibria of the demand and supply model to predict the time patterns of the actual market behavior would not be a surprising event; certainly the predictive accuracy of the model cannot be taken for granted.

Second, do the classical concepts of stability apply at all? Do markets reflect instabilities of any sort? Both classical models are insensitive to major features of the environments that might be important from the point of view of models derived from game theory. They depend only upon parameters found in the demand and supply representation of the economic environment, i.e., the shape and position of the aggregate curves, but are silent about variables related to strategic behavior. Consideration of the complexities of human expectations and strategic decisions are absent. Thus models derived from principles of game theory could lead to completely different predictions. Since the classical concepts of demand and supply were not derived from first principles of individual decisions and since the arguments in support of the classical notions of stability are typically only arguments based on analogy to physics, there are ample reasons to doubt the ability of the models to predict what might be observed in markets.

Third, if markets exhibit properties of instability, which of the classical models best captures what is observed? Previous research on the forward falling case yielded positive answers to the first two questions. The answer to the third was that in the forward falling demand case the Marshallian concept fits the dynamics of the process and the Walrasian concept is rejected. Thus, the issue is whether or not the same results hold in the backward bending case.

IV. Parameters: The Economic Environments

The economic environment includes the preferences and endowments of agents. In these experiments agents can be identified as suppliers and demanders. Since the supply is more complicated it will be discussed first. Individual and market demand will be discussed second. In some circumstances price ceilings are imposed. The nature and reasons for them will be reviewed third.

A. Supply

The backward bending supply curve creates a technical problem for experimentation. The nature of the supply requires the use of a two-dimensional nonlinear preference map with negative income effects. The dollar payoff of subjects depended nonlinearly on two variables, francs held at the end of a period, and x units are supplied to the market by the individual. In a sense, the x units supplied were like labor, the supply of which gave negative utility, and the units of francs held at the end of a period are similar to the consumption of "other things." All sales were in terms of francs so the francs held by the subject at the end of a period depended upon the number of units sold and the franc prices. Suppose, for example, an individual sells x units at a price of P francs per unit. Franc income would then be $F = Px$ and the dollar payoff to the subject at the end of the period would be $U(x,F)$ where $U(.,.)$ is the dollar incentive function.

Three different types of seller agents existed in the markets. There were two agents of each type giving a total of six suppliers. The general functional form[5] of an individual seller agent's incentive function, *U (x,F),* is in US cents and has a functional form as follows when

k = the seller type, 0, 1, 2 and

F = total francs held at the end of the period by the individual

x = the number of units supplied by the individual.

(3) $U(x,F) = -150$ if $(x,F) = (0,0)$

$$U(x,F) = \begin{cases} 400 - \Bigg[\dfrac{19{,}600\,x^2}{13} + 1{,}008\,kx - \dfrac{520{,}800}{13}x + \\ \quad + 168\,F + 235\,k^2 + 312{,}325 + \dfrac{6{,}591\,kF}{100\,x} - \\ \quad - \dfrac{5{,}239\,F}{2x} + \dfrac{2{,}197\,F^2}{400\,x^2} - 15{,}813\,k \Bigg]^{1/2} \\ \quad \text{if } F \le -\dfrac{33{,}600\,x^2}{2{,}197} + \dfrac{3{,}100\,x}{13} - 6kx \\ \\ 400 - 14.947\,x \text{ if otherwise} \end{cases}$$

The indifference curves of the incentive function for an individual of type $k = 1$ are shown in Figure 2. Some of the prominent features are in need of explanation. First, the lack of convexity is a significant departure from the assumptions of most models. We were unable to find examples of convex preferences that have properties of linear backward bending supply over the interesting ranges and which approached zero supply at sufficiently high prices. We suspect that nonconvexity is a necessary condition (for the differentiable cases) for these properties. A second feature is that the indifference curves become vertical with sufficiently large quantities. This is only a convenience. Notice that the "otherwise" statement of the utility function is defining vertical indifference curves. The inequality condition involving F describes the arch in the (x,F) plane below which the utility values given by the formula are real and above which utility values do not depend on F, so indifference curves are vertical.[6] The third feature of the function is the fact that $U(0,0)<0$. That is, individuals who do nothing lose money. This feature drives the competitive supply to a positive quantity at a zero price. Individuals would prefer to supply a positive amount at $P = 0$ rather than supply nothing.

Figure 2 is used to explain the theory/technology using one of the actual dollar payoff functions as an example. The level surfaces in the *(x,F)* space are as shown. The dollar figure represents the dollar payoff to the subjects should his/her end of period (franc inventory, units supplied) vector be on the indifference curve. Suppose, for example, the franc price in the market was a constant 54 francs per unit. The budget constraint represented by line B shows franc incomes as a function of x sales. Dollar income is maximized at a sale of x =10 units and franc income 540, which yields a dollar income of about $2.40. So, the number of units supplied by this individual at a P of 54 francs is 10. According to the theory of competitive supply this procedure of maximizing utility/dollars at different franc prices can be used to trace out the individual supply function giving x supplied by the individual as a function of P.

The competitive model, when applied to the incentive functions defined above, yields linear supply functions that are negatively sloped for each individual. In particular, the supply functions of individuals, where P is the market price and k is the individual type, are given by

(4) $x_k(P) = (3/700)(3100-13P - 78k)$

When there are two agents of each type and $k \in 0,1,2$ the market supply, X_s, given by an application of the competitive model is

(5) $X_s = \Sigma_{k=0}\, 2\, x_k = (9/350)(3022 - 13P) = 77.1 - .3343P.$

However, because only integral amounts can be supplied, the supply function becomes vertical at one unit. Beyond a certain level of francs, the individual supplier is indifferent about the additional amount of francs that the one unit sells for as was discussed above.

B. Demand

Each market had six demanders, two of each of three different types, {1,2,3}. Two different demand conditions existed across the markets which are indicated as conditions D_1 and D_2. The redemption values are given in Table I. Again, the exception is experiment 030589.

By application of competitive theory to these individual incentive functions, market demand curves can be derived. The aggregates of individual demands are shown as D_1 and D_2 in the two panels of Figure 1.

C. Price Ceilings

A feature of the unstable equilibria of mathematical/theoretical models is that small perturbations away from an unstable equilibrium will precipitate a series of movements away from that equilibrium and toward the nearest stable equilibrium. The natural question to pose is whether or not we can observe the market phenomena that are captured by this property of the model.

Price ceilings were used to force economic activity into regions near but not at the unstable equilibria of the models. Such ceilings were in the form of a restriction that neither bids or asks could be above some fixed, ceiling amount. As will be discussed later in this paper, prices in the experimental markets seemed capable of "sticking" at both stable and unstable equilibria. When this happened a ceiling was imposed below the equilibria to see if the market activity stayed at the ceiling or moved away from the ceiling in the direction of the model's nearest stable equilibria.

The price ceilings were at either 120 or 60 depending upon the demand conditions. As will be discussed later, these ceilings were always just below a Walrasian unstable (Marshallian stable) equilibrium.

D. Information

All information about costs and redemption values was private. The cost schedules of suppliers were constant. However, each period a new indifference map was supplied so the supply parameters could have changed for some without other agents knowing that something happened.

Similarly, demanders were given fresh redemption value sheets each period. When demand changes occurred each individual demander knew his/her own redemption changed so he/she could correctly guess that the market demand had changed. However, suppliers had no such clues. If the information controls worked, demand changes occurred without suppliers knowing that a change of parameters had occurred. Their only source of information would have been the market activity.

V. Model Structure: Equilibria, Dynamics, and Econometric Specification

Under conditions D_1, there are four interesting equilibria, points A,B,C and F. Under condition D_2 there are four interesting equilibria, A,B,C and E. Some controversy might exist about the nature of the boundary equilibria C, H and E, but for completeness these will be added to the set of possibilities without such detailed comment.[7] Table II is a summary of the points that are Walrasian stable (Marshallian stable) under conditions D_1. When the demand is shifted the Walrasian (Marshallian) stable points under D_1 become Marshallian (Walrasian) under D_2. In some cases the shift in demand causes points to disappear as equilibria.

A comment should be added about the definition of equilibria. The Marshallian definition is not the same as the Walrasian. The natural definition under the Walrasian structure is a price such that quantity demanded equals quantity supplied. Under Marshall it is a quantity at which demand price equals supply price. Take for example a quantity of 54 under condition D_1. The demand price for the 54th unit is 85 but the demand price for the 55th unit is 45. The supply price for the 54th unit is 69 but the supply price of the 55th unit is 66. Under such conditions, can one say that demand price equals supply price at the 54^{th} unit as required by the Marshallian definition of equilibrium? The convention adopted in this paper is to define such points as equilibria because no gains from trade are possible at the 55^{th} unit.

In modern terminology the Walrasian concept is similar to the intersection of demand and supply correspondences. The Marshallian concept is similar to a quantity that "supports" a competitive price. The concepts of equilibria can also be defined as limits of appropriate dynamic processes.

The existence of discontinuities creates some ambiguities and since discontinuities make a natural appearance in any experimental environment, judgments must be made about the best way to extend the model. In this paper, the dynamic models will be developed in the context of continuous functions. Let the demand equation be and the supply equation be piecewise linear and in the linear portions it would be

(6) $X_d - a_d - b_d P_d = 0$

and the supply equation be piecewise linear and in the linear portions

(7) $X_s - a_s - b_s P_s = 0$

where X_d and X_s are the demand and supply quantities and P_d and P_s represent demand and supply prices. The Walrasian model of dynamics is

(8) $P = P_s = P_d$

$dP/dt = F(X_d - X_s) = F(a_d - a_s + (b_d - b_s) P)$

$F'(\bullet) > 0.$

That is, if quantity demand at a price is greater than quantity supplied then prices will increase.

The Marshallian model of dynamics is

(9) $X = X_d = X_s$

$dP/dt = G(P_d - P_s) = G\,(a_s/b_s - a_d/b_d + [1/b_d - 1/b_s]\,X)$

$G'(.)>0.$

That is, if demand price at a quantity is greater than supply price then quantity increases. It is important to notice that a_d, a_s , b_d, and b_s are all known to the experimenter since they are parameters chosen as part of the experimental design. Consequently, P_d, P_d X_d and X_s are computable under appropriate assumptions.

The following assumptions will be the bases for the econometric measurements.

Assumption 1. The time t refers to an experimental period.

Assumption 2. X_t = the observed number of transactions in period t.

Assumption 3. P_t, = the observed average price in period t.

Assumption 4. The speeds of adjustment are linear, i.e., $F\,(X_z - X_s) = \beta_M(P_d - P_s)$

Under assumptions 1 through 4 the following models can be estimated. For the Walrasian dynamics the model is

(10) $P_{t+1} - P_t = \alpha_w + \beta_w(X_d - X_s) + \varepsilon_w$

$= \alpha_w + \beta_w[d_d - a_s + (b_d - b_s)P_t] + \varepsilon_w$

For the Marshallian dynamics the econometric model is

(11) $X_{t+1} - X_t = \alpha_M + \beta_M(P_d - P_s) + \varepsilon_m$

$= \alpha_M + \beta_M\,(a_s/b_s - a_d/b_d + [1/b_d - 1/b_s]\,X_t\,) + \varepsilon_M$

under both views of dynamics the estimated parameters should be $\alpha_M = \alpha_W = 0$, β_M , $\beta_W >$ 0. This follows from the assumption that $F'(\bullet) > 0$ and $G'(\bullet) > 0$ and that the demand and supply equilibria be the limit of the dynamic process. Of course, given the parameters of these environments, the downward sloping supply, one would expect $\beta_W \beta_M < 0$. That is, if the dynamics are accurately predicted by one of the models (i.e., $\beta > 0$) it is natural to anticipate the dynamics predicted by the other model to be exactly wrong.

In the analysis below the two models (10) and (11) will be estimated separately.

VI. Research Strategy and Experimental Designs

A total of six experimental markets were conducted. All subjects were students at the California Institute of Technology. All markets operated within the framework of an electronic Multiple Unit Double Auction (Plott and Gray, 1990). A control pilot experiment was conducted utilizing the multiple oral double auction. The data are not reported here.[8]

The relationships among experiments, periods, and environments are in Table III. The research design reflects the fact that subject time and research funding are scarce resources. Thus, in some cases parameters were changed before full equilibration was established in any statistical sense, such as low price variance. Instead, sequences of treatments were employed, which produced data on the dynamics and movement of the system in response to the underlying circumstances. Thus the experimental design reflected a process of sequential decisions in which the choice of treatment variables in any one experiment reflected the accumulation of evidence from the experiments conducted previously and judgements about when "sufficient" data had been collected.

The first two experiments began with the D_2 parameters. This reflected an expectation based on Plott and George (1992) that the system would be near the Marshallian stable equilibrium in the middle of the parameter space. The first two experiments destroyed this presumption and also created three additional presumptions, which influenced the overall research design. First, the markets will equilibrate. If the markets are maintained under stationary conditions, the data will "tighten" around the equilibrium once they are "near." Secondly, the data can "sit" near prices that are unstable according to the Walrasian model.

The fact that markets might be able to "sit" at an unstable equilibrium motivated the invention of some sort of vehicle that would "perturb" the market away from the equilibrium. The tool employed for this purpose was price ceilings. The dynamic models suggest that such perturbations will cause market movement to the nearest stable equilibria.

Price ceilings were used to explore two special phenomena. The first was convergence to a price of zero. Such a phenomenon would seem to be unusual and unexpected by the scientific community. To the extent that differential weight is placed on a model's ability to predict the existence of phenomena outside the ordinary realm of experiences, such treatments are important. The second phenomenon is the response of markets to the removal of nonbinding price ceilings. Theory suggests that removing nonbinding ceilings should have no effect but experiments have demonstrated that they do. Isaac and Plott (1981) discovered the existence of such efforts. The possible influences of nonbinding price controls are explored more here.

The sequential decision nature of the design resulted in the following rules governing experiments later in the series. Begin the parameters with a stable Walrasian equilibrium near the center of the parameter space and let it remain unchanged until price variance was "small." Change the parameters to switch the stability properties of the equilibria. If the prices move up let them continue until the upward movement is well established and then impose a price ceiling below the (now) unstable Walrasian equilibrium from which the prices moved. If the prices remain the same do nothing for several periods to determine if they will move themselves. If no movement occurs impose the price ceiling below the unstable Walrasian equilibrium (to see if prices move further down to the stable equilibrium below). Remove the ceiling. After a few periods to determine the direction of price movements and observe if a new equilibrium emerged, change demand parameters to the original set to switch stability conditions. If the removal of the ceiling caused prices to jump to the highest equilibrium, impose a price ceiling below the lowest unstable Walrasian equilibrium to see if the prices would converge to zero. If time allowed remove the nonbinding ceiling.

VII. Results

The results all suggest one overriding conclusion. The Walrasian model works. Even unexpected phenomena closely correspond to reasonable interpretations of the Walrasian model. The results are listed below in a way that makes that correspondence clear. In addition, the results record the existence of phenomena that the model does not address but are nevertheless of interest for an understanding of the market price discovery process.

Figures 3 through 7 contain the time series of prices. Each figure contains the stable Walrasian equilibria represented by solid lines and the stable Marshallian equilibria represented by dotted lines. Of course, each of the equilibria has the property that when it is stable according to one model, it is unstable by the other (if it exists as an equilibrium in the other). Vertical lines represent periods and the double vertical lines represent periods in which some parameter change was implemented in the experiment. Price ceilings were imposed in some periods and these are represented by the heavy dashed lines.

The time series of volumes are in Table IV. The volumes are shown for each period of each experiment. In addition, the table contains the distances from stable Walrasian and stable Marshallian equilibria, which are closest to the final values of the period before a condition change. The first three results address the central questions posed by the research. The competitive model captures the equilibration properties of the backward bending supply case and it is the Walrasian model that does the job. The Marshallian model is rejected in favor of the Walrasian model.

The first parts of experiments 030589, 030889 and 040989 in Figures 3, 4 and 5 provide good illustrations. In Figure 3 prices in the first two periods were near the unstable Walrasian (stable Marshallian) equilibrium point "A" from Figure 1. However over the eleven periods of stationary conditions prices can be seen moving away from point "A" to point "B", which is stable Walrasian (unstable Marshallian). In Figure 4 prices again begin near point "A" but this time converge upward toward point "D", which is a stable Walrasian equilibrium, and then the parameters are changed in the 7th period so point "A" is Walrasian stable and the market quickly converges toward it. In Figure 5 the experiments begin with point "A" implemented as a stable Walrasian equilibrium (unstable Marshallian) and the market converges to it during the five periods of stable parameters. In this experiment point "A" becomes unstable Walrasian after a parameter change beginning with the sixth period and as can be seen that during periods six through nine there is no movement away from the point even though it is now Walrasian unstable. At the beginning of the tenth period a price ceiling was placed on the market just below the equilibrium price "A" and as can be seen the prices immediately began to move below the ceiling falling away to the nearest stable Walrasian equilibrium point "B".

The first result addresses the first question that was posed about the applicability of the competitive model in such a complex experimental environment. Recall that for several reasons discussed in Section II one might doubt the ability of any form of competitive equilibria model to capture behavior in the backward bending case. The result states that the equilibrium notions of the competitive model survive to the experimental test.

Result 1

In all cases the data move to prices near the equilibria of the model.

Support. The statement does not distinguish between stable and unstable equilibria. For purposes of analysis the data are taken to be the last period of a treatment. The difference between the average price (total volume) in the period and the nearest equilibrium of the model (stable or unstable) is computed. If the high price periods are omitted, because the approach is always from below and the number of periods is always small, the average deviation from the model is less than two francs for the price and less than one and one unit on the quantity. The largest error is 17 francs (with a volume error of 6) and fifteen units on the volume (with a price error of 11 francs) The hypothesis that the data are near a competitive equilibrium cannot be rejected. □

The next two results are statements that say the Walrasian model and not the Marshallian model captures the nature of the equilibration process. It is important to note that these result address market movement and not market "resting" since, as was illustrated in Figure 5 and discussed above, the markets can remain at an unstable equilibrium. That is the essence of Result 2.

Result 2

Environmental perturbations that cause movement in market prices result in price patterns near a stable Walrasian equilibrium.

Support. A total of twenty treatments were implemented during the five experimental sessions. Each of these conditions existed one or more periods. Examining only the final periods under a treatment condition, the data were closer to a stable Walrasian equilibrium than to a stable Marshallian equilibrium in seventeen of the twenty periods. Each of the three cases in which the final period prices were closer to the stable Marshallian equilibrium than the stable Walrasian equilibrium, were special in the sense that prices started at the stable Marshallian equilibrium and moved very little during the periods. □

The next result is based on the econometric estimates of equations (10) and (11). These are reported in Table V and are summarized by Result 4. The result simply says that through the eyes of those models the movements are in the direction predicted by Walrasian dynamics.

Result 3

Estimates of the dynamic models can only be interpreted as resulting from Walrasian (as opposed to Marshallian) dynamics.

Support. Both models as represented in equations (10) and (11) were estimated. The estimates are reported in Table V. Recall that the slope coefficient should be positive and the intercept term should be zero. For the Walrasian model the estimated slope coefficients in every experiment are positive and significant. The intercept terms in the estimates for the Walrasian model are not significantly different from zero in two cases and in the three other cases the intercept term is significantly negative. By contrast, the slope coefficient for the Marshallian model is negative in all cases and it is significantly negative in four of the five experiments. The intercept term of the Marshallian model is significantly different from zero in four of the five experiments. Thus, the data move in the direction predicted by the Walrasian model and move in a direction opposite of that predicted by the Marshallian model. Because the models give opposite predictions one can interpret both sets of estimates as supporting the presumption that the Walrasian model and not the Marshallian model describes market movement. □

The next result can be interpreted as an extreme test for the Walrasian dynamics model. As was discussed above, under conditions D_1 a stable Walrasian equilibrium exists at a price of 0. Since one does not ordinarily see positive supplies at zero prices, the model predicts the existence of unusual phenomena that is worthy of exploration. The result is captured most dramatically in Figures 6 and 7. In Figure 6 in period 18 a price ceiling was imposed just below point "B" which is an unstable Walrasian equilibrium. Notice that all previous prices had been at the highest stable Walrasian equilibrium and in period 18 both a parameter change and a price ceiling were implemented. Prices begin near the ceiling and the slowly decay to near zero where the nearest stable Walrasian equilibrium exists. The sequence of conditions is replicated in Figure 7, period 18 and the prices decay over the four periods to the stable Walrasian equilibrium at a price of zero until the ceiling is removed in the final period.

The next result simply records the fact that the phenomenon of convergence to a zero price was observed even though other equilibria exist. For example, according to the Marshallian model prices at the ceiling constitute an equilibrium.

Result 4

A price of zero and positive volume, when it exists as a stable Walrasian equilibrium, can be observed occurring in a market even though three nonzero-price equilibria exist.

Support. The appropriate data are from experiment 041089, period, 22 in which the price is 5 and volume is 73. This compares favorably to the nearest stable Walrasian equilibria which has a price of zero and quantity of 78. Another example is experiment 041389, period 21, in which the price is zero and the quantity is 76. Again, this compares favorably to (0, 78). In both cases a convergence toward the equilibria is evident. □

The final two results record the existence of phenomena that were unanticipated. The Result 5 demonstrates that the natural price variability around an unstable equilibrium may be insufficient to cause dynamic movement away from that equilibrium. A reasonable conjecture is that there exists natural neighborhoods of stability around an otherwise unstable equilibrium. This phenomena can be seen in Figure 3 (periods 12 and 13), Figure 5 (Periods 6 -9, and Periods 17-21). Result 6 demonstrates that beliefs and expectations can override all of the "local" dynamics of the model and push prices exactly opposite to the direction anticipated by the model. It is interesting precisely because it demonstrates the overriding power of expectations and beliefs.

Result 5

Market activity can be maintained at an unstable equilibrium.

Support. The previous results establish the proposition that market activity when perturbed by price controls or by parameter changes will converge to the nearest stable Walrasian equilibria. That is, Walrasian stability in a model predicts the dynamics of price movement. Three parameter changes (experiment 030589, period 12; experiment

040989, periods 6 and 17) caused very little movement in market activity. The periods following those perturbations remained at the unstable Walrasian equilibria for several periods until a new set of parameters caused movements. □

Result 6

Removal of nonbinding controls can cause market activity to jump from one stable equilibrium to another one (the jump was over an unstable equilibrium).

Support. The relevant periods are experiment 041089 (Figure 6, period 15), and experiment 041389, (Figure 7, periods 16 and 22). In all three cases removing a nonbinding ceiling caused the market to jump away from the stable equilibrium to which it had converged to near a previously experienced, more efficient, stable equilibrium. The previously experienced nature of the equilibria is probably important since the removed ceiling in an experiment in which the system had not previously attained the more efficient stable equilibrium, experiment 040989, period 15, caused no such large jump. □

VIII. Summary of Conclusions

One result of this study (Result 1) is to buttress the results of previous experiments that demonstrate that markets have dynamic properties that can produce both stable and unstable equilibria. Such results demonstrate that markets in disequilibrium are governed by a set of laws and these laws are captured, in part, by classical notions. The results also underscore the need for a more intense research focus on concepts of disequilibrium.

However, the central results of the study reflect a discovery about the nature of stability and the equilibration process in relation to classical models. The nature of market stability is not captured by market level aggregates such as market demand and market supply. The results reported here, when joined with the results of other experiments [Plott and George; Plott and Smith] demonstrate that in order to detect whether or not a market is stable, it is necessary to inquire about the reasons why the aggregates have particular shapes. Simply measuring the slopes of aggregate demands and supplies is not sufficient. It is necessary to know why they are sloped the way they are. If the perverse slopes are due to an externality then one set of conditions (Marshallian) applies and if the perverse slopes are due to income effects the opposite (Walrasian) analysis is needed (Results 2, 3 and 4).

The study also reports discoveries that suggest that a successful theory cannot be based on the local properties of the aggregates alone. Expectations play role in stability and expectations may have global properties as opposed to local properties. This phenomenon was dramatically demonstrated when the removal of a nonbinding price ceiling caused the markets to "jump over" an unstable equilibrium (Result 5). That is, markets that are equilibrated at a stable equilibrium have the capacity to leave a stable equilibrium, jumping over an unstable equilibrium and then equilibrate at a different stable equilibrium. This phenomenon was observed on three occasions with the removal of price controls, which had been used to force the system away from a more efficient stable equilibrium. On the one occasion in which a nonbinding price ceiling was removed from a market that had not experienced the more efficient, stable equilibrium, the system responded slightly to the removal of the control but then remained at the stable equilibrium.

The study provided an opportunity to observe a market resting at an unstable equilibrium. Superficially, the patterns of activity do not seem to be different from systems that are resting at a stable equilibrium. Thus, no obvious signal existed to hint that the market could suddenly move for no obvious reason. Perhaps further research will produce such indicators.

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[1]A third of stability, related to the concept of "cobweb" cycles was introduced in the more modern literature. The robustness of the cobweb model to different patterns of individual behavior can be questioned on the basis of theory (Carlson, 1968; Auster, 1970) and in experimental tests of the model it has been rejected (Carlson, 1967; Johnson and Plott, 1989).

[2]Henderson and Quandt (1980), contains a discussion of the two concepts. The currently standard development follows the theory as found in Arrow and Hahn (1971) or Frank Hahn (1987).

[3]The first attempt to test classical adjustment models can be found in Smith (1965). His data analysis lead to a rejection of the Walrasian adjustment processes, which in his setting was functionally equivalent to the Marshallian concept. Both predict a constant rate of change to the unique competitive equilibrium that exists in Smith's parameters. However, subsequent examinations of Smith's data using better econometric techniques, reversed Smith's conclusions and found support for the Walrasian model of adjustment in Smith's data (Nelson, 1980).

[4]While the Marshallian model of externalities can apply to fads, different models have also been developed in the literature (Karni and Levin, 1994).

[5]An exception is experiment 030589 in which the parameters were different from all others. In this experiment the demand and supply were tangent at point A so the equilibrium was Walrasian stable at prices above A and Walrasian unstable at prices below A.

[6]A slight discontinuity of the function occurs here but it causes no problems in the analysis that follows.

[7]Typically these are justified as equilibria by appeal to the related theory of dynamics. Take point H for example. It is not an equilibrium in the sense that demand price equals supply price. Yet, since supply prices is greater that demand price at a quantity of zero the market pressures, according to the Marshallian model, is for quantity to fall. Since it is on the boundary it can move no further and by convention is an equilibrium.

[8]Nothing was observed that motivated a more substantial exploration of the oral procedures.

Table I
Redemption Values for Demanders

	Condition D_1			Condition D_2		
Units Type (number)	a (2)	b (2)	c (2)	a (2)	b (2)	c (2)
1	164	164	160	410	430	390
2	160	164	156	350	330	370
3	152	160	156	270	290	310
4	152	156	152	230	210	250
5	149	149	149	170	190	130
6	149	145	141	110	76	90
7	145	145	141	76	76	72
8	136	141	136	72	68	72
9	125	105	136	68	64	68
10	25	45	85	64	64	64
11	0	0	5	60	56	60
12				60	56	52
13				56	52	52
14				44	48	48
15				44	44	48

Table II
Equilibria Locations and Stability Properties
(Prices are in francs)

	$D_1D_1D_1$		$D_2D_2D_2$	
Point	Coordinates	Stability Properties	Coordinates	Stability Properties
	(P, Q)		(P, Q)	
A	(149,28)	Walrasian Stable Marshallian Unstable	(145-151,28)	Walrasian Unstable Marshallian Stable
B	(61-67, 56)	Walrasian Unstable Marshallian Stable	(64,56)	Walrasian Stable Marshallian Unstable
C	(0, 61)	Walrasian Stable Marshallian Unstable	(0,61)	No Walrasian Equilibrium No Marshallian Equilibrium
D	(370-390, 6)	No Walrasian Equilibrium No Marshallian Equilibrium	(370-390,6)	Walrasian Stable Marshallian Unstable
E*	(0, 90)	No Walrasian Equilibrium No Marshallian Equilibrium	(0,90)	Walrasian Unstable Marshallian Stable
H*	(164,0)	No Walrasian Equilibrium Marshallian Stable	(164,0)	No Walrasian Equilibrium No Marshallian Equilibrium

*Other interpretations exist for these boundary cases.

Table III
Parameters by Experiment by Period

Experiment	$D_1D_1D_1$		$D_2D_2D_2$	
	Periods	Ceiling	Periods	Ceiling
030589			1-11	no
	12-15	no		
040889			1-6	no
	7-12	no		
	13-21 different demand parameters	no		
040989	1-5	no		
			6-9	no
			10-14	yes
			15-16	no
	17-21	no		
041089	1-4	no		
			5-9	no
			10-14	yes
			15-16	no
	17-22	yes		
041389	1-8	no		
			9-12	no
			13-15	yes
			16-17	no
	18-21	yes		
	22	no		

Table IV
Volume in All Period of All Experiments and Distance from Equilibria

Experiment	030589			040889			040989			041089			041389		
Period	Volume	WE	ME	Volume	WE	ME	Volume	WE	ME	Volume	WE	ME	Volume	WE	ME
1	26	56	28	21	6	28	40	28	56	29	28	56	53	28	56
2	30	56	28	8	6	28	32	28	56	32	28	56	45	28	56
3	29	56	28	7	6	28	33	28	56	29	28	56	48	28	56
4	30	56	28	6	6	28	29	28	56	31	28	56	45	28	56
5	31	56	28	7	6	28	27	28	56	16	6	28	44	28	56
6	35	56	28	7	6	28	29	56	28	8	6	28	40	28	56
7	38	56	28	16	28	56	28	56	28	7	6	28	37	28	56
8	47	56	28	20	28	56	28	56	28	8	6	28	33	28	56
9	57	56	78	24	28	56	28	56	28	9	6	28	17	6	28
10	61	56	78	27	28	56	38	56	28*	37	56	28*	8	6	28
11	62	56	78	31	28	56	38	56	28*	41	56	28*	6	6	28
12	60	62	56	29	28	56	49	56	28*	43	56	28*	6	6	28
13	58	62	56	20			53	56	28*	46	56	28*	35	56	28*
14	57	62	56	21			56	56	28*	51	56	28*	50	56	28*
15	62	62	56	23			51	56	28	10	6	28	55	56	28*
16				24			53	56	28	6	6	28	14	6	28
17				25			50	28	56	6	6	28	6	6	28
18				25			53	28	56	60	62	56*	64	62	56*
19				27			53	28	56	62	62	56*	76	62	56*
20				29			55	28	56	63	62	56*	78	62	56*
21				31			55	28	56	70	62	56*	76	62	56*
22										73	62	56*		28	56

*The price associated with this volume is above the price ceiling.

Table V
Coefficient Estimates (t- statistics)
Walrasian: $P_{t+1} - P_t = \alpha_W + \beta_W [D(P_t) - S(P_t)] + \varepsilon_t$
Marshallian: $X_{t+1} - X_t = \alpha_M + \beta_M [P_D(X_t) - P_S(X_t)] + \varepsilon_t$

	Experiments				
	030589	040889	040989	041089	041389
N	13	10	18	17	18
Walrasian:					
$\hat{\alpha}_W$	-6.392 (-2.94)	10.975 (1.08)	-3.807 (-1.73)	2.991 (0.86)	-32.555 (-1.49)
$\hat{\beta}_W$	0.352 (1.45)	2.587 (2.43)	1.043 (3.16)	0.925 (3.34)	4.068 (2.50)
R^2	0.161	0.424	0.416	0.427	0.325
DW	0.560	1.870	1.569	2.322	2.598
Marshallian:					
$\hat{\alpha}_M$	2.889 (1.95)	2.592 (1.39)	1.439 (1.45)	2.335 (3.05)	10.893 (2.22)
$\hat{\beta}_M$	-0.002 (-0.03)	-0.034 (-2.12)	-0.128 (-2.47)	-0.027 (-3.47)	-0.217 (-3.26)
R^2	.00001	0.360	0.303	0.446	0.450
DW	1.037	1.158	1.369	2.275	2.207

Figure 1
Demand and Supply Parameters

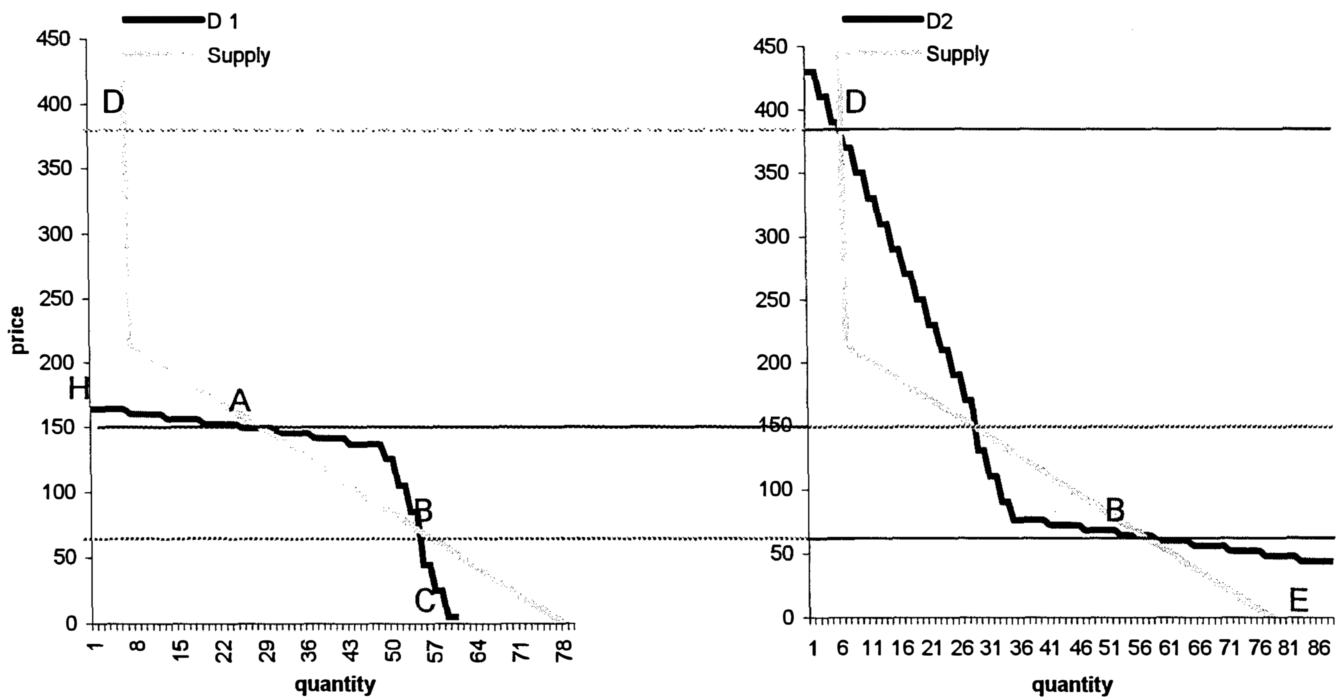

Figure 2
Indifference Curves for Seller: Level Surfaces of Dollars as a Function of Francs and Units
Agent k = 1

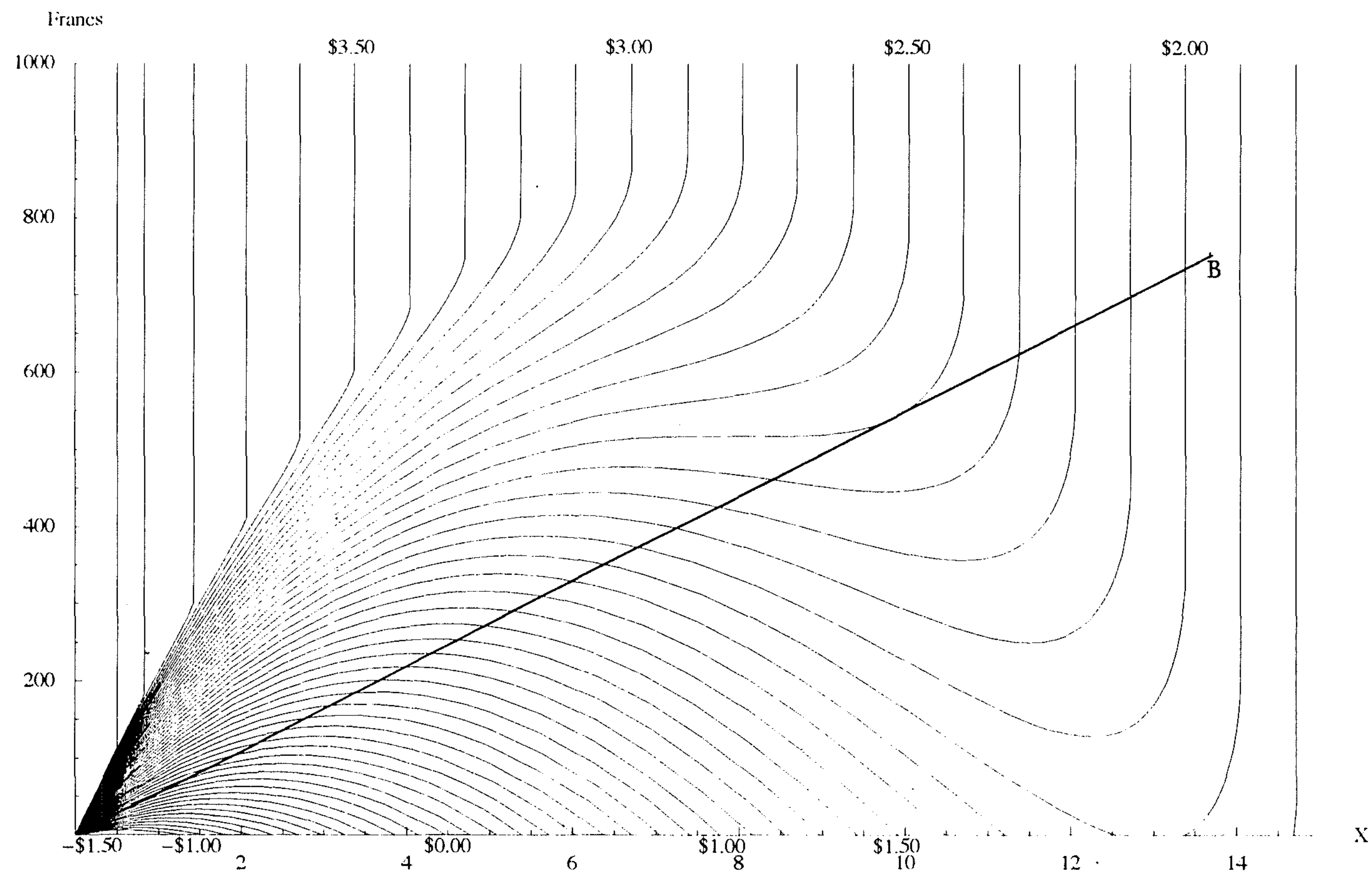

Figure 3
Time Series of Trades and Competitive Equilibrium Prices: Experiment 030589

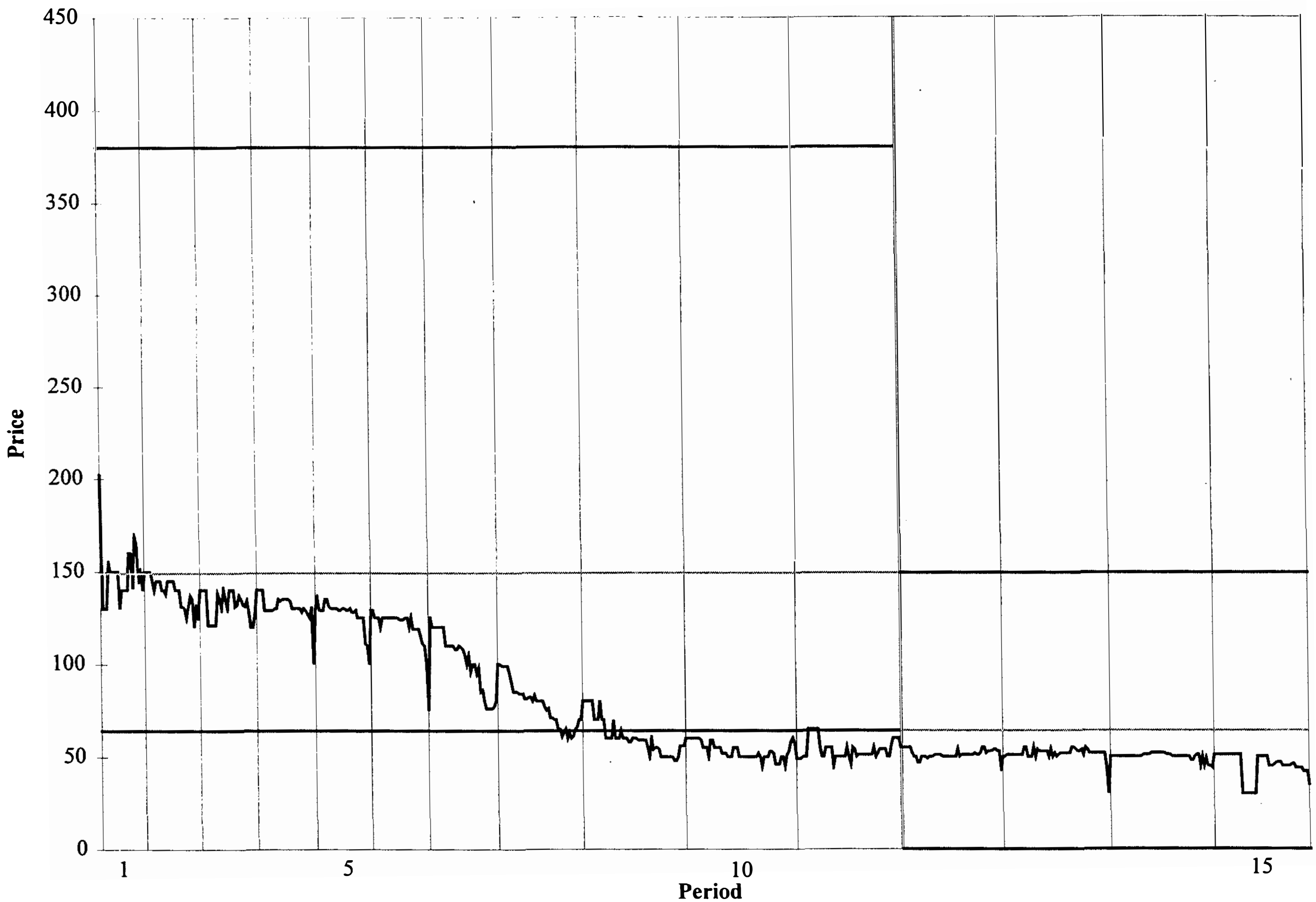

Figure 4
Time Series of Trades and Competitive Equilibrium Prices: Experiment 040889

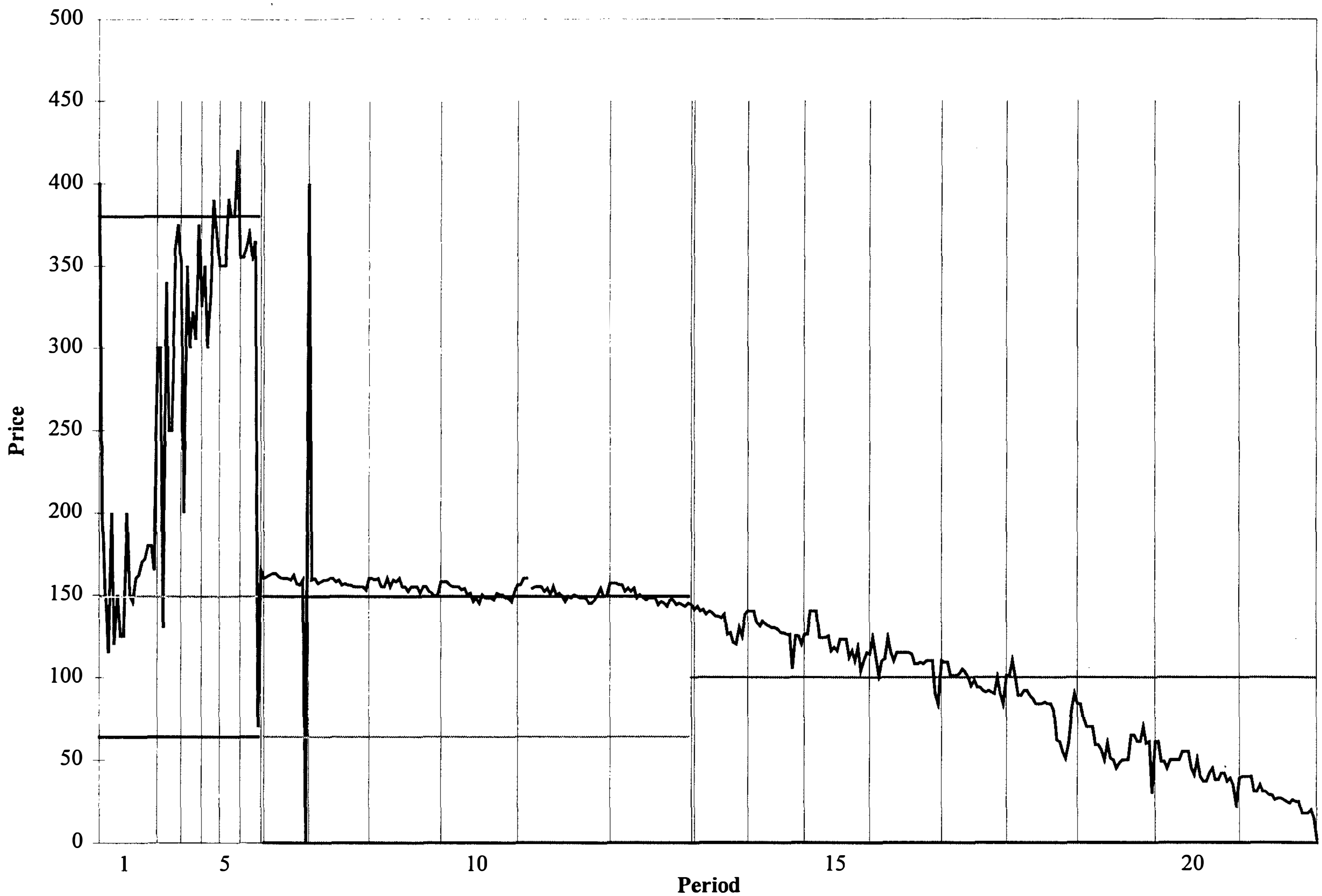

Figure 5

Time Series of Trades and Competitive Equilibrium Prices: Experiment 040989

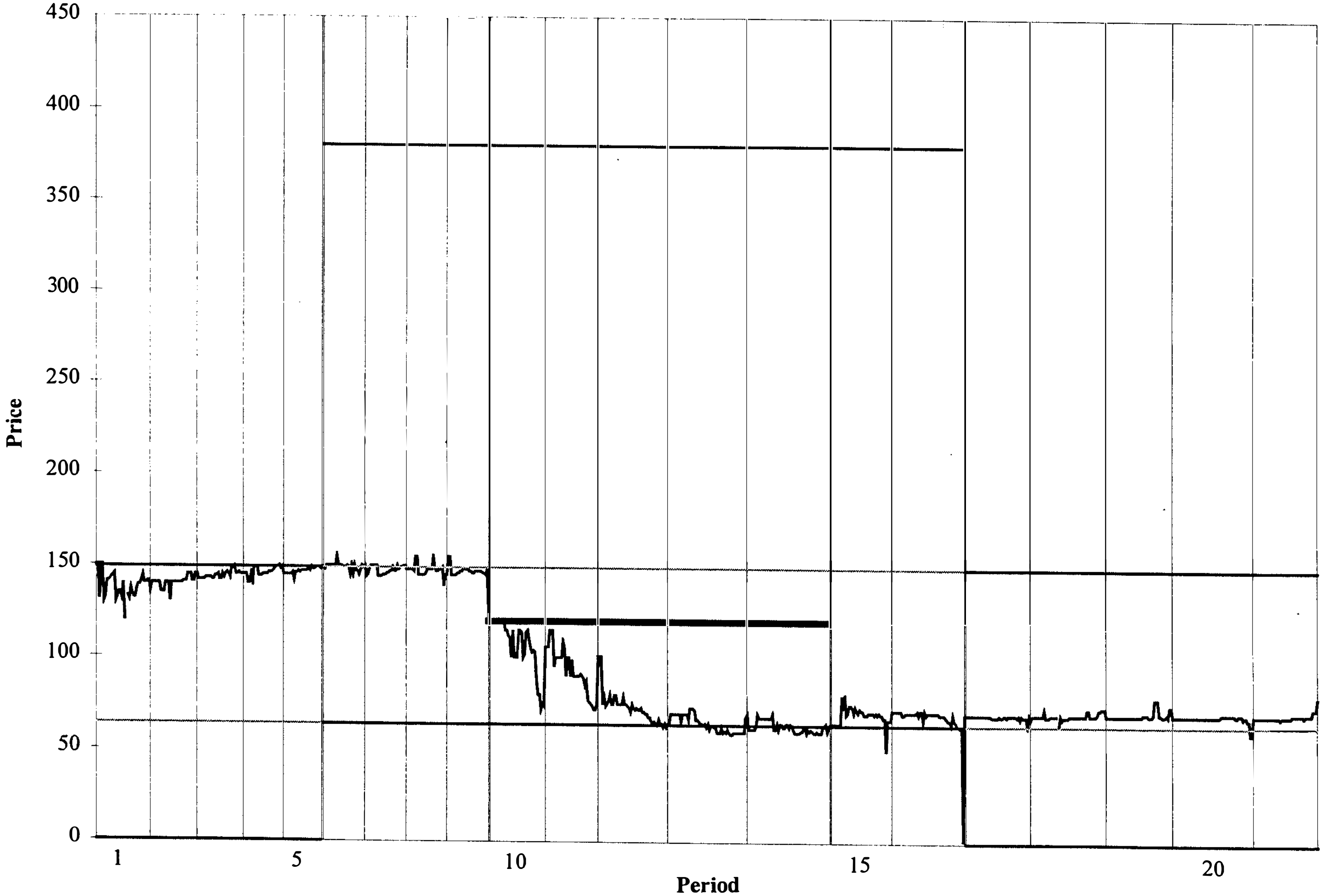

Figure 6
Time Series of Trades and Competitive Equilibrium Prices: Experiment 041089

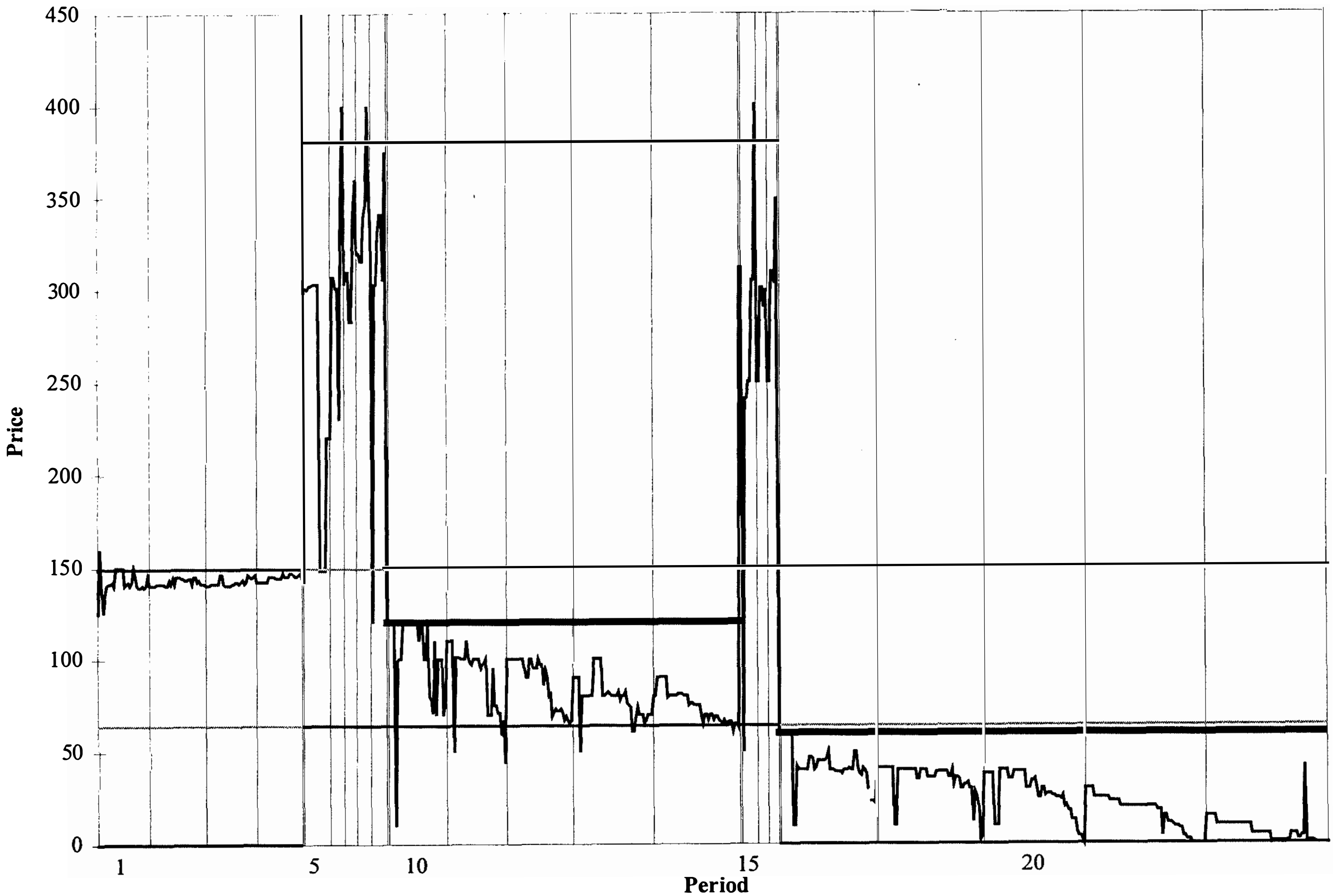

Figure 7
Time Series of Trades and Competitive Equilibrium Prices: Experiment 041389