References

1. S. Yamamoto and T. Makimoto, J. Appl. Phys. **45,** 882 (1974).
2. G. Hepner and B. Désormière, Rev. Tech. Thomson-CSF **6,** 1115 (1974).
3. G. Hepner, B. Désormière, and J. P. Castéra, Appl. Opt. **14,** 1479 (1975).
4. P. Kaminow, W. L. Mammel, and H. P. Weber, Appl. Opt. **13,** 396 (1974).
5. A. Reisinger, Appl. Opt. **12,** 1015 (1973).

Electromagnetic analysis of a laser resonator filled with a flowing medium

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Rapidly growing demand of high-power laser applications has stimulated extensive investigation of flowing molecular gas lasers. In this frame, mode control of resonators filled with a flowing medium has attracted a great deal of numerical modeling and analysis.[1,2] These approaches neglect the effect of motion on the field distribution. Motion only enters the problem by modifying the form of the rate equations ruling the device kinetics. Although one can expect the quantitative changes introduced by the motion on the mode patterns to be negligible on a practical ground, the discussion of an exact solution, besides being interesting *per se,* can provide a direct evidence for the above-mentioned approximation.

To work out an exactly solvable case, we assume a nondispersive medium moving uniformly and steadily along a z direction perpendicular to the optical axis of a resonator made of two or more highly reflecting mirrors, in a stable or unstable configuration. In addition, we assume that the medium fills the resonator completely while maintaining a uniform velocity profile, in spite of possible departures from a cylindrical geometry. We have to find solutions of Maxwell's equations, supplemented by the appropriate constitutive relations, satisfying metallic boundary conditions.

Nonstatic fields, in the absence of charges and currents, are described by two Maxwell equations:

$$\nabla \times \mathbf{H}(\mathbf{r},t) = [\partial/(\partial t)]\, \mathbf{D}(\mathbf{r},t), \tag{1}$$

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -[\partial/(\partial t)]\, \mathbf{B}(\mathbf{r},t), \tag{2}$$

together with the transverse boundary condition:

$$\mathbf{n}(\mathbf{r}) \times \mathbf{E}(\mathbf{r},t) = 0, \tag{3}$$

$\mathbf{n}(\mathbf{r})$ being a unit vector normal to the mirror surfaces and the Minkowski constitutive relations for an uniformly moving dielectric[3]

$$\mathbf{D} = \varepsilon\, \mathsf{A}\cdot\mathbf{E} + \mathbf{b} \times \mathbf{H}, \tag{4}$$

$$\mathbf{B} = \mu\, \mathsf{A}\cdot\mathbf{H} - \mathbf{b} \times \mathbf{E}. \tag{5}$$

Here A is the dyadic

$$\mathsf{A} = (\mathsf{I} - \mathsf{V}) + \mathsf{V}, \tag{6}$$

where I is the unit dyadic, $\mathsf{V} = (\mathbf{v}/v)(\mathbf{v}/v)$, with $\mathbf{v}$ indicating the medium velocity,

$$\alpha = (1-\beta^2)/(1-n^2\beta^2),\ (\beta = v/c), \tag{7}$$

and

$$\mathbf{b} = b\mathbf{v}/v = (\beta/c)\,\frac{n^2-1}{1-n^2\beta^2}\,\frac{\mathbf{v}}{v}. \tag{8}$$

If we now transform the space coordinates and the field vectors according to the relations

$$\mathbf{r} = \mathsf{A}^{1/2}\cdot\mathbf{r}', \tag{9}$$

$$\mathbf{E}(\mathbf{r},t) = T\mathsf{A}^{-1/2}\cdot\mathbf{E}'(\mathbf{r}',t), \tag{10}$$

$$\mathbf{H}(\mathbf{r},t) = T\mathsf{A}^{-1/2}\cdot\mathbf{H}'(\mathbf{r}',t), \tag{11}$$

where T is the linear operator $\exp(bz'\,\partial/\partial t)$, one easily obtains[4]

$$\nabla' \times \mathbf{H}'(\mathbf{r}',t) = [\partial/(\partial t)]\, \mathbf{D}'(\mathbf{r}',t), \tag{12}$$

$$\nabla' \times \mathbf{E}'(\mathbf{r}',t) = -[\partial/(\partial t)]\, \mathbf{B}'(\mathbf{r}',t), \tag{13}$$

with

$$\mathbf{D}' = \varepsilon\alpha\mathbf{E}', \tag{14}$$

$$\mathbf{B}' = \mu\alpha\mathbf{H}', \tag{15}$$

and the boundary condition

$$\mathbf{n}'(\mathbf{r}') \times \mathbf{E}'(\mathbf{r}',t) = 0, \tag{16}$$

$\mathbf{n}'$ being an unit vector normal to the mirror surfaces transformed in accordance with Eq. (9). Equations (12)–(16) are completely equivalent to those for the medium at rest, once the dielectric and magnetic susceptibilities are multiplied by the factor α, and the geometry of the resonator is scaled by a factor $\alpha^{1/2}$ in the directions normal to the flow velocity. Accordingly, the effects of motion can be fully analyzed by referring to an equivalent resonator filled with a medium at rest.

As a simple application, the longitudinal frequencies of a plane-mirror resonator $\nu_m = mc/(2nL)$, with $n = c(\varepsilon\mu)^{1/2}$ and m an integer, are modified into $\nu_m{}' = \nu_m/\alpha^{1/2}$.

For small values of β (in practice v does not exceed sonic velocities), α differs from unity by the quantity $(n^2-1)\beta^2$, so the geometry of the equivalent cavity does not noticeably differ from the original one. Therefore, the motion does not practically affect the mode patterns but for the presence of the factor $T = \exp(2\pi i\nu_m' bz)$, strictly related to the well-known Fizeau drag effect.

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References

1. A. E. Siegman and E. A. Sziklas, Appl. Opt. **14,** 1874 (1975).
2. Yu. A. Ananev, V. P. Trusov, and V. E. Sherstobitov, Sov. J. Quantum Electron. **6,** 928 (1976).
3. A. Sommerfeld, *Electrodynamics* (Academic, New York, 1952).
4. S. Solimeno, Alta Freq. **43,** 671 (1974).